CONTENTS

FOREWORD

This booklet contains reports written by Examiners on the work of candidates in certain papers. **Its contents are primarily for the information of the subject teachers concerned**.

MATHEMATICS

GCE Advanced Level and GCE Advanced Subsidiary Level

Paper 9709/01
Paper 1

General comments

The performance of candidates on this paper was very pleasing. There were relatively few responses from candidates who had not been prepared for the paper. The standards of numeracy, algebra and presentation were good and most questions were easily within the reach of most candidates. The two topics that presented most problems were the idea of 'unit vector' and 'range of a quadratic function'. Some Centres would help both their candidates and the Examiners by advising candidates not to divide the page into two columns.

Comments on specific questions

Question 1

This proved to be a successful starting question with most candidates scoring highly. Apart from a few numerical errors, especially in part **(ii)**, common errors were to take the sum to infinity as $\frac{a}{r-1}$, to use the formula for the tenth term instead of the sum of ten terms or to use $S_{10} = \frac{a(1-r^9)}{1-r}$ or even $\frac{a(1-r)^{10}}{1-r}$.

Answers: **(ii)** $\frac{3}{4}$; **(ii)** 242.

Question 2

The majority of candidates obtained a correct solution, but errors in integration were common. Several candidates failed to realise that $\sqrt{x} = x^{\frac{1}{2}}$ and many failed to realise the need to divide by 3, the differential of (3*x* + 1). A large number of candidates automatically assumed that the value of the integral at the lower limit of 0 was 0 and could be ignored. It was disappointing to see a few weaker candidates assuming that $\sqrt{3x+1}$ could be rewritten as $\sqrt{3x}+\sqrt{1}$.

Answer: $1\frac{5}{9}$.

Question 3

A minority of candidates realised that division by $\cos^2\theta$ led directly to a quadratic in $\tan\theta$. Other solutions were more unwieldy, including replacing $\sin^2\theta$ by $1-\cos^2\theta$ or $4\cos^2\theta$ by $4(1-\sin^2\theta)$ and later using $\frac{1}{\cos^2\theta} = \sec^2\theta = 1+\tan^2\theta$. Others divided by $\sin^2\theta$ to obtain a quadratic in $\frac{1}{\tan\theta}$ and hence proceeded to the answer. A common error was to divide through by $\cos^2\theta$ and to assume that the right-hand side was 0 instead of 4. Surprisingly, of candidates obtaining $\tan\theta$ = 1 or −4, a large proportion rejected any solutions from the negative value.

Answers: **(i)** $\tan^2\theta + 3\tan\theta - 4 = 0$; **(ii)** 45°, 104.0°.

Question 4

This was extremely well answered with most candidates showing a good understanding of the binomial expansion. Occasionally $(2x)^3$ was given as $2x^3$ but generally part **(i)** was correct. In part **(ii)** many candidates failed to realise that the term in x^3 came from the sum of two different terms in the expansion.

Answers: **(i)**160; **(ii)** −20.

Question 5

A majority of attempts were correct, and most other candidates' attempts obtained the method marks available. The most common error came from misuse (or misunderstanding) of the concept of radians. 0.8 radians was often converted to 144° and often used in a calculator in degree rather than radian mode. A few weaker candidates assumed triangle *OCD* to be right-angled in their calculation of *CD*. Use of the formulae $s = r\theta$ and $A = \frac{1}{2}r^2\theta$ was very good as was the trigonometry needed to calculate either the length of *CD* and occasionally the height of the triangle *OCD* from *O* to the mid-point of *CD*.

Answers: **(i)** 21.5 cm^2; **(ii)** 20.6 cm.

Question 6

Part **(i)** was usually correct with most candidates correctly eliminating *y* to obtain a quadratic in *x*. Surprisingly, this often appeared as $x^2 - x - 6 = 0$ instead of $x^2 + x - 6 = 0$. Algebraic errors such as replacing $y = 9 - \frac{6}{x}$ by $xy = 9 - 6$ were also common. Part **(ii)**, however, was very poorly answered, mainly through the candidates' misunderstanding of the term 'perpendicular bisector'. Most candidates realised the need to use 'm_1m_2=−1' to obtain the necessary gradient but a minority of attempts realised that the required line passed through the mid-point.

Answers: **(i)** (2, 6) and (−3, 11); **(ii)** $y = x + 9$.

Question 7

This was confidently answered and a source of high marks. The differentiation of $y = \frac{18}{x}$ and subsequent calculation of the equation of the normal was accurate and most candidates realised the need to substitute *y* = 0 to obtain the correct value of *x*. Similarly in part **(ii)**, most candidates realised the need to integrate πy^2 and, apart from errors in sign, most were correct. The use of the limits 4.5 to 6 was again surprisingly accurate with only a few errors, usually by using 6 to 4.5. The fact that the answer was given was a definite help to candidates having difficulty over signs.

Answers: **(i)** *y* = 2*x* − 9; **(ii)** 18π.

Question 8

Part **(i)** presented a few difficulties with candidates often failing to appreciate that the arc length needed for the perimeter was πr and not $2\pi r$. Several candidates also included the base (2*r*) twice in the expression relating *r* and *h*. Use of 'area = $2rh + \frac{1}{2}\pi r^2$' was generally correct and a majority of attempts obtained the given expression for *A*. Differentiation of this and the subsequent solution of $\frac{\mathrm{d}A}{\mathrm{d}r} = 0$ was very pleasing and the vast majority obtained either *r* = 1.12 or $\frac{8}{4+\pi}$. The most common error was in collecting terms in *r* with $\frac{8}{4-\pi}$ being seen in many solutions. Most also obtained a correct second differential, though $\frac{\mathrm{d}^2A}{\mathrm{d}r^2} = 8 - 4 - \pi$ was a surprisingly common error. Only a very small handful of solutions were seen in which candidates looked at the sign of $\frac{\mathrm{d}A}{\mathrm{d}r}$.

