CONTENTS

FOREWORD		
MATHEMATICS	2	
GCE Advanced Level	2	
Paper 9709/01 Paper 1	2	
Paper 9709/02 Paper 2		
Paper 9709/03 Paper 3	7	
Paper 9709/04 Paper 4	9	
Paper 9709/05 Paper 5		
Paper 9709/06 Paper 6		
Paper 9709/07 Paper 7		

FOREWORD

This booklet contains reports written by Examiners on the work of candidates in certain papers. Its contents are primarily for the information of the subject teachers concerned.

MATHEMATICS

GCE Advanced Level

Paper 9709/01

Paper 1

General comments

Candidates generally found the Paper to their liking. It gave all candidates the opportunity to demonstrate what they had been taught and there were parts of questions that allowed the more able candidates to show their potential. There were however a few really poor scripts, and it was clear that these candidates should not have been entered for the examination. Standards of numeracy and algebraic manipulation were good and the majority of scripts were well-presented and easy to mark. **Questions 2**, **6** and **9** presented candidates with most problems, implying that it is the "trigonometry" sections of the Syllabus in which candidates show least confidence. Candidates should be aware of the instruction that requires non-exact answers to be expressed to three significant figures. It is not acceptable, for example, to express the sum of a series, 542.5 in **Question 4**, as 543.

Comments on specific questions

Question 1

Failure to cope with the "-" sign in $(2x - \frac{1}{x})$ was common, but the majority of candidates realised the need to find the 4th term of the expansion and correctly evaluated ${}_5C_3 \times 2^2 \times (-1)^3$. A significant number of candidates however took the term in $\left(\frac{1}{x}\right)$ to be the 2nd term - that is, ${}_5C_1 \times (2x)^4 \times \left(-\frac{1}{x}\right)$.

Of the minority preferring to remove the "2x" from the bracket, $(2x)^5$ was often replaced by $2x^5$.

Answer: - 40.

Question 2

This was poorly answered, even by many of the very good candidates who all too often started by attempting to express sin 3*x* or cos 3*x* in terms of sin *x* or cos *x*. Even when candidates recognised the need to use "tan = sin \div cos", there were many scripts in which the "3" was cancelled to leave tan *x* instead of tan 3*x*. Others replaced tan 3*x* by 3tan *x* at a later stage. Of the minority who obtained tan 3*x* = -2, many offered only a negative solution (-21.1°) and only a few realised that there were three solutions in the range 0° to 180°.

Answers: 38.9°, 98.9°, 158.9°.

Question 3

This was a straightforward question that posed only a few problems. Candidates showed confidence in their ability to both differentiate and integrate negative powers of x. Omission of the constant of integration was the only common error.

Answers: (a) $4 - \frac{12}{x^3}$; (b) $2x^2 - \frac{6}{x} + c$.

There were a large number of completely correct answers. Most candidates correctly evaluated a = 1.5 and recognised the need to find the total number of terms in the progression. Although the majority used "a + (n - 1)d" a second time, there were others who incorrectly used "a + nd" and several who used the longer method of equating $\frac{n}{2}(2a + (n - 1)d)$ with $\frac{n}{2}(a + l)$. Candidates generally preferred to use the first of these equations to find the sum of all the terms in the progression, though about a quarter of all solutions used the simpler form of $\frac{n}{2}(a + l)$. Several candidates lost the final accuracy mark by offering the exact answer of 542.5 as 543.

Answer: 542.5.

Question 5

This was very well answered and usually a source of high marks. Virtually all candidates realised the need to form two linear simultaneous equations, the solution of which was nearly always correct. In part (ii), apart from a small minority who took ff(x) as $[f(x)]^2$, candidates confidently coped with a(ax+b)+b either algebraically or numerically. Omission of the "+b" in the expression for ff(x) was rare.

Answers: (i) a = 2, b = -3; (ii) 2.25.

Question 6

It was rare to obtain a completely correct solution. Sketches of $y = 3\sin x$ were disappointing, for apart from a significant number who either omitted the question or scored zero, there were far too many offerings in which: the curve was shown as a series of straight lines; the curve failed to pass through the origin; there was no evidence of $-3 \le y \le 3$, either marked on the diagram or implied in the working. In part (ii), only a

small percentage of candidates realised the need to substitute the point $(\frac{1}{2}\pi, 3)$ into the equation y = kx. In

part (iii) only a few solutions were seen in which the candidate realised that the other point was the minimum point of the curve. On the positive side, candidates coped well with the use of radians.

Answers: (i) Sketch; (ii)
$$k = \frac{6}{\pi}$$
; (iii) $(-\frac{1}{2}\pi, -3)$.

Question 7

This proved to be an easy question that presented the majority of candidates with full marks. Evaluating the gradient of L_1 as 2 or $\pm \frac{1}{2}$ was seen, as was the use of the perpendicular gradient as $\frac{1}{m}$. Surprisingly, most errors came in attempting to express $y - 4 = \frac{1}{2}(x - 7)$ in the form y = mx + c prior to solving simultaneous equations. Only a few solutions were seen in which the incorrect formula was used for finding the distance

equations. Only a few solutions were seen in which the incorrect formula was used for finding the distance between two points.

Answers: (i) 2y = x + 1; (ii) 4.47 or $\sqrt{20}$.

Question 8

A large number of candidates incorrectly expressed **BA** as either **b** – **a** or as **b** + **a**. Of greater concern however was the very large proportion who took **BA** as **a**.**b**. These same candidates repeated this for **BC** and tried to merge the two scalar products. With all other candidates however, use of the scalar product was, as last year, very good and most candidates realised that to test for perpendicularity, there was no need to evaluate the moduli of the two vectors concerned. In part (ii), most candidates struggled to show that the two vectors were parallel. Some candidates had been taught to use the vector product and were generally successful; others used scalar product to obtain an angle of 0°. Other candidates had difficulty in expressing

why it was that $\begin{pmatrix} -2\\4\\2 \end{pmatrix}$ and $\begin{pmatrix} -5\\10\\5 \end{pmatrix}$ were parallel.

In both parts, marks were lost through a lack of explanation. Many candidates thought it sufficient in part (i) to show that the scalar product was zero, without ever mentioning that this proved that the angle was 90° . Similarly in part (ii), candidates showed the vectors to be $2\mathbf{k}$ and $5\mathbf{k}$ without ever saying what this in fact

proved. Also in part (ii), many candidates failed to express the answer as a ratio, leaving answers as $\frac{2}{5}$, or

0.4 or even 40%, or even the wrong way round as 5:2.

