CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Subsidiary and Advanced Level

MARK SCHEME for the March 2016 series

9709 MATHEMATICS

9709/32 Paper 3 (Pure Mathematics), maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1		Use law of the logarithm of a power, quotient or product	M1	
		Remove logarithms and obtain a correct equation in x, e.g. $x^2 + 4 = 4x^2$	A1	
		Obtain final answer $x = 2/\sqrt{3}$, or exact equivalent	A1	[3]
2		Use tan($A \pm B$) formula and obtain an equation in tan θ	M1	
		Using $\tan 45° = 1$, obtain a horizontal equation in tan θ in any correct form	A1	
		Reduce the equation to $7\tan^2\theta - 2\tan\theta - 1 = 0$, or equivalent	A1	
		Solve a 3-term quadratic for tan θ	M1	
		Obtain a correct answer, e.g. $\theta = 28.7°$	A1	
		Obtain a second answer, e.g. $\theta = 165.4°$, and no others	A1	[6]
		[Ignore answers outside the given interval. Treat answers in radians as a misread (0.500, 2.89).]		
3	(i)	Consider sign of $x^5 - 3x^3 + x^2 - 4$ at $x = 1$ and $x = 2$, or equivalent	M1	
		Complete the argument correctly with correct calculated values	A1	[2]
	(ii)	Rearrange the given quintic equation in the given form, or work *vice versa*	B1	[1]
	(iii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 1.78	A1	
		Show sufficient iterations to 4 d.p. to justify 1.78 to 2 d.p., or show there is a sign change in the interval (1.775, 1.785)	A1	[3]
4	(i)	Substitute $x = -\frac{1}{2}$ and equate to zero, or divide by $(2x+1)$ and equate constant remainder to zero	M1	
		Obtain $a = 3$	A1	[2]
	(ii) (a)	Commence division by $(2x + 1)$ reaching a partial quotient of $2x^2 + kx$	M1	
		Obtain factorisation $(2x+1)(2x^2 - x + 2)$	A1	[2]
		[The M1 is earned if inspection reaches an unknown factor $2x^2 + Bx + C$ and an equation in B and/or C, or an unknown factor $Ax^2 + Bx + 2$ and an equation in A and/or B.]		
	(b)	State or imply critical value $x = -\frac{1}{2}$	B1	
		Show that $2x^2 - x + 2$ is always positive, or that the gradient of $4x^3 + 3x + 2$ is always positive	B1*	
		Justify final answer $x > -\frac{1}{2}$	B1(dep*)	[3]
5	(i)	State or imply $\mathrm{d}x = \sqrt{3}\sec^2\theta\ \mathrm{d}\theta$	B1	
		Substitute for x and dx throughout	M1	
		Obtain the given answer correctly	A1	[3]

	(ii)	Replace integrand by $\frac{1}{2}\cos 2\theta + \frac{1}{2}$	**B1**	
		Obtain integral $\frac{1}{4}\sin 2\theta + \frac{1}{2}\theta$	**B1**√	
		Substitute limits correctly in an integral of the form $c\sin 2\theta + b\theta$, where $cb \neq 0$	**M1**	
		Obtain answer $\frac{1}{12}\sqrt{3}\pi + \frac{3}{8}$, or exact equivalent	**A1**	[4]
		[The f.t. is on integrands of the form $a\cos 2\theta + b$, where $ab \neq 0$.]		
6	**(i)**	*EITHER*: State correct derivative of $\sin y$ with respect to x	**B1**	
		Use product rule to differentiate the LHS	**M1**	
		Obtain correct derivative of the LHS	**A1**	
		Obtain a complete and correct derived equation in any form	**A1**	
		Obtain a correct expression for $\frac{dy}{dx}$ in any form	**A1**	
		OR: State correct derivative of $\sin y$ with respect to x	**B1**	
		Rearrange the given equation as $\sin y = x / (\ln x + 2)$ and attempt to differentiate both sides	**B1**	
		Use quotient or product rule to differentiate the RHS	**M1**	
		Obtain correct derivative of the RHS	**A1**	
		Obtain a correct expression for $\frac{dy}{dx}$ in any form	**A1**	[5]
	(ii)	Equate $\frac{dy}{dx}$ to zero and obtain a horizontal equation in $\ln x$ or $\sin y$	**M1**	
		Solve for $\ln x$	**M1**	
		Obtain final answer $x = 1/\mathrm{e}$, or exact equivalent	**A1**	[3]
7	**(i)**	Separate variables and attempt integration of one side	**M1**	
		Obtain term $-\mathrm{e}^{-y}$	**A1**	
		Integrate $x\mathrm{e}^x$ by parts reaching $x\mathrm{e}^x \pm \int \mathrm{e}^x \, dx$	**M1**	
		Obtain integral $x\mathrm{e}^x - \mathrm{e}^x$	**A1**	
		Evaluate a constant, or use limits $x = 0, y = 0$	**M1**	
		Obtain correct solution in any form	**A1**	
		Obtain final answer $y = -\ln(\mathrm{e}^x(1-x))$, or equivalent	**A1**	[7]
	(ii)	Justify the given statement	**B1**	[1]