Answers: **(i)** $h = 4 - r - \frac{1}{2}\pi r$; **(iii)** 1.12 or $\frac{8}{4+\pi}$; **(iv)** maximum.

Question 9

This question caused most candidates some difficulty and it was rare to see a completely correct solution. The majority of candidates showed no familiarity or understanding of the term 'unit vector'. Many candidates interpreted $\overrightarrow{AB}$ incorrectly – usually as either **a** + **b** or as **a** − **b**. In part **(ii)** most candidates recognised the need to use a scalar product and correctly evaluated this as $10 - p$. Unfortunately this was not always equated to 0, with many candidates not realising that the denominator of **a.b** could be ignored or even that cos 90°=0. In part **(iii)** $\overrightarrow{AD}$ was often taken as **a** + **d** or as **a** − **d** and a minority of candidates recognised that the length of the vector $-2\mathbf{i} - 3\mathbf{j} + (q + 1)\mathbf{k}$ was $\sqrt{4+9+(q+1)^2}$. Many solutions were also seen in which $(q + 1)$ appeared as $(q - 1)$. Even many good candidates solved the resulting quadratic $(q+1)^2 = 36$ as $q = 5$ only.

Answers: **(i)** $\begin{pmatrix} \frac{1}{3} \\ -\frac{2}{3} \\ \frac{2}{3} \end{pmatrix}$; **(ii)** 10; **(iii)** 5 or −7.

Question 10

(i) This was well answered and proved to be a source of high marks. Most candidates set $x^2 - 2x$ to 15 (though occasionally 16 was seen) and obtained end values of −3 and 5. Most obtained the correct interval, though $x > -3$ and $x > 5$, or $x < -3$ and $x < 5$ were common errors.

(ii) Very few candidates offered a completely correct solution and many missed this part out completely. Only a small proportion realised that because the function f was quadratic, it was necessary to find the stationary value of the function. Only a few realised that $f(x) \geqslant -1$. More candidates realised that, because f is a quadratic function, it was not one-one and therefore did not have an inverse over the domain of real numbers. Of those attempting to find the stationary value of f, about equal numbers of candidates used calculus to those completing the square.

(iii) Nearly all candidates coped correctly with finding an expression for gf(x) and only a few misinterpreted gf(x) with fg(x). The vast majority either used the formula for solving a quadratic equation or looked directly at the value of $b^2 - 4ac$ and it was rare for candidates not to obtain "−8" and to correctly deduce that the equation had no real solutions.

(iv) The sketch graphs were generally of a pleasing standard, though a common error was to draw the graphs of g and g^{-1} as two parallel lines. A majority of attempts realised that the two lines were symmetrical about the line $y = x$ and either drew this line on the diagram or explained the relationship in words.

Answers: **(i)** $x < -3$ and $x > 5$; **(ii)** $f(x) \geqslant -1$, f does not have an inverse.

Paper 9709/02

Paper 2

General comments

Candidates' responses to the paper displayed a very wide range of ability. A significant number of scripts scored marks of 45 or above, and Examiners were delighted by the degree of mathematical skill and understanding of the syllabus displayed by such candidates. However, many other candidates were not equal to the demands posed by the majority of the questions they attempted, and only able to record a total in single figures.

There was no evidence of candidates lacking sufficient time to attempt all the questions. Those questions which were well answered included **Questions 3**, **4 (i)**, and **6**, and those causing widespread difficulty were **Questions 2 (iii)**, **4 (iii)**, **5 (ii)** and **7**. Responses to **Questions 1**, **4 (ii)**, **5 (i)** and **5 (iii)** were mixed. It is disappointing when questions apparently providing a straightforward and familiar test of topics are not answered with much conviction.

Examiners were pleasantly surprised by a marked improvement in candidates' familiarity with iteration and integration, for example, but improvements in such areas were offset by many faults in algebraic and even arithmetic manipulation.

Candidates' work was almost always neat and decipherable and the logic behind their solutions clearly displayed.

Comments on specific questions

Question 1

The topic was novel to the paper, but a clear instruction to use logarithms was invariably followed. To the Examiners' surprise, a large minority of the candidates fell victim to the mathematical gaffe $x\ln 2 = y\ln 5 \Rightarrow \frac{x}{y} = \frac{\ln 2}{\ln 5}$.

Answer: 2.32.

Question 2

(i) The iteration was basically good, but many candidates worked to three, or even two, decimal places throughout. To find α to three decimal places requires intermediate calculations correct to at least four, and preferably five, decimal places. This point was stressed in all previous reports but has not yet been generally acted upon. A surprising number of solutions were rounded from 3.14155, or 3.1416, to 3.141.

(ii) Most candidates could not cope; often attempts were made to bring in the solution to part **(i)**, saying that α = 3.142 satisfied the equation $\alpha = \frac{1}{5}\left(4\alpha + \frac{306}{\alpha^4}\right)$ and working out each side approximately.

Answers: **(i)** 3.142; **(ii)** $x = \frac{1}{5}\left(4x + \frac{306}{x^4}\right)$.

Question 3

(i) Virtually all candidates set $x = 3$ and equated the polynomial to zero, but many candidates reduced the equation $9a + 9 = 0$ to a solution $a = +1$.

(ii) Most candidates factorised the revised cubic expression correctly, but several omitted to state that $x = +3$ is a root of $f(x) = 0$.

Answers: **(i)** $a = -1$; **(ii)** $x = -2, -\frac{1}{2}, 3$.

Question 4

(i) This was generally well attempted, though many solutions featured an equation $\tan\alpha = \left(\frac{4}{3}\right)^{-1}$, rather than $\tan\alpha = \frac{4}{3}$, and hence found α = 36.87, instead of 53.13.

