MATHEMATICS

Paper 9709/01
Paper 1

General comments

In general this paper was well received by the majority of candidates and there were some excellent scripts. Virtually all questions were accessible to most candidates, though **Questions 1**, **8** and **9(i)** proved to be more taxing. The standard of presentation was generally good, though there were still many scripts in which Examiners had to work hard to find answers. Centres and candidates should be reminded of the difficulties caused when a page is divided into two columns and separate questions answered in each column.

Comments on specific questions

Question 1

This question was poorly answered. Knowledge of the surd values for sin 60° and sin 45° are in the syllabus and candidates should realise that giving decimal answers to questions requiring exact values will not earn full marks. It was disappointing that many candidates failed to sketch the triangle *ABC* and consequently obtained relatively incorrect angles which were then used either in the sine rule or by using two right-angled triangles.

Answer: $6\sqrt{6}$ or equivalent surd form.

Question 2

The trigonometric identities $\tan x = \dfrac{\sin x}{\cos x}$ and $\sin^2 x + \cos^2 x = 1$ were accurately used and the majority of candidates correctly solved the identity in part **(i)**. In part **(ii)**, a surprising number of candidates attempted to solve $2\cos^2 x + 3\cos x - 2 = 3$, though in general most candidates obtained $\theta = \cos^{-1}\left(\dfrac{1}{2}\right)$ and realised that $\cos^{-1}(-2)$ led to no further solutions.

Answer: **(ii)** 60º, 300º.

Question 3

Part **(i)** was very well answered with only a few candidates experiencing problems with using x^2 instead of x in the binomial expansion. Part **(ii)** proved to be more difficult with many candidates using $(1+x^2)^2$ as either $(1+x^2)$ or as $(1+x^4)$. Most candidates realised the need to consider more than one term in evaluating the coefficient of x^4.

Answers: **(i)** $32 + 80x^2 + 80x^4$; **(ii)** 272.

Question 4

This proved to be a source of high marks for most candidates. Apart from a few elementary algebraic errors, candidates had little difficulty in eliminating *y* and solving the resultant quadratic equation correctly, though many wasted time in giving both the *x*- and *y*-values of the points of intersection. Similarly in part **(ii)**, most realised the need to set the differential of the curve to zero or to complete the square and set the bracket $(x - 2)$ to 0.

Answers: **(i)** 2, $1\frac{1}{2}$.

Question 5

In part **(i)**, most candidates realised the need to use trigonometry to evaluate angle *POT* and Pythagoras' Theorem to find *OT* and hence *QT*. Common errors were to use angle *OTP* instead of angle *POT*, to use the incorrect trigonometric ratio or surprisingly to evaluate 13 − 5 as 7. Use of the formulae $s = r\theta$ in part **(i)** and $A = \frac{1}{2}r^2\theta$ in part **(ii)** was generally sound, though a significant number of candidates failed to realise the need to express the angle in radians and not degrees. In both parts **(i)** and **(ii)**, many candidates expressed 1.176 radians as 1.2 radians and obtained inaccurate answers.

Answers: **(i)** 25.9 cm; **(ii)** 15.3 cm².

Question 6

It was pleasing to note that, unlike previous years, the vast majority of candidates correctly recognised the notations of f ′ and f^{-1}. In part **(i)**, most candidates correctly used the chain rule, though the answers of $3(3x + 2)^2$ and $9(3x + 2)^2 - 5$ were common. Whilst many candidates scored the last mark by stating that the function was increasing because f ′(*x*) was positive, very few actually stated that $(3x + 2)^2 > 0$ and therefore f was an increasing function. The most common error was to substitute one or more values and to state that because f was positive for these values, then it was always positive. Apart from the occasional sign or algebraic error, the expression in part **(ii)** for $f^{-1}(x)$ was very well done. Very few candidates however realised that the domain of f^{-1} was the same as the range of f, and that since $x > 0$, the range of f was the set of values greater than or equal to 3.

Answers: **(i)** $9(3x + 2)^2$; **(ii)** $\frac{\sqrt[3]{x+5}-2}{3}$, $x > 3$.

Question 7

It is pleasing to note that there was very little confusion over whether to use arithmetic or geometric progressions. Parts **(i)** and **(ii)** were nearly always correctly answered, thought the premature approximation of expressing $\frac{2}{3}$ as 0.67 or as 0.7 affected the final answers. Part **(iii)** presented more difficulty with many candidates failing to realise the need to find the second and third terms of the geometric progression (54 and 36 respectively), before expressing these as '*a*' and '*a* + 3*d*'. The use of the sum of 10 terms was generally accurate.

Answers: **(i)** $\frac{2}{3}$; **(ii)** 243; **(iii)** 270.

