UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the May/June 2008 question paper

9709 MATHEMATICS

9709/03 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the May/June 2008 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only - often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

<u>Penalties</u>

MR -1	A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures - this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA -1	This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1	**EITHER**	State or imply non-modular inequality $(x-2)^2 > (3(2x+1))^2$, or corresponding quadratic equation, or pair of linear equations $(x-2) = \pm 3(2x+1)$	B1	
		Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations	M1	
		Obtain critical values $x = -1$ and $x = -\frac{1}{7}$	A1	
		State answer $-1 < x < -\frac{1}{7}$	A1	
	OR	Obtain the critical value $x = -1$ from a graphical method, or by inspection, or by solving a linear equation or inequality	B1	
		Obtain the critical value $x = -\frac{1}{7}$ similarly	B2	
		State answer $-1 < x < -\frac{1}{7}$	B1	[4]
		[Do not condone ≤ for <; accept $-\frac{5}{35}$ and – 0.14 for $-\frac{1}{7}$.]		
2	**EITHER**	State or imply $e^x + 1 = e^{2x}$, or $1 + e^{-x} = e^x$, or equivalent	B1	
		Solve this equation as a quadratic in $u = e^x$, or in e^x, obtaining one or two roots	M1	
		Obtain root $\frac{1}{2}(1+\sqrt{5})$, or decimal in [1.61, 1.62]	A1	
		Use correct method for finding x from a positive root	M1	
		Obtain $x = 0.481$ and no other answer	A1	
		[For the solution 0.481 with no working, award B3 (for 0.48 give B2). However a suitable statement can earn the first B1 in addition, giving a maximum of 4/5 (or 3/5) in such cases.]		
	OR	State an appropriate iterative formula, e.g. $x_{n+1} = \frac{1}{2}\ln(1+e^{x_n})$ or $x_{n+1} = \frac{1}{3}\ln(e^{x_n} + e^{2x_n})$	B1	
		Use the iterative formula correctly at least once	M1	
		Obtain final answer 0.481	A1	
		Show sufficient iterations to justify its accuracy to 3 d.p., or show there is a sign change in the value of a relevant function in the interval (0.4805, 0.4815)	A1	
		Show that the equation has no other root	A1	[5]
3	**(i)**	State or imply $r = a \operatorname{cosec} x$, or equivalent	B1	
		Using perimeters, obtain a correct equation in x, e.g. $2a \operatorname{cosec} x + ax \operatorname{cosec} x = 4a$, or $2r + rx = 4a$	B1	
		Deduce the given form of equation correctly	B1	[3]
	(ii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 0.76	A1	
		Show sufficient iterations to 4 d.p. to justify its accuracy to 2 d.p., or show that there is a sign change in the value of $\sin x - \frac{1}{4}(2+x)$ in the interval (0.755, 0.765)	A1	[3]
4	**(i)**	Use $\tan(A \pm B)$ formula correctly at least once to obtain an equation in $\tan\theta$	M1	
		Obtain a correct horizontal equation in any form	A1	
		Use correct exact values of tan 30° and tan 60° throughout	M1	
		Obtain the given equation correctly	A1	[4]

	(ii)	Make reasonable attempt to solve the given quadratic in tan θ	M1	
		Obtain answer $\theta = 24.7°$	A1	
		Obtain answer $\theta = 95.3°$ and no others in the given range	A1	[3]
		[Ignore answers outside the given range.]		
		[Treat answers in radians as MR and deduct one mark from the marks for the angles.]		
5	**(i)**	Find modulus of $2\cos\theta - 2i\sin\theta$ and show it is equal to 2	B1	
		Show a circle with centre at the point representing **i**	B1	
		Show a circle with radius 2	B1	[3]
	(ii)	Substitute for z and multiply numerator and denominator by the conjugate of $z + 2 - \mathrm{i}$, or equivalent	M1	
		Obtain correct real denominator in any form	A1	
		Identify and obtain correct unsimplified real part in terms of $\cos\theta$, e.g. $(2\cos\theta + 2)/(8\cos\theta + 8)$	A1	
		State that real part equals $\frac{1}{4}$	A1	[4]
6	**EITHER**	State $x^2\frac{dy}{dx} + 2xy$, or equivalent, as derivative of x^2y	B1	
		State $y^2 + 2xy\frac{dy}{dx}$, or equivalent, as derivative of xy^2	B1	
	OR	State $xy(1 + \frac{dy}{dx})$, or equivalent, as a term in an attempt to apply the product rule	B1	
		State $(y + x\frac{dy}{dx})(x + y)$, or equivalent, in an attempt to apply the product rule	B1	
		Equate attempted derivative of LHS to zero and set $\frac{dy}{dx}$ equal to zero	M1	
		Obtain a horizontal equation, e.g. $y^2 = -2xy$, or $y = -2x$, or equivalent	A1√	
		Explicitly reject $y = 0$ as a possibility	A1	
		Obtain an equation in x (or in y)	M1	
		Obtain $x = a$	A1	
		Obtain $y = -2a$ only	A1	[8]
		[The first M1 is dependent on at least one B mark having been earned.]		
		[SR: for an attempt using $(x + y) = 2a^3 / xy$, the B marks are given for the correct derivatives of the two sides of the equation, and the M1 for setting $\frac{dy}{dx}$ equal to zero.]		
		[SR: for an attempt which begins by expressing y in terms of x, give M1A1 for a reasonable attempt at differentiation, M1A1√ for setting $\frac{dy}{dx}$ equal to zero and obtaining an equation free of surds, A1 for solving and obtaining $x = a$; then M1 for obtaining an equation for y, A1 for $y = -2a$ and A1 for finding and rejecting $y = a$ as a possibility.]		