(ii) Candidates fell into two categories, with around half of all solutions correctly noting that $\theta + \alpha = \sin^{-1}\left(\frac{4.5}{R}\right)$ and correctly obtaining a first solution for α; however many stopped at this point or obtained a false second solution equal to $(180° - \alpha_1)$, where α_1 is the first solution. Other candidates did not link part **(ii)** to part **(i)** and attempted to square the given equation in part **(ii)** or divided it by $\cos\theta$ without reducing the right-hand side to a correct form $4.5\sec\theta$. These candidates often spent a great deal of time in a fruitless search for a solution.

(iii) Almost no-one scored here. What was required was to convert the given expression to that obtained in part **(i)** and then note that the maximum value of $\sin(\theta + \alpha)$ is 1, and hence the minimum value of the expression in part **(iii)** is equal to (7 – *R*). Again, this is a technique whose importance was stressed in earlier Reports.

Answers: **(i)** $R = 5$, $\alpha = 53.13°$; **(ii)** 11.0°, 62.7°; **(iii)** 2.

Question 5

(i) Differentiation was surprisingly poor. Many derivatives contained only a single term. Even those who obtained $y' = (1 - x)e^{-x}$ were often unable to correctly solve $y' = 0$.

(ii) A large proportion of attempts were by use of 3, 4 or even only 1 strip. Examiners took a benign view of the correct use of more than 2 strips, but basic arithmetic let down many candidates, regardless of how many strips were used, e.g. $y(0) = 1$ (and not 0). Many candidates tried to integrate exactly, despite the instruction to use the trapezium rule.

(iii) Very poor explanations were the norm, many being mere statements. A good rough diagram is always recommended.

Answers: **(i)** 1; **(ii)** 0.50; **(iii)** under-estimate.

Question 6

(i) This was generally well attempted, but many candidates still use $\frac{dy}{dx} = \frac{dx}{dt} \div \frac{dy}{dt}$ or $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dx}{dt}$.

(ii) There were many excellent solutions, but many candidates failed to set $t = 1$ in the given expressions for x and y.

(iii) As Examiners stress each year, it is *not* the case that $\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right)$; almost all solutions featured this error, and candidates are better advised to instead use the first derivative and to examine the signs of $\frac{dy}{dx}$ at $t = 1$, 3, for example, or at other values of t on either side of the value $t = 2$ where the stationary point occurs.

Answers: **(ii)** $x + y = 7$; **(iii)** $y = 4$, a minimum point.

Question 7

There were several excellent solutions, but most candidates avoided this question or scored only in part **(i)**.

(i) Expansions were often poor, including the form $\cos(2x + x) \equiv \cos 2x + \cos x$, or a lesser error $\cos 3x \equiv \cos x \cos 2x + \sin x \sin 2x$. Many candidates failed to correctly use the formulae $\cos 2x \equiv \cos^2 x - \sin^2 x \equiv 2\cos^2 x - 1 \equiv 1 - 2\sin^2 x$ (any of these 3 forms) and $\sin 2x \equiv 2\sin x \cos x$.

(ii) Few solutions began by noting that $\cos^3 x \equiv \frac{1}{4}(\cos 3x + 3\cos x)$, using part **(i)**. Instead, various spurious integrals of $\cos^3 x$ were presented, including $\frac{1}{4}\cos^4 x$, $\frac{\cos^4 x}{4\sin x}$ or $-3\cos^2 x \sin x$ (by differentiation). The instruction 'Hence' should have suggested use of the given result in part **(i)**.

Papers 8719/03 and 9709/03

Paper 3

General comments

There was a considerable variety of standard of work by candidates on this paper and a corresponding very wide spread of marks. The paper appeared to be accessible to candidates who were fully prepared and no question seemed to be of undue difficulty, though completely correct solutions to both parts of **Question 11** (vector geometry) were infrequent. Adequately prepared candidates seemed to have sufficient time to attempt all the questions and the presentation of their work was usually satisfactory. However, there were some very weak, often untidy, scripts from candidates who clearly lacked the preparation necessary for work at the level demanded by this paper. All questions discriminated well. The questions or parts of questions on which candidates generally scored highly were **Question 4** (algebra), **Question 7** (iteration), and **Question 9 (i)** (partial fractions). Those on which scores were low were **Question 1** (trigonometry), **Question 5** (trigonometry), **Question 6** (differential equation) and **Question 11 (ii)** (vector geometry).

It is clear from the responses to **Question 5** and **Question 10** that some candidates do not understand the meaning of the term 'exact'. Previous reports have drawn attention to this misunderstanding. Examiners also found that sign errors were common reasons for loss of marks in **Questions 3**, **5**, **7**, **9** and **10**.

The detailed comments that follow inevitably refer to common errors and can lead to a cumulative impression of poor work on a difficult paper. In fact there were many scripts showing a good and sometimes excellent understanding of all the topics being tested.

Where numerical and other answers are given after the comments on individual questions, it should be understood that alternative forms are often possible and that the form given is not necessarily the sole 'correct answer'.

Comments on specific questions

Question 1

Candidates who knew the relationship of the secant function to the cosine function scored well, often using a sketch of the graph of $y = \cos x$ as an aid. However, it was clear that many lacked a correct understanding of the secant function, for they presented erroneous sketches resembling in some cases those of functions such as $y = \text{cosec}\, x$ or $y = -\cos x$.

Question 2

The majority of candidates found the correct critical values for this inequality, but failure to derive the correct solution to the problem was frequent. Examiners often saw the answer given incorrectly as $-\frac{1}{3} < x < -1$.

Answer: $-1 < x < -\frac{1}{3}$.

Question 3

Most candidates showed an appreciation of the process of implicit differentiation and this question was generally well answered. The most frequent errors were failure to differentiate the constant term 3, and a sign error when simplifying the derivative of $-4xy$.

Answer: 2.