Answers: (i) Proof; (ii) Proof, 2:5.

Question 9

This was poorly answered and many candidates showed a serious misunderstanding of the use of radian

measure. In part (i), a significant number of candidates used the formula $\frac{1}{2}r^2\theta$ with $\theta = 179^\circ$. At least a half of all attempts failed to cope with the larger sector and left the answer as 32 cm². There were very few correct answers to part (ii) with many candidates interpreting "perimeter" as "arc length", or using the angles of the two sectors as θ and 1, or θ and 179, instead of θ and $\pi - \theta$. Only about a quarter of all attempts at part (iii) were correct. Many candidates failed to recognise the need to use trigonometry. It was apparent that a large number of candidates were unaware of the exact values of sin 60° or cos 30° since a considerable number failed to realise the significance of "exact" in the wording of the question. Decimal answers checked against (24 + 8 $\sqrt{3}$) were not acceptable for the final answer mark.

Answers: (i) 68.5 cm²; (ii) 0.381; (iii) Proof.

Question 10

This proved to be a source of high marks for most candidates. The differentiation and integration of $\sqrt{5x+4}$ was generally well done, though about a third of all attempts failed to include the " \times 5" in part (i) and

the "÷5" in part (iii). A small number of weaker candidates took $\sqrt{5x+4}$ as $(5x+4)^{-\frac{1}{2}}$ or as $(5x+4)^{-1}$ or even as $\sqrt{5x}+2$. Most candidates successfully recognised that part (ii) required the link between connected rates of change. In part (iii), at least a third of all attempts assumed that the lower limit of "0" could be ignored. Despite these errors, there were a large number of completely correct solutions.

Answers: (i) $\frac{5}{6}$; (ii) 0.025; (iii) 2.53 or $\frac{38}{15}$.

Question 11

Solutions to this question, particularly to part (v) showed considerable improvement from previous Papers. In part (i) the majority of candidates realised that $b = \pm 4$ and worked accordingly. Many attempts in part (i) expressed " $8x - x^2$ " as " $x^2 - 8x$ ", proceeded to " $(x - 4)^2 - 16$ " and then wrote the answer as " $16 - (x - 4)^2$ " with no explanation. Candidates should be made aware of the need to give full explanations of their working. Part (ii) was generally correct with the majority of candidates preferring the safety of calculus rather than relying on their answer to part (i). In part (iii) most candidates realised the need to bring the "-20" to the other side of the equation and to solve equal to zero. Only about a half of all solutions obtained the correct range, even when the end-points -2 and 10 were obtained. Many solutions were seen in which the set of values for x was stated as either " $x \le -2$ and $x \le 10$ " or " $x \ge -2$ and $x \ge 10$ ". Part (iv) was poorly answered. Most candidates seemed to realise that the domain of g^{-1} was the same as the range of g, but failed to realise that this could be stated from their answer to part (i). They were more successful with their answer to the range of g^{-1} since the domain of g was given. Answers to part (v) were pleasing with nearly two-thirds of all attempts realising that the answer to part (i) was needed to enable the inverse of g to be obtained. Simple algebraic errors were responsible for the loss of the final accuracy mark.

Answers: (i) $16 - (x - 4)^2$, a = 16, b = -4; (ii) (4, 16); (iii) $-2 \le x \le 10$; (iv) Domain $x \le 16$, range $g^{-1}(x) \ge 4$; (v) $g^{-1}(x) = 4 + \sqrt{16 - x}$. Paper 9709/02

Paper 2

General comments

A wide range of ability was displayed in candidates' responses to the Paper. Few marks of 40 or more were scored, though the Examiners were most impressed by the level of expertise displayed by candidates in these scripts. However, there were a substantial number of scripts in which candidates proved unequal to the challenge of more than one or two of the seven questions; such candidates often recorded marks in single figures.

Candidates seemed to have sufficient time to attempt all the questions, but often struggled to cope effectively with **Questions 5**, 6 and 7 (iii), in particular. Conversely, **Question 1** and 4 (ii) produced an excellent response from the overwhelming majority of candidates.

There are two areas where the Examiners recommend that candidates ought to especially concentrate on when preparing for future examinations. Firstly, candidates should familiarise themselves with the formulae sheets (list MF9) which form part of the examination provision; many examples of poor differentiation and integration techniques, and results in particular, could have been avoided had candidates been familiar with the list. Secondly, candidates should work through carefully previous 9709/02 Papers, and thus become aware of the range of topics tested and the difficulty levels of questions.

Examiners are concerned by the high proportion of candidates who currently appear poorly equipped to make any meaningful headway with most (or all) of the questions set; this was especially true of **Questions 4 (i), 5 (i)** and **(ii), 6 (ii)** and **7 (iii)**, which were rarely successfully attempted and yet were similar to problems set in previous 9709/02 Papers.

Comment on specific questions

Question 1

This was a popular question with candidates, and proved most successful with those who squared each side of the initial inequality to yield a linear equation or inequality for *x*. A very high proportion of solutions were, however, marred by the erroneous statement that -10x > -15 implies x > 1.5 (instead of the correct x < 1.5).

Those candidates who adopted an 'ad hoc', less systematic, approach based on $\pm (x - 4) > \pm (x + 1)$ were rarely successful, often arriving at results such as 4 > 1. Very few graphical solutions were seen; these were invariably very successful.

Answer: x < 1.5.

Question 2

- (i) Expanding the right hand side and comparing coefficients posed no real problem for candidates, but the majority were convinced that $a^2 = 9$ implied a = +3. Strictly, $a = \pm 3$, and the correct choice of sign comes from the further result 2a = 6. Few investigated beyond $a^2 = 9$ to obtain a second form for *a*.
- (ii) Having correctly found that a = 3, albeit usually by accident, most candidates could then cope perfectly well with the two resulting quadratic equations $x^2 + 3x + 1 = 0$ and $x^2 3x 1 = 0$; however, a small minority returned to the original equation $x^4 9x^2 6x 1 = 0$ and tried, always unsuccessfully, to solve it, either by (wrongly) treating it as a quadratic equation in x^2 or by setting $x = \pm 1, \pm 2, \pm 3,...$ and seeking integer roots.