7	(i)	State or imply the form $A+\dfrac{B}{x+1}+\dfrac{C}{x+3}$	B1	
		State or obtain $A = 1$	B1	
		Use correct method for finding B or C	M1	
		Obtain $B = \frac{1}{2}$	A1	
		Obtain $C = -\frac{3}{2}$	A1	[5]
	(ii)	Obtain integral $x+\frac{1}{2}\ln(x+1)-\frac{3}{2}\ln(x+3)$	B2√	
		[Award B1√ if only one error. The f.t. is on A, B, C.]		
		Substitute limits correctly	M1	
		Obtain given answer following full and exact working	A1	[4]
		[SR: if A omitted, only M1 in part **(i)** is available, then in part **(ii)** B1√ for each correct integral and M1.]		
8	(i)	State $\dfrac{y}{TN}=\dfrac{\mathrm{d}y}{\mathrm{d}x}$, or equivalent	B1	
		Express area of PTN in terms of y and $\dfrac{\mathrm{d}y}{\mathrm{d}x}$, and equate to $\tan x$	M1	
		Obtain given relation correctly	A1	[3]
	(ii)	Separate variables correctly	B1	
		Integrate and obtain term $-\dfrac{2}{y}$, or equivalent	B1	
		Integrate and obtain term $\ln(\sin x)$, or equivalent	B1	
		Evaluate a constant or use limits $y = 2$, $x=\frac{1}{6}\pi$ in a solution containing a term of the form a/y or $b\ln(\sin x)$	M1	
		Obtain correct solution in any form, e.g. $-\dfrac{2}{y}=\ln(2\sin x)-1$	A1	
		Rearrange as $y=2/(1-\ln(2\sin x))$, or equivalent	A1	[6]
		[Allow decimals, e.g. as in a solution $y=2/(0.3-\ln(\sin x))$.]		
9	(i)	Either use correct product or quotient rule, or square both sides, use correct product rule and make a reasonable attempt at applying the chain rule	M1	
		Obtain correct result of differentiation in any form	A1	
		Set derivative equal to zero and solve for x	M1	
		Obtain $x=\frac{1}{2}$ only, correctly	A1	[4]
	(ii)	State or imply the indefinite integral for the volume is $\pi\int e^{-x}(1+2x)\,\mathrm{d}x$	B1	
		Integrate by parts and reach $\pm e^{-x}(1+2x)\pm\int 2e^{-x}\,\mathrm{d}x$	M1	
		Obtain $-e^{-x}(1+2x)+\int 2e^{-x}\,\mathrm{d}x$, or equivalent	A1	
		Complete integration correctly, obtaining $-e^{-x}(1+2x)-2e^{-x}$, or equivalent	A1	
		Use limits $x=-\frac{1}{2}$ and $x = 0$ correctly, having integrated twice	M1	
		Obtain exact answer $\pi(2\sqrt{e}-3)$, or equivalent	A1	[6]
		[If π omitted initially or 2π or $\pi/2$ used, give B0 and then follow through.]		

10	**(i)**	State a vector equation for the line through A and B, e.g. $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + s(\mathbf{i} - \mathbf{j})$	B1	
		Equate at least two pairs of components of general points on AB and l, and solve for s or for t	M1	
		Obtain correct answer for s or t, e.g. $s = -6, 2, -2$ when $t = 3, -1, -1$ respectively	A1	
		Verify that all three component equations are not satisfied	A1	[4]
	(ii)	State or imply a direction vector for AP has components $(-2t, 3 + t, -1-t)$, or equivalent	B1	
		State or imply cos 60° equals $\dfrac{\overrightarrow{AP}.\overrightarrow{AB}}{\lvert\overrightarrow{AP}\rvert.\lvert\overrightarrow{AB}\rvert}$	M1*	
		Carry out correct processes for expanding the scalar product and expressing the product of the moduli in terms of t, in order to obtain an equation in t in any form	M1(dep*)	
		Obtain the given equation $3t^2 + 7t + 2 = 0$ correctly	A1	
		Solve the quadratic and use a root to find a position vector for P	M1	
		Obtain position vector $5\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ from $t = -2$, having rejected the root $t = -\frac{1}{3}$ for a valid reason	A1	[6]