Question 4

Nearly all candidates obtained the correct quadratic equation in *y*, and many were successful in solving it. Some solutions went no further than this. Those that continued usually involved the correct method for finding *x*, reaching a ratio of logarithms. A common error in the evaluation of this ratio was to work with 1.62, the rounded value of the positive root of the quadratic equation. This leads to a final answer of 0.696 to 3 significant figures in which the last digit is incorrect, and illustrates the danger of making a premature approximation in numerical work.

Answers: **(i)** $y^2 - y - 1 = 0$; **(ii)** 0.694.

Question 5

In part **(i)**, a minority completed the proof quickly, showing that both sides of the identity were equivalent to $\frac{1}{4}\sin^2 2\theta$. However, the majority either lacked an overall strategy or else failed to complete the solution by making errors, usually of sign, in their working. Part **(ii)** was also poorly answered. Though nearly all used the given identity to change the integrand, failure to integrate correctly was surprisingly common. Errors in evaluating the integral were also frequent, e.g. using 60° as a limit instead of $\frac{1}{3}\pi$. Many candidates gave an approximate decimal value rather than the exact answer requested.

Answer: **(ii)** $\frac{1}{24}\pi + \frac{\sqrt{3}}{64}$.

Question 6

Though some excellent answers were seen, Examiners felt this question was not well answered. A substantial number of candidates could not separate variables correctly and made little or no progress. Those that did separate variables and integrate accurately usually included a constant and evaluated it correctly. However, the error of taking an expression of the form ln *a* = *b* + *c* to be equivalent to $a = \mathrm{e}^b + \mathrm{e}^c$ was encountered quite frequently at this point. An expression for $\ln(y^3 + 1)$ having been obtained, some solutions ended when y^3 was expressed in terms of *x* and were thus incomplete.

Answer: $y = (2\mathrm{e}^{3x} - 1)^{\frac{1}{3}}$.

Question 7

Examiners felt that all three parts of this question were generally well answered. However, in part **(iii)** there were candidates who appeared to be unable to compose a correct sequence of calculator operations for evaluating $\frac{2x^3 - 1}{3x^2 + 1}$. Also a small number of candidates who had correctly obtained a sequence converging to –0.68 nevertheless went on to state that the root was 0.68.

Answer: **(iii)** –0.68.

Question 8

In part **(i)** many candidates obtained the correct roots. A fairly common error was to take $\sqrt{-3}$ to be equal to 3i. Whereas the method for finding the modulus of a complex number seemed to be well known, candidates did not always show a proper appreciation of the nature of the argument of a complex number.

Answers: **(i)** $\frac{1}{2} + \mathrm{i}\frac{\sqrt{3}}{2}$, $\frac{1}{2} - \mathrm{i}\frac{\sqrt{3}}{2}$; **(ii)** 1, $\frac{1}{3}\pi$, 1, $-\frac{1}{3}\pi$.

Question 9

Part **(i)** was answered extremely well in general. In part **(ii)** most candidates attempted to find the expansion by combining the binomial expansions associated with the partial fractions. The expansion of $-2(x+1)^{-1}$ was usually found correctly, but errors, often of sign, were made frequently when forming the expansions of $-1(x-1)^{-1}$ and $4(x-2)^{-1}$.

Answer: **(i)** $-\frac{1}{x-1}+\frac{4}{x-2}-\frac{2}{x+1}$.

Question 10

The first two parts were answered quite well in general. The exact coordinates of the maximum point were requested but some candidates ignored or did not fully understand the request, giving approximate decimal answers instead. Those that tried to give exact answers sometimes found it difficult to give a correct simplified exact value for *y* when $x = \mathrm{e}^{\frac{1}{2}}$. In the final part, many of the attempts at integration by parts were weak. At the outset serious errors such as setting *u* = ln *x* and $\frac{\mathrm{d}v}{\mathrm{d}x} = x^2$ were common, and even those who correctly set $\frac{\mathrm{d}v}{\mathrm{d}x} = x^{-2}$ often made slips in the work that followed. Here again the request for an exact answer was not always met.

Answers: **(i)** 1; **(ii)** $x = \mathrm{e}^{\frac{1}{2}}, y = \frac{2}{\mathrm{e}}$; **(iii)** $1-\frac{2}{\mathrm{e}}$.

Question 11

In part **(i)** candidates used a variety of methods to find the equation of the plane containing *P*, *Q*, and *R*, and the standard of work was generally good. The second part discriminated well. The most popular approach was to form the equation of the perpendicular from *S* to the plane, find *N* the point of intersection of this line with the plane, and then calculate the length of *SN*. Very few candidates seemed be using a rough sketch as an aid. It is possible that such a procedure might have saved them from major errors such as thinking that the normal to the plane was perpendicular to *OP* and *OQ*, or that the point *N* lay on the line *PQ*, or that the point of intersection of *SP* with the plane was *N*.

Answers: **(i)** $2x + 3y - 6z = 8$; **(ii)** $\mathbf{i} + 2\mathbf{j}$.

Paper 9709/04

Paper 4

General comments

In each of the first five questions the proportion of candidates scoring full marks was disappointingly low. In the case of **Question 4** this feature prevailed because of lack of accuracy. **Question 6** was a fruitful source of marks, even for some of the weaker candidates.

As expected **Question 7** proved more difficult. Nevertheless a significant number of candidates scored full marks.

Comments on specific questions

Question 1

This question was fairly well attempted although many candidates omitted either the weight or the vertical component of the applied force in part **(ii)**. Some candidates used $\frac{5}{12}$ or $\frac{5}{13}$ as the angle α in degrees, rather than its tangent or sine.

Another common error was to introduce $F = \mu R$ into parts **(i)** and **(ii)**. It is not until part **(iii)** that this formula applies.

Answers: **(i)** 12 N; **(ii)** 16 N; **(iii)** 0.75.

