

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level

MATHEMATICS

9709/02

Paper 2 Pure Mathematics 2 **(P2)**

May/June 2005

1 hour 15 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **3** printed pages and **1** blank page.



1 Solve the inequality $|x| > |3x - 2|$. [4]

2 (a) Use logarithms to solve the equation $3^x = 8$, giving your answer correct to 2 decimal places. [2]

(b) It is given that

$$\ln z = \ln(y + 2) - 2 \ln y,$$

where $y > 0$. Express z in terms of y in a form not involving logarithms. [3]

3 The sequence of values given by the iterative formula

$$x_{n+1} = \frac{3x_n}{4} + \frac{2}{x_n^3},$$

with initial value $x_1 = 2$, converges to α .

(i) Use this iteration to calculate α correct to 2 decimal places, showing the result of each iteration to 4 decimal places. [3]

(ii) State an equation which is satisfied by α and hence find the exact value of α . [2]

4 The polynomial $x^3 - x^2 + ax + b$ is denoted by $p(x)$. It is given that $(x + 1)$ is a factor of $p(x)$ and that when $p(x)$ is divided by $(x - 2)$ the remainder is 12.

(i) Find the values of a and b . [5]

(ii) When a and b have these values, factorise $p(x)$. [2]

5 (i) By differentiating $\frac{1}{\cos \theta}$, show that if $y = \sec \theta$ then $\frac{dy}{d\theta} = \sec \theta \tan \theta$. [3]

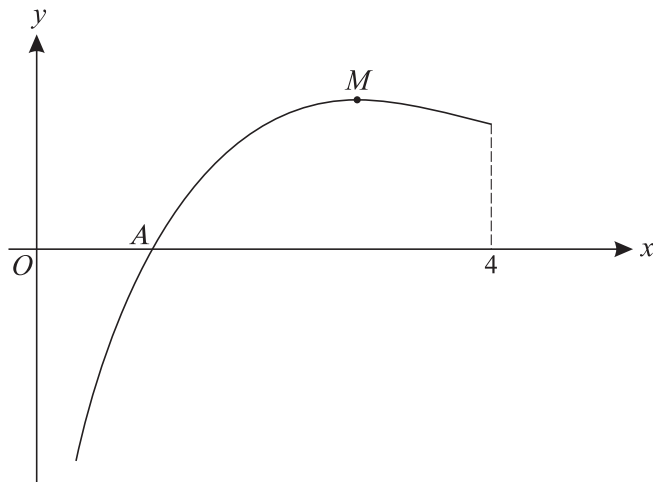
(ii) The parametric equations of a curve are

$$x = 1 + \tan \theta, \quad y = \sec \theta,$$

for $-\frac{1}{2}\pi < \theta < \frac{1}{2}\pi$. Show that $\frac{dy}{dx} = \sin \theta$. [3]

(iii) Find the coordinates of the point on the curve at which the gradient of the curve is $\frac{1}{2}$. [3]

6



The diagram shows the part of the curve $y = \frac{\ln x}{x}$ for $0 < x \leq 4$. The curve cuts the x -axis at A and its maximum point is M .

(i) Write down the coordinates of A . [1]

(ii) Show that the x -coordinate of M is e , and write down the y -coordinate of M in terms of e . [5]

(iii) Use the trapezium rule with three intervals to estimate the value of

$$\int_1^4 \frac{\ln x}{x} dx,$$

correct to 2 decimal places. [3]

(iv) State, with a reason, whether the trapezium rule gives an under-estimate or an over-estimate of the true value of the integral in part (iii). [1]

7 (i) By expanding $\sin(2x + x)$ and using double-angle formulae, show that

$$\sin 3x = 3 \sin x - 4 \sin^3 x. \quad [5]$$

(ii) Hence show that

$$\int_0^{\frac{1}{3}\pi} \sin^3 x \, dx = \frac{5}{24}. \quad [5]$$

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