Question 8

This question proved to be difficult for many candidates. It was pleasing that only a very small number of candidates confused fg with gf or took fg to mean f $\times$ g. Whilst most candidates realised the need to use the discriminant on a quadratic equation, the algebra usually proved too difficult for them. Common errors were to take $4\left(\frac{9}{2-x}\right)=\frac{36}{8-4x}$ or to express $\frac{36}{2-x}-2k=x$ as $36-2k=x(2-x)$. A surprising number overlooked the 'x' on the right-hand side of the equation and then attempted to use '$b^2 - 4ac$' on the resulting linear equation. The algebraic errors of either expressing $-2x + 2kx$ as $-x(2 + 2k)$ or $-4k + 36$ as $-(4k + 36)$ were very common. A considerable number of solutions took the discriminant as positive rather than zero and obtained a range of values for k. In part **(ii)**, the fact that each value of k led to an equation with equal roots was lost on most candidates. Expressing the roots as factors instead of numerical values was a further error.

Answers: (**i**) 5 or –7; (**ii**) $x = -4$ or 8.

Question 9

Part **(i)** caused a lot of problems. Many candidates ignored the instruction 'by integration' and attempted to work backwards. The integration of $-kx^{-3}$ was often expressed as $-\frac{1}{2}kx^{-2}$ or as $\pm kx^{-2}$ and in many attempts the constant of integration was omitted. Those candidates who realised that the substitution of (1, 18) and (4, 3) led to two simultaneous equations for k and c were generally successful. Part **(ii)** presented fewer problems and the integration required was accurately done. Most candidates used limits correctly though sign errors in evaluating $(-10 + 3.2) - (-16 +2)$ were common.

Answer: **(ii)** 7.2.

Question 10

Although this question was well answered, part **(i)** presented most difficulty with many candidates taking the scalar product as ±1 instead of 0. The arithmetic manipulation in obtaining an angle of 40º in part **(ii)** was impressive. Part **(iii)** caused some difficulty with some candidates still taking vector $\overrightarrow{AB}$ as $\mathbf{a} - \mathbf{b}$ instead of $\mathbf{b} - \mathbf{a}$ and others failing to realise the need to find the modulus of vector $\overrightarrow{AB}$ before equating to 3.5. Many candidates obtained the equation $(p - 2)^2 = 1.5^2$ and then obtained $p - 2 = 1.5$ instead of $p - 2 = \pm 1.5$.

Answers: **(i)** –2; **(ii)** 40°; **(iii)** 0.5 or 3.5.

Question 11

Part **(i)** proved to be a straightforward question for most candidates, though common errors were to take the product of the gradients of perpendicular lines as 1 instead of −1 or to take the gradient of a line as 'x-step ÷ y-step'. Part **(ii)** was poorly answered (or ignored) by a large number of candidates who failed to realise that X(4, 6) was the mid-point of BD. Many candidates misread (or misinterpreted) 'kite' as 'parallelogram' and found D as (14, 6). Part **(iii)** was usually well done, with most candidates realising that the answer did not depend upon part **(ii)**. Premature approximation of the lengths of AB and BC before doubling and adding often led to the loss of the final accuracy mark.

Answers: **(i)** (4, 6); **(ii)** (6, 10); **(iii)** 40.9.

MATHEMATICS

Paper 9709/02
Paper 2

General comments

The overall standard of scripts was high and a noticeable improvement on the quality from previous sessions was evident. Only **Question 2** proved very difficult for the majority of candidates. Other questions produced an excellent response in some cases and an acceptable one in others. Examiners again wish to to stress the vital importance of working through previous papers with a view to candidates familiarising themselves with the nature of the questions that can be expected in future. Poorer candidates struggled basically due to poor manipulative skills and lack of understanding of the key rules and results of differentiation and integration.

Candidates work was generally clearly and neatly set out, thus helping Examiners to accurately allocate marks. There were no signs of candidates lacking time to finish the paper.

Comments on specific questions.

Question 1

Many among those who adopted the method of squaring each side of the inequality failed to do so on the right hand side. Others could spot the case $x < 1$ but could proceed no further. Some candidates, who presented otherwise good solutions, showed some uncertainty at the end of the question as to the direction of one or both inequality signs. The simplest technique is to take one simple case, e.g. $x = 0$, and see if this value satisfies the initial inequality; if it does so, it must be included in the solution set.

Answer: $-\frac{1}{3} < x < 1$.

Question 2

Almost every candidate successfully took logarithms of the left hand side, but very few could do so on the right hand side; expressions such as $x\ln 6$ were extremely common, rather than the correct ($\ln 2 + x\ln 3$). Those who wrote $x\ln 4 = x\ln 6$ failed to realise that this implied (incorrectly) that $x = 0$; instead they contrived to obtain a numerical non-zero value for x. The Examiners stress that more time and effort needs to be put into practising this type of problem.

Answer: 2.41.

Question 3

Most candidates obtained, on integration, a linear combination of sin $2x$ and cos x, though some made sign errors or incorrectly found $2\sin 2x$; the correct indefinite integral was ($\frac{1}{2}\sin 2x - \cos x$). The question asked for an exact value of the definite integral, but many candidates used approximate values for $\frac{1}{2}\sin\left(\frac{\pi}{3}\right)$ and $\cos\left(\frac{\pi}{6}\right)$.

