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MATHEMATICS **9709/12**

Paper 1 **October/November 2016**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

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This document consists of **8** printed pages.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

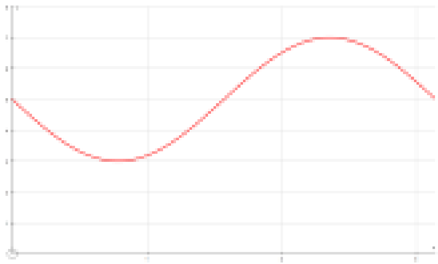
PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	$(y)=8(4x+1)^{\frac{1}{2}}$ ÷ ½ ÷ 4 $(+c)$ Uses $x = 2$ and $y = 5$ $c=-7$	**B1** **B1** **M1** **A1**	[4]	Correct integrand (unsimplified) without ÷4 ÷4. Ignore c. Substitution of correct values into an integrand to find c. $y=4\sqrt{4x+1}-7$
2 (i)	2sin2x = 6cos2x tan2x = k → tan2x = 3 or k =3	 **M1** **A1**	[2]	 Expand and collect as far as tan2x = a constant from sin ÷ cos soi cwo
(ii)	$x =$ $(\tan^{-1}(\textit{their } k)) \div 2$ (71.6° or −108.4°) ÷2 x = 35.8°, −54.2° x = 0.624^c, – 0.946^c x = 0.198π^c, – 0.301π^c	**M1** **A1 A1**√	[3]	Inverse then ÷2. soi. √ on 1st answer +/ − 90° if in given range but no extra solutions in the given range. Both SR A1A0
3 (i)	$2x^2-6x+5>13$ $2x^2-6x-8(>0)$ $(x =)$ −1 and 4. $x>4$, $x<-1$	 **M1** **A1** **A1**	[3]	 Sets to 0 + attempts to solve Both values required Allow all recognisable notation.
(ii)	$2x^2-6x+5=2x+k$ → $2x^2-8x+5-k(=0)$ Use of b^2-4ac → −3 **OR** $\frac{dy}{dx}=4x-6$ $4x-6=2$ $x=2$ $x=2 \rightarrow y=1$ Using *their* (2,1) in $y=2x+k$ or $y=2x^2-6x+5$ $\rightarrow k=-3$	 **M1*** **DM1** **A1** **M1*** **DM1** **A1**	[3] [3]	 Equates and sets to 0. Use of discriminant Sets (their $\frac{dy}{dx}$) = 2 Uses *their* x = 2 and *their* y = 1

4	Term in $x = \frac{nx}{2}$	B1		Could be implied by use of a numerical *n*.
	$(3-2x)(1+\frac{nx}{2}+\ldots) \rightarrow 7 = \frac{3n}{2} - 2$	M1		(Their 2 terms in *x*) = 7
	$\rightarrow n = 6$	A1		
	Term in $x^2 = \frac{n(n-1)}{2}\left(\frac{x}{2}\right)^2$	B1		May be implied by (their *n*) × (their *n-1*) ÷ 8.
	Coefficient of $x^2 = \frac{3n(n-1)}{8} - \frac{2n}{2}$	M1		Considers 2 terms in x^2.
	$= \frac{21}{4}$	A1	[6]	aef
5	$A(a, 0)$ and $B(0, b)$	B1		soi
	$a^2 + b^2 = 100$	M1*		Uses Pythagoras with their *A* & *B*.
	M has coordinates $\left(\frac{a}{2}, \frac{b}{2}\right)$	B1√		√ on their *A* and *B*.
	M lies on $2x + y = 10$ $\rightarrow a + \frac{b}{2} = 10$	M1*		Subs into given line, using their M, to link *a* and *b*.
	Sub $\rightarrow a^2 + (20-2a)^2 = 100$ or $\left(10 - \frac{b}{2}\right)^2 + b^2 = 100$	DM1		Forms quadratic in *a* or in *b*.
	$\rightarrow a = 6, b = 8.$	A1	[6]	cao

6 (i)	$\frac{r}{10} = \sin 0.6$ or $\frac{r}{10} = \cos 0.97$ or $BD = \sqrt{200 - 200\cos 1.2}\,(= 11.3)$ $r = 10 \times 0.5646$, $r = 10 \times \sin 0.6$, $r = 10 \times \cos 0.971$ or $r = ½\,BD$ $\rightarrow r = 5.646$ AG	**M1** **A1**	[2]	Or other valid alternative.
(ii)	Major arc = $10(\theta)$ (= 50.832) $\theta = 2\pi - 1.2$ (= 5.083) or C = $2\pi \times 10$, Minor arc = 1.2×10 Semicircle = 5.646π (= 17.737) Major arc + semicircle = 68.6	**M1** **B1** **A1**	[3]	$\theta = 2\pi - 1.2$ or $\pi - 1.2$ Implied by 5.1
(iii)	Area of major sector $= ½10^2(\theta)$ (= 254.159) Area of triangle *OBD* $= ½10^2\sin 1.2$ (= 46.602) Area = semicircle + sector + triangle (= 50.1 + 254.2 + 46.6) = 351	 **M1** **M1** **A1**	[3]	 $\theta = 2\pi - 1.2$ or $\pi - 1.2$ Use of $½ab\sin C$ or other complete method
7 (i)	$\frac{dy}{dx} = \frac{-3}{(2x-1)^2} \times 2$	**B1** **B1**	[2]	B1for a single correct term (unsimplified) without ×2.
(ii)	e.g. Solve for $\frac{dy}{dx} = 0$ is impossible.	**B1**√	[1]	Satisfactory explanation.
(iii)	If $x = 2$, $\frac{dy}{dx} = \frac{-6}{9}$ and $y = 3$ Perpendicular has $m = \frac{9}{6}$ $\rightarrow y - 3 = \frac{3}{2}(x - 2)$ Shows when x=0 then y=0 AG	**M1*** **M1*** **DM1** **A1**	[4]	Attempt at both needed. Use of $m_1m_2 = -1$ numerically. Line equation using (2, their 3) and their m.
(iv)	$\frac{dx}{dt} = -0.06$ $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt} \rightarrow -\frac{2}{3} \times -0.06 = 0.04$	 **M1 A1**	[2]	