Question 2

This question was poorly attempted, many candidates simply adding the magnitudes of the given forces to obtain *R*, and others finding it as $\sqrt{300^2 + 100^2 + 250^2}$.

Among the work of candidates who found components *X* and *Y* the mistakes were many and varied. They included errors of sign, muddles with sine and cosine, the use of 100 degrees in a right angle (so that sin 30° appears where cos 70° might have done), the inclusion of 'components' of the 300 N and the 100 N in *X* and *Y* respectively, and the omission of the component of the 250 N from either *X* or *Y*. Some candidates produced components of the equilibrant instead of those of the resultant.

Most candidates who found values for *X* and *Y* used them correctly to find an answer for *R*, but a significant minority obtained an answer for α by using cos $\alpha = \frac{100}{196.6}$.

Some candidates did not understand 'anticlockwise' from the force of magnitude 100 N and correct work throughout the question was at times accompanied by wrong answers for α such as 70.7, 160.7, 199.3 and 340.7.

Answer: $R = 197$, $\alpha = 19.3$.

Question 3

Almost all candidates obtained *AC* correctly but many gave the answer for *AB* as 130 m.

The most common error in part **(ii)** was to show the graph having a positive slope for $t > 15$. This occurred even among candidates who scored all three marks in part **(i)**.

The answer *AB* = 130 m in part **(i)** was not necessarily a barrier to a completely correct graph in part **(ii)**.

Answers: **(i)** 70 m, 10 m.

Question 4

This question was well attempted, although some candidates gave answers for the speed instead of the kinetic energy.

The Examiners accepted answers of 1.40 J and 0.680 J although the values 1.4 and 0.68 are exact.

In part **(i)** many candidates used the principle of conservation of energy to find *v* and then used KE = $\frac{1}{2}mv^2$, failing to realise that the answer can come directly from *mgh*. A consequence of this was a loss of accuracy in many cases. Among candidates who did realise that the answer can come directly from *mgh*, some used $h = 2.5$ instead of $h = 0.7$.

Common errors in part **(ii)** included $R = 0.2g$ and $F = 0.15$, and the use of 'Gain in KE = Loss in PE – Force'.

Many candidates failed to obtain the correct final answer because of premature approximation of the angle of inclination, or because *F* was taken as 0.29 instead of 0.288.

Answers: **(i)** 1.4 J; **(ii)** 0.68 J.

Question 5

Although many candidates inappropriately used formulae that apply only to motion with constant acceleration, many did integrate $v(t)$ to find an expression for the displacement. However candidates should be aware not only that integration is necessary, but also that in contextual questions the role of the constant of integration is vital. Unfortunately this was not the case for very many candidates.

In part **(iii)** some candidates recovered from the absence of the constant of integration by (implicitly) arguing that when the distance travelled, $10t^2 - 0.25t^4$ (for $0 < t < \sqrt{20}$), is 36, the particle is at O, thus obtaining the correct equation. Some candidates, however, incorrectly equated the expression for this distance travelled with –36.

Many candidates failed to realise that the relevant equation is a quadratic in t^2. Some candidates factorised $10t^2 - 0.25t^4$ and then erroneously equated each factor to 36.

Some candidates got as far as $t^2 = 4, 36$, without proceeding to square roots, and some forgot that they were dealing with t^2 and gave the answers as $t = 4, 36$. Another common error was to write $t = 4, 6$ after obtaining the correct values for t^2.

Answers: **(i)** $10t^2 - 0.25t^4 - 36$; **(ii)** 60 m; **(iii)** 2, 6.

Question 6

Most candidates made a scoring attempt at part **(i)** of this question and very many gave a completely correct answer. The most common errors were in the sign of the 400, or the absence of the 400, on using Newton's second law.

Most candidates realised that $a = 0$ and thus the driving force is 400 N in part **(ii)**. The given answer was easily confirmed thereafter.

Most candidates took the direct route to the answer in part **(iii)**, via 'time taken = $\frac{1500000}{20000}$'. Success by first finding the distance travelled depended, as in part **(ii)**, on realising that $a = 0$ and thus the driving force is 400 N.

A common error when using a basically correct method was to take 1500 kJ as 1500 J, leading to the answer 0.075 s.

Answers: **(i)** 20 ms^{-1}; **(iii)** 75 s.

Question 7

In part **(i)** many candidates equated $30t - 5t^2$ with 25 and obtained the relevant values $t = 1$ and $t = 5$. However most of these candidates failed to interpret the roots of the equation to give the required duration as 4 s.

Among the candidates who found the time to maximum height and the time to height 25 m, and then subtracted, very few doubled the 2 s obtained to account for the time that P_1 is ascending *and* descending.

In part **(ii)** very few candidates dealt with the 25, so that many wrote $30t - 5t^2 = 10t - 5t^2$. Some realised that something was amiss, and changed the equation to $30t - 5t^2 = 10t + 5t^2$. Those who did use the 25 almost always did so inappropriately.

Some candidates used $s_1 = 20t - 5t^2$ where t in this case is the time after P_1 has passed the top of the tower. Very few such candidates found $s_2 = 5 - 5t^2$ as the corresponding equation for P_2.

Those candidates who used $v^2 = u^2 - 2gs$ almost always failed to distinguish between v_1 and v_2 on substituting into $s_1 = s_2 + 25$. Among those who were successful in obtaining a correct equation in v_2 only, almost all obtained $v_2 = 2.5$ instead of $v_2 = -2.5$.

Candidates who showed understanding in parts **(i)** and **(ii)** usually made a scoring attempt in part **(iii)**, but the majority of candidates were already defeated by the earlier parts.

Answers: **(i)** 4 s; **(ii)** 17.5 ms^{-1} and –2.5 ms^{-1}; **(iii)** 1.75 s.