Answers: (i)
$$a = 3$$
; (ii) $x = -\frac{3 \pm \sqrt{5}}{2}$, $\frac{3 \pm \sqrt{13}}{2}$.

- (i) Few successful attempts were made at integrating the function e^{2x} ; many candidates *differentiated* e^{2x} .
- (ii) Follow through marks were usually earned, but many candidates erroneously used $ln(a + b) \equiv ln(a) + ln(b)$.

Answers: (i)
$$\frac{1}{2}$$
 (e^{2p} - 1); (ii) $p = \frac{1}{2} \ln 11 \approx 1.20$.

Question 4

- (i) There were many good solutions; unsuccessful attempts were caused by an error in signs in the denominator of the left hand and/or right hand side expansions or, more seriously, candidates used tan(A + B) = tan A + tan B.
- (ii) Those who erred as above in part (i) failed to score, but an overwhelming majority of candidates picked up full marks.

Answers: (ii) 18.4°, 71.6°.

Question 5

- (i) Few examples of correct sketches of both graphs were seen, though many solutions featured *one* good graph. Both the functions $\ln x$ and $(2 x^2)$ have well documented basic shapes and the Examiners were surprised that the graph of the former, in particular, was unfamiliar to so many candidates.
- (ii) Many attempts were not based on the given values 1.0 and 1.4, and featured attempts to do the work of part (iii). All that was required was to compare the values of f(1.0) and f(1.4), where $f(x) = \pm (\ln x 2 + x^2)$, and to comment that f(1.0), f(1.4) have different signs.
- (iii) Although more successfully attempted, it was noticeable that many solutions featured oscillating values 1.31 and 1.32; the key to successful iteration is to work, at early and intermediate stages, to *more than* the number of decimal places required in the final answer. Here, for an answer correct to two decimal places, one should work to four places during successive interactions.

Answer: (iii) 1.31.

Question 6

- (i) Around half of all solutions failed to use the chain rule or used an incorrect format based on that rule.
- (ii) Very few candidates were successful; many used an interval of $h = \frac{180}{8} = 22.5$ instead of

$$\frac{1}{2} \cdot \frac{\pi}{4} = \frac{\pi}{8}$$
, even though the question explicitly referred to angle x being measured in radians.

Also, instead of using two strips, with vertical ordinates measured at the 3 values x = 0, $\frac{\pi}{8}$, $\frac{\pi}{4}$, a substantial proportion of solutions were based on the use of two or four ordinates, or more.

(iii) There were a substantial number of correct deductions, often based on excellent sketches. Other solutions usually featured the right answer but without any reason.

Answers: (i) $y = \frac{-\sec^2 x}{(1 + \tan x)^2} < 0$; (ii) 0.57; (iii) over-estimate.

- (i) Responses were disappointing; many candidates were unable to differentiate $x(\theta)$ or $y(\theta)$ and often $\frac{dy}{dx}$ was set equal to $\frac{dx}{d\theta} \div \frac{dy}{d\theta}$.
- (ii) A majority of solutions failed due to an inability to calculate the basic trigonometrical functions $\sin \theta$, $\cos \theta$ and $\cot \theta$ at $\theta = \frac{\pi}{4}$, though setting up the equation of the tangent was usually done successfully.
- (iii) Very few attempted this part of the question. Those that did failed to note that $\frac{dy}{dx} = 0$, and so

 $\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ at the points in question.

Answers: (ii) $y = x + 3 - \frac{\pi}{2} \approx x + 1.43$; (iii) (π , 3) and (3π , 3).

Paper 9709/03 Paper 3

General comments

There was a considerable variety of standard of work by candidates on this Paper and a corresponding very wide spread of marks from zero to full marks. The Paper appeared to be accessible to candidates who were well prepared and no question seemed to be of undue difficulty. Moreover adequately prepared candidates seemed to have sufficient time to attempt all questions. However there were some very weak, often untidy, scripts from candidates who clearly lacked the preparation necessary for work at the level demanded by this Paper. All questions discriminated to some extent. The questions or parts of questions on which candidates generally scored highly were **Question 2** (integration by parts), **Question 8** (i) (stationary point) and (iii) (iteration), and **Question 9** (i) (vector geometry). Those on which scores were low were **Question 4** (ii) (algebra), **Question 5** (complex numbers), **Question 6** (ii) (series expansion) and **Question 10** (iii) (trigonometrical integral).

The detailed comments that follow inevitably refer to common errors and can lead to a cumulative impression of poor work on a difficult Paper. In fact there were many scripts showing a good and sometimes excellent understanding of all the topics being tested.

Where numerical and other answers are given after the comments on individual questions, it should be understood that alternative forms are often possible and that the form given is not necessarily the sole 'correct answer'.

Comments on specific questions

Question 1

Errors of sign and in the values of $\cos 60^\circ$, $\sin 60^\circ$, $\cos 30^\circ$, $\sin 30^\circ$, prevented some candidates from reaching an equation in $\cos x$ only, but generally this question was well answered.

Answer: (ii) 125.3°.

Most candidates approached the integration by parts correctly. Errors in integrating e^{2x} , and in simplification were quite frequent, but the main source of error was the failure to appreciate the correct meaning in this context of the adjective 'exact'.

Answer:
$$\frac{1}{4}(e^2 + 1)$$
.

Question 3

Very few candidates realised that x = 1 was the only critical value in relation to this inequality. Many answers involved a further value, usually $x = \frac{5}{3}$. Examiners also noted that, while some candidates investigated the

related inequality obtained by squaring both sides of the given inequality, a substantial number dropped the modulus sign and mistakenly squared only one side. Fully correct solutions were rare and often obtained with the assistance of a sketch graph.

Answer: x < 1.

Question 4

Many candidates answered part (i) well, using either the factor theorem with x = 2, or long division.

There were very few completely satisfactory solutions to part (ii). By contrast, there were many fallacious attempts at a proof, e.g. those based on a set of instances of non-negative values of f(x), or the claim that f(2) = 0 and 2 > 0 together implied that f(x) > 0 for all x. The three correct methods seen were arguments based on (a) an exhaustive discussion of the stationary points and graph of y = f(x), (b) a discussion of the nature of the zeros of $x^2 - 4x + 4$ and $x^2 + 2x + 2$ together with a proof that both expressions only took non-negative or positive values, and (c) completing the squares and writing f(x) as $(x - 2)^2 ((x + 1)^2 + 1)$. Most attempts to use method (a) or (b) omitted some essential detail. Method (c) was usually successfully completed.