Answer: $1 - \frac{1}{4}\sqrt{3}$.

Question 4

This was done better than any other question, with almost all candidates very familiar with the procedures required. A few evaluated p(+1) and p(+2) instead of the correct p(–1) and p(–2), with others equating p(–2) to zero rather than 5.

A few others obtained the two correct simultaneous equations in *a* and *b*, but made sign or numerical errors in solving them.

Answers: 2, –3.

Question 5

(i) This was generally well done, though a few sign errors occurred in the forms for $R\cos\alpha$ and $R\sin\alpha$. Others obtained tan α = 5 instead of the correct 0.2. Often an approximate value 5 was gived for *R*; the question aksed for the exact value.

(ii) Most candidates obtained one sensible value for θ based on their *R* and α values from part **(i)**. However the majority thought that there was only one value for θ, or added 180° to their first θ-value. The correct process takes account of the basic feature that if a cosine takes on a positive value, then this produces values in the first and the fourth quadrant for the corresponding angle

Answers: **(i)** $\sqrt{26}\cos(\theta + 11.31°)$; **(ii)** 27.0°, 310.4°.

Question 6

(i) Almost everyone successfully differentiated *y* to obtain y' = (x – 1)e^x, but many candidates could not deduce that *x* = 1 is the only valid solution to (x – 1)e^x = 0.

A few used the approximate corresponding value *y* = –2.72 instread of the (requested) exact value.

(ii) Here one can argue (a) from *y*-values to the left and right of *x* = 1, or (b) do likewise with the values of y', or (c) obtain $y''(1)$ and deduce from its sign if a minimum or a maximum occurs. Examiners were pleased to see all three techniques successfully used and especially by the high number of correct expressions for $y''(x)$.

Answers: **(i)** (1, –e); **(ii)** minimum.

Question 7

(i) This was well handled bar the occasional sign error, e.g. the derivative of $-xy$ put as $-xy' + y$. The weaker candidates, though few in number, did no differentiation whatever.

(ii) Here y' = 0 so *x* = 2*y*. Other put both 2*y* – *x* and *y* – 2*x* equal to zero. Many candidates realised that *x* = 2*y* was crucial, but then come to a halt instead of substituting this relation back into the equation of the curve to obtain x^2 = 4 or y^2 = 1.

Answers: **(ii)** (2, 1) and (–2, –1).

Question 8

(i) Only a few candidates could not successfully integrate both parts of the integrand to obtain $\frac{1}{2}x^2 + \ln x$, but a few lost a factor of 2 in one term when simplifying. This part was very well done.

(ii) Few candidates realised that it is necessary to form a function $f(a) = a - \sqrt{(13 - 2\ln a)}$, or $g(a) = a^2 - (13 - 2\ln a)$, and to evaluate f(3) and f(3.5), or g(3) and g(3.5). The two values differ in sign, proving that the root of f(*a*) = 0 (or g(*a*) = 0) lies between 3.0 and 3.5.

(iii) This was very well done, though some iterated only 3 times (4 iterations are needed) or failed to round off their final a_4 = 3.2613. Some candidates worked only to 2 or 3 decimal places when iterating.

Answer: **(iii)** 3.26.

MATHEMATICS

Paper 9709/03

Paper 3

General comments

The variation in the standard of work on this paper was considerable and resulted in a wide spread of marks. This proved to be a challenging paper for many candidates. However, well prepared candidates appeared to have sufficient time to answer all questions and no question seemed to be of undue difficulty. The questions or parts of questions that were done well were **Question 3** (iteration) and **Question 4** (trigonometry). Those that were done least well were **Question 5** (complex numbers), **Question 7** (partial fractions), **Question 8(i)** (differential equation), **Question 9(ii)** (integration) and **Question 10(ii)** (vector geometry). Overall the main weakness was in the algebraic work. Marks were lost not only because of errors in manipulation, but also because of the use of methods that were unsound or incorrect. For example, in **Question 7**, the majority of candidates attempted to find constants A and B such that the linear expression $A(x+3)+B(x+1)$ is identically equal to the quadratic expression x^2+3x+3; an impossible task.

In general the presentation of work was good but there are still candidates who present their work in a double column format. This makes marking difficult for Examiners and it would be helpful if Centres could continue to discourage the practice. Secondly, though the rubric for the paper informs candidates of 'the need for clear presentation in your answers', there are some who do not show sufficient steps or make clear the reasoning that leads to their answers. This occurs when they are working towards answers or statements given in the question paper, for example as in **Question 4(ii)** and **Question 7(ii)**, but also when candidates state the solution to a question without showing the method by which they arrived at it, for example as in **Question 2**. The omission of essential working may result in the loss of marks.

Where numerical and other answers are given after the comments on individual questions, it should be understood that alternative forms are often acceptable and that the form given is not necessarily the sole 'correct answer'.

Comments on specific questions

Question 1

Some candidates answered this well. The most common error was to omit the squaring of 3 when forming a non-modular quadratic inequality or equation. Though this leads to irrational critical values, it did not seem to deter candidates from continuing further. Some candidates, having found the correct critical values, and in some cases having drawn a sketch graph on which the required interval was clear, nevertheless gave the solution as $-\frac{1}{7}<x<-1$.