8 (a) (i)	$200+(15-1)(+/-5)$ $= 130$	**M1** **A1**	[2]	Use of nth term with $a = 200$, $n = 14$ or 15and $d = +/- 5$.
(ii)	$\frac{n}{2}\left[400+(n-1)(+/-5)\right] = (3050)$ $\rightarrow\ 5n^2 - 405n + 6100\ (= 0)$ $\rightarrow\ 20$	**M1** **A1** **A1**	[3]	Use of S_n a=200 and $d = +/- 5$.
(b) (i)	$ar^2, ar^5 \rightarrow r = ½$ $\frac{63}{2} = \frac{a\left(1-½^6\right)}{½} \rightarrow a = 16$	**M1 A1** **M1 A1**	[4]	Both terms correct. Use of $S_n = 31.5$ with a numeric r.
(ii)	Sum to infinity = $\frac{16}{½} = 32$	**B1**√	[1]	√ for their a and r with $\lvert r \rvert < 1$.
9 (i)	$-4 - 6 - 6 = -16$ $\sqrt{x_1^2 + y_1^2 + z_1^2}$ or $\sqrt{x_2^2 + y_2^2 + z_2^2}$ $3 \times 7 \times \cos\theta = -16$ $\rightarrow\ \theta = 139.6°$ or 2.44^c or 0.776π	**M1** **M1** **M1** **A1**	[4]	Use of $x_1x_2 + y_1y_2 + z_1z_2$ on their $\overrightarrow{OA}$ & $\overrightarrow{OB}$ Modulus once on either their $\overrightarrow{OA}$ or $\overrightarrow{OB}$ All linked using their $\overrightarrow{OA}$ & $\overrightarrow{OB}$
(ii)	$\overrightarrow{AC} = \text{c} - \text{a} = \begin{pmatrix} 0 \\ 8 \\ 6 \end{pmatrix}$ Magnitude = 10 Scaling $\rightarrow \frac{15}{\textit{their}10} \times \begin{pmatrix} 0 \\ 8 \\ 6 \end{pmatrix} = \begin{pmatrix} 0 \\ 12 \\ 9 \end{pmatrix}$	**B1** **M1** **A1**	[3]	For 15 × *their* unit vector.
(iii)	$\begin{pmatrix} 2+2p \\ 6-2p \\ 5-p \end{pmatrix}$ $\rightarrow -2(2 + 2p) + 3(6 - 2p) + 6(5 - p) = 0$ $\rightarrow\ p = 2¾$	**B1** **M1** **A1**	[3]	Single vector soi by scalar product. Dot product of ($p\,\overrightarrow{OA} + \overrightarrow{OC}$) and $\overrightarrow{OB} = 0$.

10 (i)	$3 \leqslant f(x) \leqslant 7$	B1 B1	 [2]	Identifying both 3 and 7 or correctly stating one inequality. Completely correct statement. **NB** $3 \leqslant x \leqslant 7$ scores B1B0
(ii)		B1* DB1	 [2]	One complete oscillation of a sinusoidal curve between 0 and π. All correct, initially going downwards, all above $f(x)=0$
(iii)	$5-2\sin 2x = 6 \;\rightarrow\; \sin 2x = -\frac{1}{2}$ $\rightarrow\; 2x = \frac{7\pi}{6}$ or $\frac{11\pi}{6}$ $\rightarrow\; x = \frac{7\pi}{12}$ or $\frac{11\pi}{12}$ 0.583π or 0.917π $\frac{\pi+0.524}{2}$ or $\frac{2\pi-0.524}{2}$ 1.83^c or 2.88^c	M1 A1 A1√	 [3]	Make $\sin 2x$ the subject. √ for $\frac{3\pi}{2}$ − 1st answer from $\sin 2x = -\frac{1}{2}$ only, if in given range SR A1A0 for both.
(iv)	$k = \frac{\pi}{4}$	B1	[1]	
(v)	$2\sin 2x = 5 - y \;\rightarrow\; \sin 2x = \frac{1}{2}(5 - y)$ $(g^{-1}(x)) = \frac{1}{2}\sin^{-1}\frac{(5-x)}{2}$	M1 M1 A1	 [3]	Makes $\pm\sin 2x$ the subject soi by final answer. Correct order of operations including correctly dealing with "–". Must be a function of x