Answer: (i) 8.

Question 5

Though some candidates found this question quite straightforward, it was generally poorly answered. Given the modulus and argument of the complex number *w*, many candidates were unable to state it in the form

x + iy immediately. Thus they embarked on a lengthy search based on $x^2 + y^2 = 1$ and $x : y = \cos \frac{2}{3}\pi : \sin \frac{2}{3}\pi$,

and quite frequently arrived at a wrong answer. Whatever the outcome, multiplication of 2i by w was often incorrectly done and though most knew how to divide 2i by w, errors, particularly of sign, were common. The plotting of points on an Argand diagram was usually well done, but part (iii) was only accessible to those who had completed part (i) correctly. The most common method here was to show that UA = AB = BU.

Answers: (i)
$$\cos \frac{2}{3}\pi + i\sin \frac{2}{3}\pi, -\sqrt{3} - i, \sqrt{3} - i.$$

Question 6

In part (i), most candidates started out with an appropriate form of partial fractions with three unknown constants. Errors in identifying the numerator of f(x) with that of the combined fractions proved costly. A thorough check of the algebraic work at this stage would have helped. Indeed since full marks in part (ii) were clearly dependent on accurate work earlier, regular checks during part (i) were desirable, for example when setting up simultaneous equations in the unknowns or when evaluating expressions. A fairly common error was to start with an inappropriate form of fractions.

Examiners were disappointed to see so many poor attempts at part (ii). Whereas most candidates could expand $(1 + 2x)^{-1}$ correctly, very few could deal with $(x - 2)^{-1}$ or $(x - 2)^{-2}$ accurately.

Answer: (i) $\frac{1}{2x+1} + \frac{4}{x-2} + \frac{8}{(x-2)^2}$ or $\frac{1}{2x+1} + \frac{4x}{(x-2)^2}$.

There were many sound solutions to part (i). A minority of candidates merely showed that the given differential equation is satisfied when initially x = 5. This did not show that x satisfies the equation at all times. In part (ii) the work was generally quite good with many candidates reaching a solution involving ln(100 - x), or equivalent. The main errors were the omission of a constant of integration and failure to give ln(100 - x) the appropriate sign.

Answers: (ii) $x = 100 - 95\exp(-0.02t)$; (iii) x tends to 100.

Question 8

Part (i) was generally well answered. The work in part (ii) was disappointing. Few candidates realised that the solution involved replacing the iterative formula with an equation in α and showing this to be equivalent

to $3 = \ln \alpha + \frac{2}{\alpha}$, or vice versa. However part (iii) was often correctly done, though some failed to carry out

sufficient iterations to establish convergence to 0.56.

Answers: (i) (2, ln 2 + 1), minimum point; (ii) $\alpha = \frac{2}{3 - \ln \alpha}$; (iii) 0.56.

Question 9

The first part was generally very well answered. Some candidates seemed not to understand what the angle between the two planes really was, for having found 40.4° correctly from the normals they followed it with its complement 49.6°.

Clearly some candidates were unprepared for part (ii) and failed to make progress. However others tackled it by a variety of methods. Some found two points on the line e.g. one with x = 0 and one with y = 0, and obtained the vector equation of the line from them. Others used the normals to the planes to obtain a direction vector for the line and completed the solution by finding a point on the line. Another method was to develop a Cartesian equation for the line by eliminating variables from the plane equations, and deduce an equation in vector form. Examiners remarked that algebraic and numerical slips were frequent here.

Answers: (i) 40.4° ; (ii) $3j + 2k + \lambda(6i - 10j - 7k)$.

Question 10

In part (i) some attempts broke down because of trivial slips in manipulation. The majority succeeded and solutions varied in length from five lines to two pages. Part (ii) was fairly well answered though some solutions failed to contain sufficient working to justify the given answer. Examiners felt that part (iii) was poorly done. The structure of the question led to the integration of $\cot x - \cot 2x$, yet many of those who had integrated $\cot x$ correctly in part (ii) could not produce a correct integral of $\cot 2x$ here. Most attempts at integrating $\csc 2x$ directly were very poor indeed, though occasionally a correct integral was obtained.

Answer: (iii) $\frac{1}{4} \ln 3$.

Paper 9709/04 Paper 4

General comments

Many candidates were well prepared for this examination and some scored very high marks. However a substantial number of candidates were ill prepared for the challenge, and scored very low marks.

The incline of difficulty within the Question Paper is reflected in candidates' work in **Question 1** and **Question 7**. Almost all candidates scored full marks in **Question 1** and almost all candidates found some difficulty with **Question 7**. Candidates generally worked through the questions in order; this is an appropriate strategy for Question Papers of this type.

Candidates should be aware of the rubric requirement that answers must be correct to three significant figures, or one decimal place in the case of angles in degrees. Many candidates gave answers to two significant figures in Question 6 (i) and Question 7 (i), and insufficiently accurate answers arising from premature approximation were often seen in Question 6 (ii) and Question 7 (iv). Although procedures are in place to prevent an unreasonable loss of marks arising from repeatedly giving insufficiently accurate but otherwise correct answers, candidates can lose marks for 2 significant figure answers and for errors arising as a result of premature approximation.

Answers were given at several points in the Question Paper, which are not reconcilable with common sense considerations. The case most obvious to Examiners of this feature was in Question 7 (i), in which very many candidates gave an answer for the speed of P at B greater than or equal to the speed of P at A $(8 \text{ ms}^{-1}).$

Comments on specific questions

Question 1

This was found to be a straightforward starter question with most candidates scoring all four marks. The most common mistake was to use v = 0.8, instead of 0.4, in applying P = Tv.

Answers: (i) 8000 N; (ii) 3200 W.

Question 2

Part (i) of this question was poorly attempted; very many candidates did not seem to understand what was required.

In some cases either one particular force was ignored in answering both parts (a) and (b), or one force was omitted in answering part (a) and a different force was omitted in answering part (b). This suggests that many candidates believe that 'resultant' means the resultant of just two forces.

Some candidates failed to distinguish between the component of the resultant, and the components of the three individual forces. In almost all such cases relevant minus signs were omitted.