Answer: $-1<x<-\frac{1}{7}$.

Question 2

There were some good solutions but many candidates failed to see the problem as solving an equation in e^x. Application of the erroneous rule ln(a + b) = ln a + ln b led some to state $x + 2x = 3x$, after which they gave up. Amongst those who found the quadratic in e^x there were some who omitted the working leading to their final answer for x. Also those candidates who prematurely rounded the positive root of the quadratic to 1.62 and obtained x = 0.482, lost the final mark.

Answer: 0.481.

Question 3

This was generally well answered. In **(i)** the solutions varied considerably in length but often were successful. Part **(ii)** proved accessible to most candidates and correct solutions were common. A few candidates attempted the iterations with their calculators in degree mode rather than in radian mode and some did not pay sufficient attention to the question's requests regarding accuracy.

Answer: **(ii)** 0.76.

Question 4

Most candidates made a correct start to part **(i)** but errors in the subsequent algebraic work were frequent. Those who planned their approach to the given answer, in particular those who quickly removed $\sqrt{3}$ from all denominators, tended to have the most success. In part **(ii)** candidates generally had the right approach but many merely found the acute angle, and there were cases where incorrect rounding led to 24.8° and 95.2° as answers.

Answer: **(ii)** 24.7°, 95.3°.

Question 5

This was very poorly answered. In part **(i)** most candidates failed to demonstrate that the modulus of $z - \mathrm{i}$ was 2. The Argand diagram sketches were occasionally correct but usually the attempt at a drawn circle suffered from having the wrong centre and/or the wrong radius. Very few candidates made a sensible start on part **(ii)** and Examiners rarely saw a completely correct solution explicitly identifying $\frac{1}{4}$ as the real part. Some candidates were clearly under the incorrect impression that it was sufficient to verify the result for one or two specific substituted values of θ.

Question 6

This was fairly well answered. Many candidates obtained a correct expression involving the first derivative, either by differentiating the product or by multiplying out the bracket and differentiating the two terms. However a few treated the constant a as a variable so the term $3a^2$ appeared in their working. Common mistakes after that stage were sign errors in the algebra and not going on to find the coordinates of the point where $y = -2x$. The possibility $y = 0$ was often overlooked and even when it was noted a valid reason for its rejection was almost invariably absent.

Answer: $(a, -2a)$.

Question 7

In part **(i)** the majority of candidates mistakenly took the form of fractions to be $\frac{A}{x+1} + \frac{B}{x+3}$ and seemed undeterred by the fact that in the course of their attempt to find A and B they were setting the given quadratic numerator of $f(x)$ identically equal to a linear expression. Those who equated coefficients ended part **(i)** with $f(x)$ equal to $\frac{3}{x+3}$ yet still moved on to part **(ii)**. The minority who adopted a correct form of fractions or divided and expressed the remainder as two partial fractions were nearly always successful. In part **(ii)** the integration was usually correctly done but, in the case of candidates with the correct integrals, the manipulation of logarithms needed to proceed from the correct substitution of limits to the given answer was not always given in sufficient detail for the final mark to be awarded.

Answer: **(i)** $1 + \frac{1}{2(x+1)} - \frac{3}{2(x+3)}$.

Question 8

Part **(i)** proved to be difficult for many candidates. The variable x was frequently used to represent the distance TN, and at times the angle PTN so that the gradient of the tangent at P was tan x. Part **(ii)** was answered quite well. The variables were usually separated correctly and the subsequent integrations were often correct, though some candidates could not integrate cot x. In their attempts at evaluating their constant of integration, some candidates failed to take into account the effect of manipulations of the indefinite integral on the constant. For example, the effect of multiplying the expression $-\frac{2}{y} = \ln(\sin x) + c$ by y converts it to $-2 = y\ln(\sin x) + cy$ and not to $-2 = y\ln(\sin x) + c'$.

Answer: **(ii)** $y = \frac{2}{1 - \ln(2\sin x)}$.

Question 9

In part **(i)** most candidates made a correct attempt to differentiate using the product or quotient rule. Success in finding the x-coordinate of the stationary point from a correct derivative was a test of the candidate's algebraic skills. There seemed to be a widespread reluctance to remove the factor of $e^{-\frac{1}{2}x}$ at an early stage in this piece of work. Its removal would have eased the work and helped to avoid some of the slips that were made.

The answers to part **(ii)** were generally poor. Many candidates did not know the formula for a volume of revolution. Some had incorrect limits or substituted the correct limits in the wrong order and the substitution of $x = -\frac{1}{2}$ proved troublesome at times. Some candidates organised the integration by parts untidily and made errors of sign and mistakes when transferring or copying pieces of work from place to place.

Answers: **(i)** $\frac{1}{2}$; **(ii)** $\pi(2\sqrt{e} - 3)$.