Papers 8719/05 and 9709/05

Paper 5

General comments

On the whole there was a good response to this paper from those who had a good understanding of mechanical ideas. On the other hand, some of the less able candidates could have helped their case by taking more care with the presentation of their work by giving more detailed explanations of what principles they were attempting to use. With the exception of **Question 2 (i)**, it was possible for all candidates, except the weakest, to make progress in all the questions. All the evidence pointed to the fact that candidates had sufficient time to tackle all the questions to the best of their ability.

Yet again, a large proportion of candidates, of all abilities, carelessly threw away marks through a cavalier treatment of the accuracy required for answers which were not exact. The Instructions on the front cover of the question paper clearly state that 3 significant figure accuracy is required, or 1 decimal place in the case of angles in degrees. Working with figures corrected to 3 significant figures does not necessarily mean that the final answer will also be correct to this accuracy. For instance in **Question 2 (ii)**, if the moment of *CDE* about *AB* is taken to be 19.6×12.1, the final answer is 6.9994.., which does not round to the required answer 7.01 cm. Or again in **Question 4 (ii)**, stating that $X = 2800\sin 16.3°$ leads to $X = 785.86..$ as opposed to the correct answer $X = 784$ N. In all calculations candidates should work with the best values given by their calculators and then round their final answer only to the required accuracy.

Another cause for concern was the lack of diagrams in the scripts. Candidates were obviously using the given diagrams on the question paper. For instance in **Question 4**, a large number of candidates, who did draw a diagram, had the force required in part **(i)** as a vertical force acting where the rod joined the beam, whilst others had components of this force at the point where the rod joined the wall. Even those who did have the force acting along the rod often had it in the wrong direction. When the opening line of a diagramless solution was of the form "Taking moments about *O*, $F \times 0.7$ = etc…", Examiners had little opportunity to give any credit for correct methods, as they had no idea where *O* and *F* were. In the statics questions the identification of which forces were acting on a body at rest, together with the directions of these forces, seemed to be a major stumbling block with many candidates. Some of the difficulties experienced will be commented on later in this report.

Comments on specific questions

Question 1

This question was answered very well by the better candidates but there was a lot of failure by many of the remainder who took moments about *M* and then omitted the upward force of the platform on the plank (1100 N). Some other candidates included in their equations the moments of the upward reactions of the plank on the man and on the child. This could not be correct as these forces were not acting on the beam.

It would probably have surprised a considerable number of candidates to know that it was possible to put nine forces on the diagram, but in practice the important ones were the four which were acting on the plank, when it was in an equilibrium position. It is essential for candidates to be aware of all the forces acting on a body in equilibrium, even though some of them may not appear in a derived equation.

It is perhaps worth mentioning that, strictly speaking, the moment $75g \times 0.9$ about the edge of the platform was not the moment of the weight of the man but the moment of the downward reaction of the man on the plank. Through a combination of Newton's Third Law of Motion and the fact that the man was in equilibrium, this reaction also had a magnitude of $75g$ Newtons.

Answer: 3.16 metres.

Question 2

Surprisingly only a limited number of candidates obtained the correct answer in part **(i)**. Most incorrectly substituted $\alpha = \frac{1}{2}\pi$ into the given formula provided on the Formula List MF9 and deluded themselves into thinking that this was the correct approach. Even those who used $\alpha = \frac{1}{4}\pi$ could not make the next step to get the distance from *CE*. Apart from the error already mentioned in the **General comments**, the remainder of the question was well done by the abler candidates. The most frequent errors by the rest were either having the incorrect distance of the centre of mass of *CDE* from *AB*, or to have the wrong area of the quarter circle.

Answer: **(ii)** 7.01 cm.

Question 3

A large number of candidates, of all abilities, failed to obtain maximum marks in this question. Candidates did not seem to appreciate that when a calculus form of acceleration was required to solve the problem, the direction of the acceleration was in the direction of x increasing. Hence, when Newton's Second Law of Motion was applied to set up the differential equation, it was necessary to have a negative sign in front of the given force. Apart from that, most candidates knew how to proceed with this type of question, despite a sprinkling of integration and calculation errors. At the end of the question it was disappointing to see many candidates stating a value for v from $v^2 = -0.75$ rather than retracing their steps to find the lost minus sign.

Answer: Speed of $P = 0.866$ ms^{-1}.

Question 4

This question exposed the weaknesses of many candidates in their lack of understanding of the nature of forces and how they affect different parts of the system. A light rod could be thought of as a rigid string with the forces acting along the length of the rod. In this question the force would have been an upward thrust to counterbalance the other three downward forces acting on the beam. Those candidates who had the force in the rod directed downwards should have realised that equilibrium of the beam was impossible if all the forces acting on it were directed downwards. Many candidates thought that the force required was a vertical force at the point where the rod joined the beam, whilst others used the components of the force at the point where the rod joined the wall. In the latter case these components were, of course, the forces of the wall on the rod and had nothing to do with the equilibrium of the beam. Candidates did not appear to appreciate that, if the equilibrium of the beam was being considered, then only the forces acting on the beam had to be taken into consideration.

It addition to the comments already made in the **General comments**, other errors were: the omission of the weight of the beam in the moments equation, taking the weight of the beam to be 680 N and assuming that Y was merely the component of the force in the rod.

Answers: **(i)** 2800 N; **(ii)** $X = 784$, $Y = 1870$.

Question 5

There was a good response to this question as most of the candidates realised that the required equation was going to be found from a consideration of energy. It was fortuitous for many that the answer was given as there was a lot of evidence of backtracking, with some of it of a very dubious nature, in order to justify a factor of 2 multiplying the EPE of one string.

An alternative approach was to apply Newton's Second Law of Motion and then solve the resulting differential equation. Unfortunately, it was very rare to see the correct initial conditions of $v = 8g$ when $x = 0$.

