MATHEMATICS **9709/07**

Paper 7 Probability & Statistics 2 **(S2)** **May/June 2008**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
                        Graph Paper
                        List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.

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1 A magazine conducted a survey about the sleeping time of adults. A random sample of 12 adults was chosen from the adults travelling to work on a train.

    (i) Give a reason why this is an unsatisfactory sample for the purposes of the survey. [1]

    (ii) State a population for which this sample would be satisfactory. [1]

A satisfactory sample of 12 adults gave numbers of hours of sleep as shown below.

    4.6   6.8   5.2   6.2   5.7   7.1   6.3   5.6   7.0   5.8   6.5   7.2

    (iii) Calculate unbiased estimates of the mean and variance of the sleeping times of adults. [3]

2 The lengths of time people take to complete a certain type of puzzle are normally distributed with mean 48.8 minutes and standard deviation 15.6 minutes. The random variable X represents the time taken in minutes by a randomly chosen person to solve this type of puzzle. The times taken by random samples of 5 people are noted. The mean time $\overline{X}$ is calculated for each sample.

    (i) State the distribution of $\overline{X}$, giving the values of any parameters. [2]

    (ii) Find $P(\overline{X} < 50)$. [3]

3 The lengths of red pencils are normally distributed with mean 6.5 cm and standard deviation 0.23 cm.

    (i) Two red pencils are chosen at random. Find the probability that their total length is greater than 12.5 cm. [3]

The lengths of black pencils are normally distributed with mean 11.3 cm and standard deviation 0.46 cm.

    (ii) Find the probability that the total length of 3 red pencils is more than 6.7 cm greater than the length of 1 black pencil. [4]

4 People who diet can expect to lose an average of 3 kg in a month. In a book, the authors claim that people who follow a new diet will lose an average of more than 3 kg in a month. The weight losses of the 180 people in a random sample who had followed the new diet for a month were noted. The mean was 3.3 kg and the standard deviation was 2.8 kg.

    (i) Test the authors' claim at the 5% significance level, stating your null and alternative hypotheses. [5]

    (ii) State what is meant by a Type II error in words relating to the context of the test in part **(i)**. [2]

5 When a guitar is played regularly, a string breaks on average once every 15 months. Broken strings occur at random times and independently of each other.

   (i) Show that the mean number of broken strings in a 5-year period is 4. [1]

A guitar is fitted with a new type of string which, it is claimed, breaks less frequently. The number of broken strings of the new type was noted after a period of 5 years.

   (ii) The mean number of broken strings of the new type in a 5-year period is denoted by λ. Find the rejection region for a test at the 10% significance level when the null hypothesis $\lambda = 4$ is tested against the alternative hypothesis $\lambda < 4$. [4]

   (iii) Hence calculate the probability of making a Type I error. [1]

The number of broken guitar strings of the new type, in a 5-year period, was in fact 1.

   (iv) State, with a reason, whether there is evidence at the 10% significance level that guitar strings of the new type break less frequently. [2]

6 People arrive randomly and independently at the elevator in a block of flats at an average rate of 4 people every 5 minutes.

   (i) Find the probability that exactly two people arrive in a 1-minute period. [2]

   (ii) Find the probability that nobody arrives in a 15-second period. [2]

   (iii) The probability that at least one person arrives in the next t minutes is 0.9. Find the value of t. [4]

7 If Usha is stung by a bee she always develops an allergic reaction. The time taken in minutes for Usha to develop the reaction can be modelled using the probability density function given by

$$f(t) = \begin{cases} \dfrac{k}{t+1} & 0 \leqslant t \leqslant 4, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

   (i) Show that $k = \dfrac{1}{\ln 5}$. [4]

   (ii) Find the probability that it takes more than 3 minutes for Usha to develop a reaction. [3]

   (iii) Find the median time for Usha to develop a reaction. [3]

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