8	**(i)**	*EITHER*:	Substitute for **r** in the given equation of p and expand scalar product	**M1**	
			Obtain equation in λ in any correct form	**A1**	
			Verify this is not satisfied for any value of λ	**A1**	
		*OR*1:	Substitute coordinates of a general point of l in the Cartesian equation of plane p	**M1**	
			Obtain equation in λ in any correct form	**A1**	
			Verify this is not satisfied for any value of λ	**A1**	
		*OR*2:	Expand scalar product of the normal to p and the direction vector of l	**M1**	
			Verify scalar product is zero	**A1**	
			Verify that one point of l does not lie in the plane	**A1**	
		*OR*3:	Use correct method to find the perpendicular distance of a general point of l from p	**M1**	
			Obtain a correct unsimplified expression in terms of λ	**A1**	
			Show that the perpendicular distance is $5/\sqrt{6}$, or equivalent, for all λ	**A1**	
		*OR*4:	Use correct method to find the perpendicular distance of a particular point of l from p	**M1**	
			Show that the perpendicular distance is $5/\sqrt{6}$, or equivalent	**A1**	
			Show that the perpendicular distance of a second point is also $5/\sqrt{6}$, or equivalent	**A1**	[3]
	(ii)	*EITHER*:	Calling the unknown direction vector $a\mathbf{i}+b\mathbf{j}+c\mathbf{k}$ state equation $2a+b+3c=0$	**B1**	
			State equation $2a-b-c=0$	**B1**	
			Solve for one ratio, e.g. $a : b$	**M1**	
			Obtain ratio $a : b : c = 1 : 4 : -2$, or equivalent	**A1**	
		OR:	Attempt to calculate the vector product of the direction vector of l and the normal vector of the plane p, e.g. $(2\mathbf{i}+\mathbf{j}+3\mathbf{k})\times(2\mathbf{i}-\mathbf{j}-\mathbf{k})$	**M2**	
			Obtain two correct components of the product	**A1**	
			Obtain answer $2\mathbf{i}+8\mathbf{j}-4\mathbf{k}$, or equivalent	**A1**	
			Form line equation with relevant vectors	**M1**	
			Obtain answer $\mathbf{r}=5\mathbf{i}+3\mathbf{j}+\mathbf{k}+\mu(\mathbf{i}+4\mathbf{j}-2\mathbf{k})$, or equivalent	**A1**$\sqrt{}$	[6]
9	**(i)**		State or obtain $A = 3$	**B1**	
			Use a relevant method to find a constant	**M1**	
			Obtain one of $B = -4$, $C = 4$ and $D = 0$	**A1**	
			Obtain a second value	**A1**	
			Obtain the third value	**A1**	[5]
	(ii)		Integrate and obtain $3x-4\ln x$	**B1**$\sqrt{}$	
			Integrate and obtain term of the form $k\ln(x^2+2)$	**M1**	
			Obtain term $2\ln(x^2+2)$	**A1**$\sqrt{}$	
			Substitute limits in an integral of the form $ax+b\ln x+c\ln(x^2+2)$, where $abc\neq 0$	**M1**	
			Obtain given answer $3-\ln 4$ after full and correct working	**A1**	[5]
10	**(a)**		Substitute and obtain a correct equation in x and y	**B1**	
			Use $\mathrm{i}^2=-1$ and equate real and imaginary parts	**M1**	
			Obtain two correct equations, e.g. $x + 2y + 1 = 0$ and $y + 2x = 0$	**A1**	
			Solve for x or for y	**M1**	
			Obtain answer $z=\frac{1}{3}-\frac{2}{3}\mathrm{i}$	**A1**	[5]

(b)	**(i)**	Show a circle with centre $-1+3\,\mathrm{i}$	**B1**	
		Show a circle with radius 1	**B1**	
		Show the line Im $z = 3$	**B1**	
		Shade the correct region	**B1**	[4]
	(ii)	Carry out a complete method to calculate the relevant angle	**M1**	
		Obtain answer 0.588 radians (accept 33.7°)	**A1**	[2]