Question 10

Part **(i)** was well answered. However some candidates equated the components of a general point on the line l to the corresponding components of $\overrightarrow{AB}$ rather than to the components of a general point on the line through A and B.

Part **(ii)** proved to be more challenging. Many candidates realised that a scalar product involving 60° was required but few were able to set up a correct equation and reduce it to the given quadratic. Those who correctly decided to work with $\overrightarrow{AP}.\overrightarrow{AB}$ often spoiled their chances of success by making algebraic slips in forming the components or the magnitude of $\overrightarrow{AP}$. While most candidates solved the given quadratic in t correctly, many did not use their solution to find a position vector for P. To earn the final mark, it was necessary to explain that $t = -2$ gives the correct position vector for P because the other value $t = -\frac{1}{3}$ corresponds to the case when angle PAB = 120°. Only a few exceptional candidates earned this mark.

Answer: **(ii)** 5**i** + 3**j** + 4**k**.

MATHEMATICS

Paper 9709/04

Paper 4

General comments

The paper was generally well attempted with a significant number of candidates scoring very high marks; however many candidates scored very low marks and were clearly not ready for examination at this level.

It is disappointing to report that the work of candidates from a few Centres was extremely poorly presented. Candidates whose work is poorly presented are prone to make mistakes, and in some cases the presentation is so poor that it is impossible for Examiners to determine just what the candidate is writing.

It is clear that in **Question 6** of this paper a very significant minority of candidates worked with the 10.4 ms^{-2} and 9.6 ms^{-2} the wrong way round, and as a result could score only a maximum of 5 of the 9 marks available. These candidates were reluctant to use a complete method and thus rarely reached this maximum.

Comments on specific questions

Question 1

(i) This is a routine exercise on the use of $v^2 = u^2 + 2as$ and was answered correctly by nearly all candidates.

(ii) Although this was intended as a routine application of $a = g\sin\alpha$, the formula was used by a minority of candidates. However many candidates used the principle of conservation of energy in an appropriate way.

Answers: **(i)** 2.5 ms^{-2}; **(ii)** 14.5.

Question 2

This question was very well attempted and many candidates scored full marks.

Answer: **(ii)** 2 kW.

Question 3

Many candidates sketched a correct triangle of forces in equilibrium, and others thought of the force of magnitude F as being equal in magnitude and opposite in direction to the resultant of the other two forces. Almost all such candidates in either category were successful in scoring all five marks.

A smaller proportion of candidates who resolved forces in the 'x' and 'y' directions were completely successful. Some could not proceed beyond $F\cos\theta = 10$, $F\sin\theta = 13$ and others made trigonometrical mistakes.

Answers: 52.4, 16.4.

Question 4

(i) A very high proportion of candidates used $a = g$ in $v^2 = u^2 + 2as$ or used $h = 2.4$ in PE = mgh, each case leading to the wrong answer for kinetic energy of 19.2 J.

(ii) Most candidates recognised that the speed of P at C is the same as that at A.

(iii) Most candidates attempted to use the principle of conservation of energy. However a very large proportion had the kinetic energy at B as being equal to the loss of potential energy of P between A and B, taking no account of the fact that P is in motion at A.

Answers: **(i)** 14.7 J; **(ii)** 6.06 ms^{-1}; **(iii)** 1.36 m.

Question 5

(i) Most candidates recognised the need to write down two equations, one relating to A and one relating to B. In many cases however the equations included one or both of $T = 4$, relating to A, and $6 - T = 0.6a$ relating to B. Sometimes the frictional force was taken as 0.5 instead of $0.6g \times 0.5$.

(ii) This part was very well attempted with most candidates scoring both marks, albeit benefiting from 'follow through' for the accuracy mark in many cases.

Answers: **(i)** 1 ms^{-2}, 3.6 N; **(ii)** 2.45 s.

Question 6

(i),(ii) Some candidates answered these parts of the question by using the principle of conservation of energy, without considering the air resistance. Many candidates did use the given deceleration and acceleration correctly and scored all five marks for the two parts.

Some candidates gave the answer as 1.3 m in part **(i)**, omitting the addition of the given 6.2 m. A significant number of candidates used $u = 5.2$ instead of $u = 0$, and/or $s = 6.2$ instead of $s = 7.5$, in applying $v^2 = u^2 + 2as$ to find the required speed.

(iii) It was expected that this part would test even the best candidates, and it was pleasing to see many correct answers. Nevertheless there were many poor attempts and many candidates did not attempt this part.

Candidates adopted a very wide variety of approaches to the question. Those who looked at it from a work/energy balance point of view offered solutions in which the work done is calculated as overall PE loss (37.2 J) minus overall KE gain (35.088 J), or total initial energy (45.312 J) minus total final energy (43.2 J), or energy loss upwards (0.312 J) plus energy loss downward (1.8 J).