Most candidates used a correct method for finding the magnitude of the resultant in part (ii), although some candidates wrote $R = \sqrt{10^2 + 10^2 + 6^2}$. Where candidates used trigonometrical methods firstly to combine two of the forces, and then to use the result of this in combination with the third force, inaccuracies often occurred.

Answers: (i)(a) 14.3 N, (b) 5.20 N; (ii) 15.2 N.

Question 3

Although intended as a straightforward demand, many candidates either omitted part (i) or identified an incorrect region. Some candidates thought they needed to sketch a (t, x) graph.

An incorrect region was not however a barrier to scoring full marks in part (ii); this part was very well attempted with almost all candidates obtaining the correct answer.

Part (iii) was much less well attempted. Although it is clear that P is moving more quickly than Q throughout the period T < t < 9, so that the gap between the two continues to widen, the answer 16 m was frequently given. So too was 9 m, from 25 - 16.

The main source of error was in using $s = \frac{1}{2}(u + v)t$ once for the whole distance travelled by *P* during the

first 9 s. Thus the incorrect answer 20 m, from 45 – 25, was common.

Other incorrect answers arose from the difference in area of two triangles, and include 9 m (25 - 16), 65 m (81 – 16) and 56 m (81 – 25).

Answers: (ii) 4; (iii) 40 m.

Part (i) of this question was well answered by most candidates.

However some candidates integrated where they should have differentiated, and some used $v = \frac{s}{t}$.

Part (ii) was less well attempted; many candidates obtained a(t) = 0.2t following $v(t) = t + 0.1t^2$. Using a(10) instead of a(0) for the initial acceleration was very common.

Some candidates set up the equation for *t* as 1 + 0.2t = 2(1 + 0.2t).

Answers: (i) 20 ms⁻¹; (ii) 5.

Question 5

Part (i) of this question was poorly attempted, perhaps not surprisingly given that very few candidates made sketches showing the forces acting on *A* and *B*.

It was expected that candidates would consider the equilibrium of each of the particles, but in many cases it was far from clear that this was the intention. In some cases it appeared that candidates were considering the equilibrium of each of the strings. Considering the equilibrium of S_1 is not a useful move, although considering the equilibrium of S_2 does lead directly to T_1 .

Many candidates introduced an acceleration in part (i) and used Newton's second law. Common incorrect answers included $T_1 = T_2 = 2$, and $T_1 = 2$, $T_2 = 4$.

Part (ii) was a little better attempted than part (i), although many candidates omitted the weight or the resistance or the tension in applying Newton's second law to each of the particles.

The absence of *T* from the equations was particularly prevalent. Thus answers for the acceleration of 8 ms⁻² for *A* and 9 ms⁻² for *B* were common, notwithstanding the impossibility of this with the string *S*₂ unbroken.

Many candidates who included the tension in their equations brought forward numerical values from part (i), thus producing two values for the acceleration, one from each of two simple equations in *a*.

Answers: (i) 4 N in S_1 , 2N in S_2 ; (ii) 8.5 ms⁻², 0.1 N.

Question 6

Part (i) was very well attempted, most candidates obtaining the correct answer. In a few cases candidates obtained 0.0375 by multiplying the mass 0.15 by the given answer in part (ii), although clearly no marks could be given for this.

In part (ii) many candidates obtained a = 0.25 from 0.0375 = 0.15a. This answer scored only one of the available marks, unless it was supported by some indication that *a* represents 'deceleration' here, or that the direction of the acceleration *a* is opposite to the direction of motion.

Notwithstanding the given result in part (ii), more candidates used a = +0.25 or a = +1.375 or a = -1.375 in the subsequent parts of the question, than used a = -0.25.

In part (iii) the most common wrong answers were:

24 m (from 5.5 x 4 +
$$\frac{1}{2}$$
 0.25 x 4²) and 11 m (from $\frac{1}{2}$ (5.5 + 0) x 4 or from 0 = 5.5 + 4a and 5.5 x 4 + $\frac{1}{2}$ a4²).

Some candidates used methods that involved calculating the speed of arrival of the block at the boundary board, including some who obtained this speed as 6.5 ms⁻¹, despite the obvious slowing down of the block.

Candidates who used an incorrect positive value for *a* in part (iii) usually continued with the same value in parts (iv) and (v). Thus it was common for candidates to obtain answers greater than 3.5 ms^{-1} for the answer in part (iv), contrary to common sense.

Some candidates used t = 4 in part (iv), again contrary to common sense. Thus 2.5 ms⁻¹ was a common wrong answer. It ought to be clear to candidates that, because the block is slowing down, it will take longer to return to *A* than it did to reach the boundary board.

Most candidates used a correct method in part (iv), although a few implicitly assumed that the speed of rebound of the block was the same as the speed of arrival at the boundary board, and calculated the total distance as 60.5 m, from $0^2 - 5.5^2 = 2(-0.25)s$.

Answers: (i) 0.0375 N; (iii) 20 m; (iv) 1.5 ms⁻¹; (v) 44.5 m.

Question 7

This question proved difficult for candidates and it was common for one or more parts to be omitted.

In part (i) very few candidates approached the problem through energy considerations. Those who did usually obtained the PE gain correctly, but this was often followed by one of two common errors linked with the kinetic energy. In the first of these the candidate simply equated the PE gain with $\frac{1}{2}mv^2$, taking no account of the initial speed and obtaining *v* as 6.58. In the second the candidate equated the PE gain with $+\frac{1}{2}m(v^2 - 8^2)$ instead of minus this quantity, obtaining *v* as 10.4.

Among the candidates who considered the acceleration of the particle and then used $v^2 = 8^2 + 2a(2.5)$, most had a = 8.66 leading to v = 10.4, rather than a = -8.66. If candidates had questioned the validity of their answer they would have realised that the speed at *B* must be less than that at *A*.

Unfortunately a very large number of candidates produced work for part (i) which bore no relationship with the question set. Answers included finding the speed of a particle projected vertically, with initial speed 8 ms⁻¹, when at a height of 2.5sin60° above the point of projection ($v^2 = 8^2 + 2(-g)(2.5sin60^\circ)$).

Another case was finding the speed of a particle projected at 60° to the horizontal with speed 8 ms⁻¹, when at a height of 2.5sin60° above the point of projection.