There is still a minority of candidates who persist in taking $g = 9.8$ or 9.81 despite the instruction on the front cover of the question paper. Here they often stubbornly stuck to it even though the given equation in the question could only be obtained by taking $g = 10$ ms^{-2}.

Answer **(ii)** $x = 4$.

Question 6

There was a good response to the first part of this question with many all correct solutions. The usual approach was to first substitute into the trajectory equation and solve the resulting equation in V^2. Most of the errors were due to the careless manipulation of this equation to find V. Some of the weaker candidates used some of the standard formulae based on the incorrect assumption that the highest point of the projectile path was 2 m.

In part **(ii)** there were many solutions with the starting point $\dot{y}_M{}^2 = (V\sin 35°)^2 - 2g \times 2$ leading to $\dot{y}_M = 7.67$. Candidates then assumed that this was the speed which the question required and then stated that the particle was moving upwards, ignoring completely that the solution of the equation was $\pm$ 7.67. This occurred so often that it is difficult to believe that so many candidates could all carelessly misread the same question. There were in fact a number of ways to find whether the particle was moving up or down at M. The most popular one from those who could do this part of the question was to use $\dot{y}_M = V\sin 35° - gT$ to give $\dot{y}_M = -7.67$. Other methods were to compare T with the time taken to reach the highest point, or to compare 25 m with half the range of the projectile.

Answers: **(i)** $V = 17.3$ ms^{-1}, $T = 1.76$ seconds; **(ii)** Downwards at 16.1 ms^{-1}.

Question 7

Candidates of all abilities coped well with part **(i)** of this question which was a straightforward example of circular motion. Errors that did occur were usually either of a trigonometric nature (confusion of sine or cosine), or having the wrong value of the radius of the circle (0.15 or 0.15tan 60°).

Less able candidates experienced more difficulty with part **(ii)**. A frequent wrong approach to **(a)** was $T\cos 45° = 5$. Paradoxically, these candidates then went on to give a correct equation involving their value of T and the normal force exerted by the surface on the particle. There were a number of solutions which had the required force perpendicular to the string which demonstrated, yet again, the uncertainty that candidates have concerning the nature of forces acting on a body. Those were errors of a mechanical nature, but many candidates who had the right ideas lost marks through carelessness by either retaining the same radius of the circle as in part **(i)** or using 0.11 as an approximation for 0.15sin 45°.

Answers: **(i)** $v = 1.5$; **(ii)(a)** 5.4 N, **(b)** 1.18 N.

Paper 9709/06

Paper 6

General comments

This paper produced a wide range of marks. Many Centres however, entered candidates who had clearly not covered the syllabus and this was reflected in the performance of these candidates.

Most candidates answered questions to a suitable degree of accuracy, and it was pleasing to observe that only a few lost marks due to premature approximation.

Candidates seemed to have sufficient time to answer all the questions, and only the weaker ones answered questions out of order. Candidates from some Centres did not appear to know anything about the normal distribution. The use of clear diagrams in answering these questions would have helped many candidates to earn more marks, as many found the wrong area for the probability.

Comments on specific questions

Question 1

This question was well done by nearly everyone. There are still some Centres who do not teach candidates to use SD mode on their calculators, and so there were pages of working for the standard deviation when all that was required was a single number from their calculator. It is to be hoped that candidates realise that when only one mark is given, they are not expected to do pages of working. The second part was well answered, with many candidates having a good knowledge of the relationship between consistency and standard deviation.

Answers: **(i)** 139, 83.1; **(ii)** team B, smaller standard deviation.

Question 2

This question was a little unusual in that giving the data in the form of quartiles could have been represented by a box-plot. However, most candidates drew a credible cumulative frequency curve. A cumulative frequency polygon was also acceptable, as were percentage curves/polygons. However, some did not label their axes, and many chose inappropriate scales in order to fill the page completely. There were scales going up in, for example, 64 or 32 or 8 or 15. While the use of these scales was not penalised, these scales invariably meant that points were plotted wrongly and thus candidates did lose marks. Most candidates realised that the cumulative frequency values were 'less than' and so subtracted to find the number of people 'more than'.

Answer: **(ii)** Between 40 and 70 if a curve was drawn, or between 60 and 70 if a polygon was drawn.

Question 3

Unfortunately this question was completely misunderstood by a large number of candidates who gave a probability of $\frac{1}{6}$ for everything. Most knew what E(X) meant and were able to pick up a mark here, providing that their probabilities in part **(i)** were less than 1.

Answers: **(i)**

x	1	2	3	4	5	6
P($X = x$)	$\frac{11}{36}$	$\frac{9}{36}$	$\frac{7}{36}$	$\frac{5}{36}$	$\frac{3}{36}$	$\frac{1}{36}$

(ii) E(X) = $\frac{91}{36}$ = 2.53.

Question 4

The first part of this question was straightforward and most candidates who had covered the normal distribution performed well on it. Continuity corrections in this part gained no marks. The second part gave candidates an opportunity to show their understanding. It involved having to think through a small problem and plan how to solve it, and was well done by the good candidates.

Answers: **(i)** 0.203; **(ii)** 481.

Question 5

This question was the worst attempted on the whole paper. After the last two years' excellent permutation and combination solutions, it was disappointing to find that candidates felt obliged to put in some permutations, combinations or factorials where in fact straight multiplication of the options was all that was required. Very few candidates read the small print at the foot of the menu, about salad and either new potatoes or french fries, and thus missed a factor of 2. Part **(b)** was the best attempted part of this question.

Answers: **(a)(i)** 90, **(ii)** 69; **(b)** 252 252.

Question 6

A large majority of candidates gained full marks for this question, and most managed to draw a respectable tree diagram.

Answers: **(ii)** 0.247; **(iii)** $\frac{5}{19}$ = 0.263.