A few candidates adopted an energy deficit approach, repeating their calculations in parts **(i)** and **(ii)**, but without the air resistance. This process yields answers of 7.552 m and $\sqrt{151.04}$ ms^{-1}, and the energy deficit (equal to the required work done) is given by

$$\frac{1}{2}0.6(151.04 - 144) \text{ or } 0.6 \times 10 \times 7.552 - 0.6 \times 9.6 \times 7.5.$$

The most successful approach was to use Newton's second law to find the magnitude of the resistive force (0.24 N for both the upwards and the downwards motion). Unfortunately incorrect values of 6.24 N (upwards) and 5.76 N (downwards) were frequently seen, as was 12.24 N (upwards) accompanied by the correct 0.24 N (downwards).

Answers: **(i)** 7.5 m; **(ii)** 12 ms^{-1}; **(iii)** 2.11 J.

Question 7

(i) There were not only very many correct answers to this part of the question, but also incorrect answers representing a range of ways in which the question can be wrongly answered. These ways included substituting t = 10, t = 20, t = 22.5, t = 30 and even t = 80 into $-0.01t^2 + 0.5t - 1$; evaluating $\frac{1}{2}(v(10) + v(30))$ and evaluating $\int_a^b(-0.01t^2 + 0.5t - 1)\,dt$, where a and b were usually 10 and 30. Sometimes the definite integral was divided by $b - a$.

(ii) As in part **(i)** there were very many completely correct answers, but also a range of wrong answers. The most common wrong answer was obtained by first finding an indefinite integral of $v(t)$, say $s(t)$, then evaluating $s(20)$, instead of $s(30) - s(10)$, as the distance of the middle section. Another common answer for the distance travelled during the interval $10 < t < 30$ was obtained from $\frac{1}{2}(v(10) + v(30)) \times 20$. Integrating the given function $v(t)$ between 0 and 80 was also very common as an answer for the whole distance.

$\frac{1}{2}5.25 \times 10$ was frequently seen as the distance travelled during the interval $0 < t < 10$.

Answers: **(i)** 5.25 ms^{-1}; **(ii)** 233 m.

MATHEMATICS

Paper 9709/05
Paper 5

General comments

The paper proved a fair test. Most candidates worked to appropriate accuracy, although a few examples of premature approximation were seen. Only a handful of candidates used g = 9.8 or 9.81. Careless errors and misreads were rarely seen.

Many candidates drew their own diagrams to assist them with their solutions.

Questions 2(ii) and **4** were found to be the most difficult ones on the paper.

Comments on specific questions

Question 1

Most candidates attempted to use $T = F = 1.5$ and $T = \frac{\lambda x}{l}$ with $\lambda = 6$ and $l = 2$ leading to $x = 0.5$. This usually resulted in the correct answer for PA.

Apart from the sole use of x, different expressions involving x were attempted and this often caused confusion. A few candidates attempted to use energy equations.

Answer: 1.3 m.

Question 2

(i) This part of the question was generally well done. Some candidates used the wrong formula for the centre of mass. With the correct formula α was often taken as $\frac{\pi}{2}$ or 45° instead of $\frac{\pi}{4}$.

(ii) Very few candidates were able to complete this part of the question. A clear diagram would have been a good aid to solving this problem. The correct triangle to use would have been triangle AMG, where G is the centre of mass and M is the mid-point of AB.

Answer: **(ii)** 15.3.

Question 3

(i) Most candidates attempted to resolve vertically at the ring. Some errors occurred because the wrong angles had been found for ORC and ORD. Generally this part of the question was well answered. A few candidates had different tensions in the two parts of the string, not realising the tension would be the same throughout the string.

(ii) Newton's second law was often applied correctly resulting in a completely correct solution. Again errors occurred when the wrong angles had been calculated. This question was generally a good source of marks.

Answer: **(ii)** 3.93.

Question 4

(i) When taking moments about B some candidates did not see that the distance of the 25 N force was 2 m and tried to calculate it, often incorrectly. Some candidates used the horizontal and vertical components of T and only used one of them in the moment equation.

(ii) Only one triangle was considered by many candidates. $5T = 60 \times \frac{4}{3}$ was seen very frequently and not $5T = 2 \times 20 + 2 \times 25 + 60 \times \frac{4}{3}$ as required.

(iii) Again only one triangle was considered by many candidates.
Vertical component = $60 - \text{their } T \times \frac{4}{3}$ was seen instead of $120 - \text{their } T \times \frac{4}{3}$.

Answers: **(i)** 18; **(ii)** 34; **(iii)** 92.8 N.

Question 5

(i) This part was generally well done.

(ii) This part was also well done.

(iii) Often only the horizontal distance was attempted so
AB^2 = (horizontal distance)^2 + (vertical distance)^2 was never seen.

Answers: **(i)** 1.2 s; **(iii)** 13.6 m.

Question 6

(i) Most candidates applied the conservation of energy principle and many candidates gained all 4 marks. A few candidates tried to use Newton's second law but usually failed to complete the method.

(ii) Many candidates used the idea that the maximum speed occurred when the acceleration was zero. They used $T = mg = 5$ and $T = \frac{\lambda x}{l} = \frac{20x}{1.25}$ which led to $\frac{20x}{1.25} = 5$ and so $x = \frac{5}{16}$. This value was then substituted into the expression for v^2 found in part **(i)** to produce the maximum value of v.