In none of the cases seen was any attempt made to suggest that the scenarios considered lead to the same answer as that actually required. Candidates cannot expect to score marks for answers to a question in which they change the scenario, unless they produce clear arguments for equivalence in the sense that the revised problem inevitably leads to the same answer. This is almost certain to be more difficult for candidates than to answer the question as set, and this is what is strongly recommended by Examiners. The difficulty here is highlighted by considering the differences in the components of the velocities at *A* and *B* in three cases.

	Velocity at A	Velocity at A		Velocity at <i>B</i>	
	Horiz. Comp.	Vert. Comp.	Horiz. Comp.	Vert. Comp.	
Question as Set	4	6.93	2.27	3.94	
Vertical Projection	0	8	0	4.55	
Oblique Projection	4	6.93	4	2.17	

In considering part (ii) the acceleration is not constant as the particle travels from *A* to the highest point, nor is the component of acceleration constant in any particular direction. These features preclude the use of $v^2 = u^2 + 2as$, and this part of the question can only be successfully undertaken by considering energy. Candidates should be encouraged to expect to need to use energy when particles move along curved paths in a vertical plane.

Unfortunately very few candidates used energy in part (ii), and $v^2 = u^2 + 2as$ was used in several different irrelevant ways.

Notwithstanding candidates' reluctance to use energy in other parts of the question, many referred to energy implicitly or explicitly in answering part (iii).

Part (iv) was reasonably well attempted, although hardly any candidates used the best strategy of applying the fact that the work done by the frictional force is equal to the KE at *A* minus the KE at *D*.

Among the candidates who considered the work done as the energy lost by the particle between *C* and *D*, some failed to include the KE at *C*. Some other candidates used $mg(2.5\sin 60^\circ) + \frac{1}{2}m(v_D^2 - v_C^2)$ as the total energy lost.

Rather more candidates used Newton's second law to find the acceleration of the particle from *C* to *D*, and then used $v^2 = u^2 + 2^{as}$. The main difficulty for candidates using this method was in finding the component of the weight down the plane. Many candidates gave this as $mgsin30^\circ$, assuming implicitly (and incorrectly) that the angle between *CD* and the horizontal is 30° .

Some candidates omitted the weight component, obtaining the acceleration as -3.5 ms^{-2} . Some candidates had the magnitude of *a* correct, but with a minus sign.

Answers: (i) 4.55 ms⁻¹; (iii) Path BC is smooth and B and C are at the same height (\Rightarrow KE the same at B and C); (iv) 5.25 ms⁻¹.

Paper 9709/05 Paper 5

General comments

There was a good response to this Paper. In the majority of scripts the work was well presented and there was no evidence that candidates were pressed for time in completing the Paper.

The dynamics questions were tackled with confidence with many all correct solutions by candidates from a fairly wide spectrum of the ability range. However, this could not be said of the statics questions where a lot of uncertainty was displayed particularly with the idea of taking moments.

Yet again marks were carelessly thrown away through failing to work correct to three significant figures. For example, in **Question 3**, the tension in the string was 467.6537..., which was then correctly rounded to 468N. Many candidates then used the value 468 to obtain the vertical component of the force at *A* as 44.7N rather than using the best value retained in the calculator to obtain 45N.

Thankfully the number of candidates still using $g = 9.81 \text{ ms}^{-2}$, despite the instructions on the front page of the Paper, continues to decline. However these candidates should have been alerted in **Question 7 (ii)** when this value of *g* was used. This gave a mass of 3.06 kg which was not the 3 kg requested in the question.

Comments on specific questions

Question 1

Although this question posed few problems for the more able candidates many of the remainder fell into error for a variety of reasons. The main one, and most serious, was the fact that candidates who were obviously taking moments about the centre of the ring, more often than not, had the term 1.5×25 appearing in the equation. No credit could be obtained as equations derived from either taking moments or resolving must include only the relevant number of terms, i.e. 2 terms if the moments were taken about the centre of the ring, or 3 terms if taken about the *y*-axis where the origin of coordinates was such that the centre of the ring was (25, 25).

Another frequent error was to assume that the masses of the ring and rod were proportional to their length (i.e. $2\pi \times 25$ and 48), despite the fact that the question did not specify that the components of the frame were made of the same material. The Method mark was allowed in this case provided all else was correct.

Answer: 2 cm.

The better candidates coped with part (i), but there were frequent errors by the rest in either calculating OQ or by using the wrong formula to find the distance of the centre of mass from O. It was depressing to find candidates taking an Advanced Level Paper who took the complement of 70° to be 30°. With the calculation of the distance of the centre of mass from O, it would be true to state that each of the first five centres of

mass given in the formula sheet MF9 appeared often, with the most frequent offender being $\frac{1}{2}r$ for the

hemispherical shell.

In part (ii) apart from a correct deduction, few candidates could give a coherent reason why the hemisphere did not fall on its plane face. There were many ambiguous statements of the type "the centre of mass falls before P". Only rarely was there a succinct statement, based on statical ideas, that after release, the resultant moment of the system about P was due to the weight of the hemisphere only and resulted in a clockwise moment.

Answers: (i) Centre of mass not between O and Q as 1.5 cm > 1.46 cm.

Question 3

It was generally appreciated that the tension in the string could only be found by taking moments, preferably about *A*. Then angle *PAB* was usually found correctly to be 30° although it was not unusual to see 26.6° from the less able candidates.

A frequent careless error was to have the weight also acting at a distance 2.5 cm from *A*. An approach by some candidates was to consider the tension as the vertical and horizontal components $T\cos 30^{\circ}$ and $T\sin 30^{\circ}$. Unfortunately the resulting moments equation often did not contain the moment of the $T\sin 30^{\circ}$ component. Hence no credit could be given as the derived equation must contain the moments of all the relevant forces.

Part (ii) presented a lot of difficulty for many candidates. One large group thought that the resultant force exerted by the wall at *A* was in the direction *AB*. Another group seemed to interpret "horizontal and vertical components" as being parallel and perpendicular to *AB*.

Answers: (i) 468 N; (ii) 234 N and 45 N.

Question 4

Candidates of all abilities scored well on this question. The fact that the differential equation was given in part (i) undoubtedly helped nearly all candidates to score maximum marks in part (ii). Only very weak candidates failed to see that it was necessary to apply Newton's Second Law of Motion to answer part (i). It was encouraging to see that the negative sign appeared in its proper place in the development of the equation as there was very little evidence of sign fiddling to get the required answer.