Question 7

This proved to be a good source of marks for those candidates who were familiar with the normal distribution and its approximation to the binomial. Some candidates thought that 'at most 2' meant 'exactly 2' or 'at least 2' and so lost a couple of marks. However, on the whole, this question was well done. In part **(iii)** the continuity correction was often used, although not always correctly. A surprising number of candidates failed to find the correct area in calculating the probability; a diagram would have helped.

Answers: **(i)** 0.398; **(ii)** 9; **(iii)** 0.972.

Papers 8719/07and 9709/07

Paper 7

General comments

This was a reasonably well attempted paper with candidates able to demonstrate and apply their knowledge on the topics examined. **Questions 3** and **7** were particularly well attempted, with even the weakest of candidates scoring well. The initial questions caused a variety of problems for some candidates, and lack of rigour in solutions was evident, particularly in **Question 1**. Timing did not appear to be a problem, with most candidates offering solutions to all questions. It was particularly pleasing to note this time that fewer marks were lost by candidates due to premature approximation and inability to round answers to three significant figures; candidates were, in general, more successful in adhering to the specified accuracy.

Comments on specific questions

Question 1

This was not, in general, a well answered question. Some candidates were able to correctly define their null and alternative hypotheses, though two-tailed tests, $p = \frac{22}{60}$ and even $\mu = \frac{1}{4}$ or $p = 15$ were commonly noted. Very few candidates found the correct value of the test statistic (1.938). Errors included failure to use a continuity correction, and in many cases an incorrect denominator was used with the factor 60 omitted. The critical value of 1.645 should then have been compared with the test statistic. In some cases it was not clear that this comparison had been done, hence marks were lost through lack of rigour. There was also much confusion in making the final conclusions. Some candidates correctly rejected the null hypothesis but then stated that her claim was not justified and she did not have a special method. Candidates must make careful conclusions (related to the question), as contradictions will negate marks even if the correct statement 'reject H_0' is seen.

Answers: **(i)** H_0: $\mu = 15$ or $p = 0.25$, H_1: $\mu > 15$ or $p > 0.25$; **(ii)** Claim justified.

Question 2

This was, again, not a particularly well attempted question. In part **(i)** many candidates correctly found the mean, though $\frac{9.5}{3}$ was the incorrect answer most often seen. Another common error was to calculate the standard deviation by adding 0.3, 0.25 and 0.35. Some candidates correctly squared the given standard deviations and found the variance as 0.275 but then failed to square root this, and thus did not get full marks as the question asked for the standard deviation to be given.

Candidates attempts to standardise were varied with much confusion between methods, for instance numerators of $36 - 9.5$ or $9 - 38$ were frequently seen, whereas $\frac{9-9.5}{\sqrt{0.06875}}$ or $\frac{36-38}{\sqrt{1.1}}$ was required. Inconsistent denominators were also seen.

Some candidates raised their probability to the power of 4 or multiplied it by 4, again demonstrating a lack of understanding.

Answers: **(i)** Mean = 9.5, Standard deviation = 0.524; **(ii)** 0.972.

Question 3

This question was particularly well attempted with many candidates gaining full marks. However, some candidates in part **(i)** carelessly calculated $2\times8 - 3\times6$ as $18 - 18$ and some changed a correct '–2' into '+2'. Errors in part **(ii)** included omitting to square 3 and 2, or subtracting the variances rather than adding them, resulting in a negative variance which was, surprisingly, not questioned by candidates. Most candidates realised that the variance of *Y* was 6, although a zero variance was occasionally seen.

Answers: **(i)** –2; **(ii)** 73.2.

Question 4

Most candidates correctly found the unbiased estimate of the population mean, but Examiners noted many errors in calculating the unbiased variance. A large number of candidates quoted a correct formula but then used 375.3 rather than 3753 for $\sum x$, and some candidates merely calculated the biased estimate. Premature approximation here caused large errors in the final answer.

Part **(ii)** was not well attempted with few candidates able to calculate a confidence interval for a proportion. There seemed to be some confusion by candidates with the method for calculating a confidence interval for the population mean, and even in cases of a correct numerical answer, μ rather than *p* was seen. Incorrect values for *z* were commonly used (e.g. 1.751 and 2.54).

Answers: **(i)** 375.3, 8.29; **(ii)** $0.133 < p < 0.247$.

Question 5

In part **(i)** the majority of candidates attempted the question by starting with the value of 52.74, and a reasonable level of success was achieved. The more straight-forward method, was to start with 0.1 and set up an equality/inequality $\frac{c-54}{3.1\sqrt{10}} = -1.282$ leading to $c = 52.74$. Candidates who used this method were generally successful though omission of $\sqrt{10}$ was common.

In part **(ii)** many candidates were unable to identify the outcome for a Type II error. The ability to quote what is meant by a Type II error was evident, but its application to the given situation was not. The main error seen was to use the value 54.

Despite these common errors, it was pleasing to note that attempts at this type of question are, in general, improving.

Answer: **(ii)** 0.103.

Question 6

Most candidates correctly used a Poisson Distribution for this question, though incorrect means were seen throughout the question. Part **(i)** was well attempted, as was part **(ii)**, though the usual problems with the interpretation of 'at least' were evident. Part **(iii)**, however, was not well attempted and few candidates successfully reached the final answer. Errors included adding P(1) and P(4) rather than multiplying, not dividing by P(5) and using incorrect means. An answer of 0.155 caused by premature approximation was occasionally noted.

Answers: **(i)** 0.161; **(ii)** 0.475; **(iii)** 0.156.

Question 7

This was a very well attempted question with many candidates scoring full marks. Examiners were particularly pleased to find that full working out was shown, in the majority of cases, in part **(i)** where the question required the given value of *c* to be shown. The main error noted by Examiners occurred in part **(iii)** where there was confusion between 'mean' and 'median'.

Answers: **(ii)** 0.576; **(iii)** $\frac{8}{3}$.