(iii) The majority of candidates solved the equation $-32x^2 + 20x + 25 = 0$ to find $x = 1.25$ and then used Newton's second law to find the acceleration.

Answers: **(ii)** 5.30 ms^{-1}; **(iii)** 30 ms^{-2}.

Question 7

This question was a good source of marks for many candidates.

(i) Some candidates used Newton's second law but with an incorrect sign appearing. The variables were separated and an integration often resulted in a logarithmic function. Quite a number of candidates did not introduce a constant of integration.

(ii) A number of candidates could not integrate $e^{-0.4t}$. Again the constant of integration was often omitted.

Answers: **(i)** 2.75; **(ii)** 4.51 m.

MATHEMATICS

Paper 9709/06

Paper 6

General comments

This paper proved to be accessible with almost all candidates showing their ability in a positive way. There were some Centres, however, who entered candidates who had clearly not covered the syllabus and a large number of these candidates performed poorly.

Premature approximation leading to a loss of marks was evident in a few papers, especially regarding the normal distribution, but most candidates realised the necessity of working with more than 3 significant figures.

Candidates seemed to have sufficient time to answer all the questions, and few candidates answered questions out of order. Clear diagrams on normal distribution questions would have helped many candidates to earn more marks, as many used the wrong area.

Comments on specific questions

Question 1

This was a very straightforward question to start the paper and almost all candidates were able to score something. In part **(ii)** most candidates found the upper quartile to be 35 but then wrote $x = 35$ and not $x = 5$. They had not clearly understood how the stem-and-leaf diagram worked.

Answers: **(i)** 24, 16; **(ii)** 5.

Question 2

A few candidates found the large numbers confusing, and had probabilities of 0.2 million. However for most this was a straightforward question. Many candidates did not use a tree diagram but preferred to work with the numbers involved; either method was acceptable. Some candidates wrote the answer to part **(iii)** as $\frac{0.28}{0.42} = 0.67$, which is only written to 2 significant figures.

Answers: **(i)** 0.2; **(ii)** 0.42; **(iii)** $\frac{2}{3}$ or 0.667.

Question 3

Many candidates find questions on permutations and combinations difficult. There were a lot of good answers seen to part **(ii)** and rather fewer to part **(i)**. Some candidates still do not know when to add their numbers or probabilities and when to multiply.

Answers: **(i)** 2 177 280; **(ii)** 90.

Question 4

This question was poorly done by many candidates who either did not look up the *z*-value backwards or ignored it altogether. Of those who did look up the *z*-value backwards, most managed to obtain 0.674 and then solved for $z = 0.674$ instead of $z = -0.674$. However values seen were 0.675, 0.67, 0.671 and anything in between. Candidates were penalised for using the wrong *z*-value. Premature approximations here also resulted in marks being lost for the final answer.

Answers: **(i)** 8.75; **(ii)** 0.546.

Question 5

Many candidates found this question difficult. They did not realise that a histogram had no gaps. Of those who attempted to find a frequency density, many did not plot it correctly. Variations were seen using frequency $\times$ class width, class width / frequency. Some candidates did the calculations correctly but read the scale wrongly on their graph when plotting. In part **(ii)** many attempted to find the mean by adding frequencies $\times$ semi-class width instead of frequency $\times$ mid-interval. Altogether, this question proved one of the worst attempted.

Answer: **(ii)** 2.1 hours.

Question 6

It was pleasing to see that candidates generally managed to understand this question and draw correct tree diagrams.

Answers: **(ii)** $\frac{1}{2}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{16}$; **(iii)** $\frac{15}{16}$.

Question 7

Many candidates added the probabilities, and since the total came to less than 1 they felt the answer was correct. In part **(ii)** some thought that ‘at least 8’ meant ‘exactly 8’ or ‘more than 8’ or ‘fewer than 8’. In such cases credit could only be given for recognising the binomial distribution. The normal approximation to the binomial was well done by the majority of candidates with only a few not using the continuity correction. Again though, many gave the answer as 0.0442 instead of 0.956. A diagram would have helped many candidates.

Answers: **(i)** 0.00563; **(ii)** 0.526; **(iii)** 0.956.

MATHEMATICS

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Paper 7

General comments

Overall, this proved to be a reasonably straight forward paper for most candidates. **Questions 2(ii)**, **6(i)** and **(ii)** and **7** were well attempted by many candidates. **Question 1(iii)** did not prove too difficult, but parts **(i)** and **(ii)** were a good discriminator for the more able. The question that proved most problematic for candidates was **Question 5**, with many candidates only able to score on part **(i)**. Thereafter many candidates were unable to understand what was required by this question.

There were many good scripts, with few candidates appearing totally unprepared for the paper. There were a few cases of candidates not adhering to the accuracy required, but not as many as in the past. Lack of time did not seem to be a problem for candidates.