Answer: (ii) 3 ms^{-1} .

Question 5

There were many excellent all correct solutions to this question, even by candidates who only performed modestly in other parts of the Paper. Considering the improvement generally on this topic, it is to be hoped that circular motion is no longer one of the great mysteries of mechanics. Some of the infrequent errors in part (i) were (a) the tension in the string in the wrong direction, (b) the omission of 8N force when resolving vertically and (c) resolving in the direction of the string and equating the forces to zero. The latter case could

not be correct as the acceleration of the aircraft has a component $\frac{v^2}{r} \cos 30^\circ$ in this direction.

Part (ii) was also very well answered and only the weakest candidates used r = 9 rather than $9\sin60^\circ$. A number of solutions were laboured through using the acceleration in the form $r\omega^2$ to find ω and then using $v = r\omega$ to find the speed. Occasionally the premature approximation of taking the radius to be 7.8 m led to an answer which was not correct to three significant figures and thus led to the needless loss of the final mark.

Answers: (i) 6 N; (ii) 9 ms^{-1} .

On the whole there was a high degree of success with parts (i) and (ii), but many of the routes to the answers were somewhat lengthy. In part (ii) for example, many went to great lengths to first calculate the time taken to reach the highest point rather than merely state that it was 5 seconds. Others, who seem to think that all projectile problems are dependent on the use of the equation of the trajectory, first found the horizontal distance at the instance when the particle was at its highest point.

Although able candidates coped well with part (iii), many of the rest failed to appreciate that it depended on recognising that, at time *T*, the vertical component of the velocity was equal to the horizontal component. Had some of them drawn a simple sketch it could have avoided the frequent error of assuming that at time *T*, the components of the velocity were $60\cos 45^\circ$ and $60\sin 45^\circ$. Across the whole ability range there were many who unnecessarily found the speed of the particle (46.9 ms⁻¹) at time *T* during the course of their calculations.

Answers: (i) 56.4°; (ii) 125 m; (iii) 1.68 seconds.

Question 7

It was a very weak candidate indeed who failed to answer part (i) correctly.

The vertical resolution of the forces in part (ii) was good with a vast improvement in performance over similar situations occurring in problems in the past. Despite the mass of the stone being given, only in a minority of solutions was there any evidence of a late adjustment when the first attempt produced, for example, a mass of 1.5kg.

One of the most frequent failures in part (iii) was to assume that AB = 10m. Examiners got so used to seeing the incorrect answer 511 J that they knew instantly where the error lay. A more disturbing error was the assumption that the extension of the string was proportional to the depth of the stone below AB.

Apart from the best candidates, the usual mark obtained in part (iv) was 2. Although the G.P.E. (= 240 J) invariably appeared in the energy equation, the E.P.E. of the string as the stone passed through the mid-point of AB did not.

Answers: (i) 39 N; (iii) 650 J; (iv) 16 ms⁻¹.

Paper 9709/06

Paper 6

General comments

This Paper elicited a wide range of marks. There were a couple of difficult parts to the questions, but these were offset by some very easy parts. Only a few candidates appreciated the misleading impression of a false zero in **Question 1**. Premature approximation was only a problem in **Questions 3** and **4**, where some candidates from certain Centres continued to work to only 1 significant figure and thus did not use their normal tables correctly.

Comments on specific questions

Question 1

The first part of this question was the worst attempted on the Paper, with many imaginative and varied (but incorrect) reasons for why the graph was misleading. The stem-and-leaf diagram was very well attempted by almost everybody. Most candidates remembered to give a key. The median was poorly attempted

however, with many candidates thinking it was the $\frac{n}{2}$ th term, even from 21 people, and of those who

obtained the correct number (the 11th), some quoted it as 9 rather than 79. Many wrote the numbers out in order to find the median, rather negating the purpose of a stem-and-leaf diagram.

Answers: (i) false zero; (ii)(b) 79.

Some candidates went straight into a binomial situation here, without realising that taking two pens is equivalent to taking one pen then taking another without replacement. Again, some managed to do part (i) correctly, with the help of the answer being given, but could not see any relationship between part (i) and part (ii). The expected value was followed through provided the probabilities summed to 1.

Answers: (ii)
$$P(0) = \frac{7}{15}$$
, $P(1) = \frac{7}{15}$, $P(2) = \frac{1}{15}$; (ii) $\frac{3}{5}$.

Question 3

This was well done with pleasing knowledge of the normal distribution. Some candidates lost marks by premature approximation, taking a *z*-value of 0.4651 to be 0.46 or 0.47. In part (ii) many candidates looked up 0.8 backwards in the tables but approximated to 2 significant figures, instead of working with 4. As usual, some candidates lost a minus sign.

Answers: (i) 0.321; (ii) 14.3.

Question 4

Part (i) of this question was pleasingly done by a large number of candidates, all of whom recognised the binomial situation. However, half of the candidates lost a mark for approximating the answer to 3 decimal places and not 3 significant figures. If 0.0829 was seen anywhere, the candidate gained full marks, but if the answer appeared straight as 0.083 then a mark was lost for premature approximation.

In part (ii) the normal approximation to the binomial was also very well done, with almost all candidates recognising the situation and applying a continuity correction.

Answers: (i) 0.0829; (ii) 0.275.

Question 5

Solutions to permutations and combinations questions continue to improve. The last part needed some thought and only the real thinkers managed to make a success of it, although most managed to gain some credit for attempting an option of some sort.

Answers: (i) 120; (ii) 186; (iii) 90.

Question 6

The first two parts were well done, with most candidates understanding what was required, and getting high marks on the probability in part (ii). Part (iii) needed some understanding, but most candidates made a beginning by realising there were two options, and many also divided by their answer to part (ii), realising it was a conditional probability question. However only the most perspicacious finally arrived at the correct answer.

Answers: (i) $\frac{3}{8}$; (ii) $\frac{17}{42}$; (iii) $\frac{10}{17}$.

Question 7

This question was very well done with many candidates achieving full marks. There was some confusion about what constituted the mid-point of the intervals, but credit was given for trying almost anything apart from an end point or class width. In working out the standard deviation credit was given for using their (albeit wrong) mid-point. Thus many method marks were gained by candidates who did not quite produce the final correct mean and standard deviation. A few candidates used 0.5, 10.5, etc as end points thus losing a mark, but nearly everyone knew how to calculate frequency density for the histogram. The graphs were very pleasingly drawn, with straight ruled lines, and scales and axes labelled.

Answers: (i) 18.4, 13.3;

(ii) frequency densities 2.2, 4.0, 3.2, 1.8, 1.0, 0.2 or scaled frequencies usually of 11, 20, 16, 9, 5, 1.

Paper 9709/07 Paper 7

General comments

The performance of candidates in this Paper was varied. A number of candidates scored very highly with well presented, clear solutions. However there were, equally, some very poor attempts from candidates who were unprepared for this examination.

Candidates performed well on **Questions 4** and **6** in particular, and often found **Questions 1** to **3** more challenging. **Question 5** on Type I and Type II errors was slightly better answered, in general, than has been the case in the past.

Comments on working to the correct level of accuracy have been made in the past; the Question Paper requires three significant figures unless otherwise stated. Whilst in general fewer candidates are losing marks because of this it is still surprising at this level that some (often very good) candidates still lose marks due to premature approximation (i.e. working to three significant figures or less in earlier stages of working) or even confusing significant figure accuracy with decimal place accuracy. This was particularly seen on **Question 2** where the answer required was 0.0834 to three significant figures and many candidates gave an answer of 0.083, and in **Question 5** the answer of 0.0234 was often given as 0.023.

Candidates did not appear to be under any time pressure to complete the Paper (despite many using a lengthy method in their attempt at **Question 1**). On the whole candidates gave clear and full solutions.

Comments on specific questions

Question 1

Many candidates did not use the straightforward way of attempting this question (mean = np, variance = npq) and instead tried to set up a probability distribution table. This caused a time penalty and often errors were made, (including some very fundamental ones with probabilities in tables totaling more than one). The main error noted on part (ii) was to multiply the variance in part (i) by 2 rather than by 4.

Some non-numerical solutions to both parts were also seen.

Answers: (i) 2.5, 1.25; (ii) 5, 5.

Question 2

Some candidates correctly appreciated that this was a significance test using a Binomial Distribution. Common errors were to calculate P(X > 10), P(X < 10), or merely P(X = 10) rather than $P(X \le 10)$ and occasionally contradictory comments were seen in the conclusions (e.g. "Reject $H_0(p = 0.6)$, therefore the player had not improved"). The majority of candidates attempted this question using a Normal Distribution which was not strictly valid as *nq* was equal to 4.8; however, some credit was given. Many errors were noted including lack of, or incorrect, continuity correction and much confusion between different methods was seen. This was not a well attempted question.

Answer: Accept claim at 10% level.

Question 3

Several careless mistakes were seen in calculating the mean, but on the whole candidates were able to calculate a confidence interval. A particularly common error was to use a wrong *z*-value. Some candidates found their own standard deviation from the sample rather than using the given value, or even used

 $\frac{n}{n-1}$ x 3. In part (ii), many candidates ignored the instructions to "use your answer to part (i)" and did

unnecessary further calculations (often incorrect). Candidates who did use their confidence interval calculated in part (i) were often not clear in their explanation. A statement such as "30 was inside the interval and therefore the claim could be accepted" was required. "It is in the interval" was too vague and could not be accepted. Other incorrect comments such as "Accept the claim because 30 is close to 31", "Reject because $30 \neq 29.4$, and $30 \neq 32.6$ " were seen showing a lack of understanding by the candidate. Most candidates scored some marks on this question, but not many gained full marks.

Answers: (i) (29.4, 32.6), 30% is inside the interval; (ii) Accept claim (at 2% level).

This question was well attempted by the majority of candidates. Errors in part (i) included using wrong limits for the integration, in part (ii) an error often seen was $\int x \, dx = x^2$ and surprisingly in part (iii) many basic errors were seen in solving the quadratic equation. Poor algebra and errors such as $m(4 - m) = 2 \Rightarrow m = 2$ or (4 - m) = 2 were too often seen. Candidates often lost the final answer mark in part (iii) by not rejecting the solution to the quadratic equation which was inadmissible.

Answer: (i) 0.0625; (ii) $\frac{2}{3}$; (iii) 0.586.

Question 5

Whilst attempts at this topic were better than in the past there were still a large number of candidates who did not attempt the question at all. Lack of clear numerical interpretation of Type I/Type II errors was still evident. Common errors noted were omission of $\sqrt{20}$ in the denominator when standardising or use of the wrong tail. In part (ii) 2.1 was often incorrectly used.

Answer: (i) 0.0234; (ii) 0.160.

Question 6

Many candidates scored well on this question. In part (i) most candidates correctly used $\lambda = 1.25$ but errors such as finding P(0, 1, 2, 3, 4), P(1, 2, 3) or 1 – P(0, 1, 2, 3) were seen. In part (ii) some candidates used an incorrect variance and omitted or used a wrong continuity correction. Many candidates correctly found the final answer of 0.123 in part (iii), though finding P(4) with $\lambda = 1.25$ and P(4) with $\lambda = 5$ and multiplying these together was a common error, as was merely finding P(4) with $\lambda = 5$.

Answers: (i) 0.962; (ii) 0.0915; (iii) 0.123.

Question 7

This was a reasonably well attempted question. In part (i) a common error of $20^2 \times 0.15^2$ or $20^2 \times 0.27^2$ when calculating the variances was noted by Examiners along with rounding errors. Some candidates considered 2A > B or A - B < -2 rather than A - B > 2, and even attempts to include a continuity correction were seen

so that A - B > 2 became A - B > 1.5. Of the candidates who found $\overline{A} \sim N$ (20.05, $\frac{0.15^2}{20}$) and

 $\overline{B} \sim N$ (20.05, $\frac{0.27^2}{20}$) very few went onto consider $\overline{A} - \overline{B} > 0.1$ ($\frac{2}{20}$) and some incorrectly used

 $\overline{A} - \overline{B} > 2$. In part (b) weaker candidates worked with 0.975 and never found the *z*-value of 1.96 and some candidates formed an incorrect equation involving an '*n*' on the numerator. Surprisingly many candidates incorrectly went from $\sqrt{n} = 14.7$ to n = 3.8, and a final answer of 217 or 216.09 was also common.

Answers: (i) 0.0738; (ii) 216.