Detailed comments on individual questions follow. Whilst the comments indicate particular errors and misconceptions, it should be noted that there were also many very good and complete answers.

Comments on specific questions

Question 1

Many candidates were unable to give a full answer to part **(i)**. To simply say that it was unsatisfactory because it was taken on a train merely echoes the question itself. Candidates needed to state that the sample was *unrepresentative* of the whole population because it was taken on a train; there are adults that do not work or who do not travel on trains who would not be represented. Many candidates answered part **(ii)** totally incorrectly thinking that they needed to find a suitable *sample* for the same population rather than a suitable *population* for the given sample - thus demonstrating a lack of understanding of the word 'population'.

Part **(iii)**, however, was reasonably well attempted, with just the usual confusion between 'biased' and 'unbiased' estimates, and confusion between the two different formulas that could be used to calculate the unbiased variance. Candidates that were most successful used the formula as given in the formula list. A few candidates confused standard deviation and variance.

Answers: **(i)** Commuters are not representative of the whole population; **(ii)** People who travel to work on this train; **(iii)** 6.17, 0.657.

Question 2

It appeared that some candidates did not know what was meant by 'distribution' and 'parameters' in part **(i)** of this question. Many candidates did not state N(48.8, $\frac{15.6^2}{5}$), or equivalent, but then went on in part **(ii)** to successfully use this distribution. Candidates thus appeared to be able to successfully solve the problem, but with, perhaps, a lack of understanding of the underlying theory.

Answers: **(i)** N(48.8, $\frac{15.6^2}{5}$); **(ii)** 0.568.

Question 3

Many candidates made a fair attempt at this question. Surprisingly few made the usually common errors of considering $2 \times R$ rather than $R_1 + R_2$ in part **(i)** and $3 \times R$ rather than $R_1 + R_2 + R_3$ in part **(ii)**. Part **(i)** was generally better attempted than part **(ii)**, but on the whole this question was a good source of marks for better candidates.

Answers: **(i)** 0.938; **(ii)** 0.993.

Question 4

Some candidates gave incorrect hypotheses at the start of this question using λ instead of μ, or omitting μ completely and stating $H_0 = 3$. Most candidates correctly used a one-tail test with very few making an error with the wrong tail. It is important when doing a significance test that all working and justification of the conclusion is shown. Many candidates found the correct test statistic, but went onto state their conclusion without showing the comparison with 1.645 (or equivalent). Others did an incorrect comparison, thus invalidating their conclusion. Some candidates' conclusions were also invalidated by statements containing contradictory comments, though this was not noted by Examiners as a particularly common occurrence this time.

Part **(ii)** required understanding of a Type II error. Many candidates were able to quote a 'text book' definition, but an answer 'in context' proved too demanding for the majority of candidates.

Answers: **(i)** $H_0 : \mu = 3$, $H_1 : \mu > 3$, Not enough evidence to support the claim; **(ii)** Say no extra weight loss when there is.

Question 5

Apart from part **(i)** this was a poorly attempted question. In part **(ii)** there was much confusion. Many candidates calculated P(4) or P(⌀ 4), many used *z*-values from a normal distribution, and even for those who correctly used a Poisson distribution with $\lambda = 4$ essential working was often omitted. The question required P(0), P(1) and P(2) to be calculated, then P(0), P(0) + P(1), and finally P(0) + P(1) + P(2) to be compared with 0.1, in order to identify the rejection region. All too often candidates merely compared the individual probabilities rather than the sum with 0.1, or if candidates were comparing the sum this was not clear. Also in many cases the comparison with 0.1 was not clearly stated thus invalidating the final answer. It appeared that many candidates were not familiar with the method of finding a rejection region for a discrete distribution. This was also highlighted by the highly common incorrect answer of 0.1 in part **(iii)**. Part **(iv)** was equally poorly attempted with many candidates attempting further calculations rather than using their previous answers.

Answers: **(ii)** 0 or 1; **(iii)** 0.0916; **(iv)** 1 is in the rejection region, there is evidence that the new guitar string lasts longer.

Question 6

A Poisson distribution was correctly used by most candidates in order to find the probabilities, though errors were made in calculating the values for λ. The most common error in part **(iii)** was to use t rather that $0.8t$ for λ when setting up the equation to solve. Some candidates set up an equation using P(1) as well as P(0), leading to an equation with no solutions for t. This was a well attempted question and a good source of marks for most candidates.

Answers: **(i)** 0.144; **(ii)** 0.819; **(iii)** 2.88.

Question 7

Most candidates managed to correctly show that *k* was $\frac{1}{\ln 5}$, though some candidates submitted solutions that were rather minimal for a question where the requirement was 'to show' a given value. Very weak candidates found the integration problematic. Having dealt with the integration correctly in part **(i)**, most candidates went on to offer reasonable solutions for part **(ii)** and **(iii)**. Common errors in part **(ii)** included finding less than 3 rather than more than 3, and in part **(iii)** solution of the equation involving ln(m + 1), where m is the median, produced many errors.

Answers: **(ii)** 0.139; **(iii)** 1.24 minutes.

