MATHEMATICS

Paper 9709/01

Paper 1

General comments

The paper achieved its main aim of giving the vast majority of candidates the opportunity of showing what they had learned. There were relatively few poor scripts and there was little evidence that candidates had had insufficient time for the paper. It is worth mentioning that in general candidates have difficulty in answering questions that require a proof or an explanation. This was particularly evident in **Question 1** and **Question 10(iv)**. The standard of presentation was generally good, but there are still Centres in which candidates divide the page into two halves and then work each separately. This makes the marking, in particular the recording of marks, very difficult.

Comments on specific questions

Question 1

The standard of the solutions varied considerably. There were many perfectly correct answers but many others in which candidates were unable to obtain a correct common denominator or to obtain a correct numerator for the two fractions on the left-hand side of the identity. Many solutions failed when $-1(1 - \sin x)\sin x$ was rewritten as $-\sin x - \sin^2 x$. It should be pointed out to candidates that it is not sufficient to express $\dfrac{2\sin^2 x}{1-\sin^2 x}$ as $2\tan^2 x$ without the intermediate line of working.

Answer: Proof

Question 2

There were very few completely correct solutions to this question. Almost all candidates realised the need to equate $kx - 4$ with $x^2 - 2x$ but only about a half of these realised the need to use $b^2 - 4ac$ on the resulting quadratic equation. Many then failed to cope with '$-2x - kx$' and it was common to see b^2 written as $-(2 - k)^2$. Although many candidates were able to obtain $k = -6$ and $k = 2$, only a small proportion correctly treated the question as an inequality.

Answer: $k > 2$ or $k < -6$

Question 3

Part **(i)** was very well answered. In part **(ii)**, most candidates realised that the coefficient of x^2 in the product of $1 + ax$ and $(2 + 3x)^5$ was $240a + 720$ and that this led to $a = -3$, though a surprising number offered the answer as $a = 3$. There were also a significant number of candidates who failed to realise that the coefficient of x^2 came from considering two products.

Answers: **(i)** $32 + 240x + 720x^2$; **(ii)** -3

Question 4

This was badly answered. In part **(i)**, most candidates realised that $c = 3$ from the intercept on the y-axis, and many were able to obtain $b = 2$ because there were two complete cycles for $0 \le x \le 2\pi$. Very few realised that the value of a could be obtained directly from the amplitude (= 9 − 3). In part **(ii)**, most candidates obtained method marks for making $\sin bx$ the subject and dividing their solution by 'b'. Of the few that did answer part **(i)** correctly, only a few were able to obtain the smallest value of x. Many offered either the answer in degrees or offered a negative value for x.

Answers: **(i)** $a = 6$, $b = 2$, $c = 3$; **(ii)** $\frac{7\pi}{12}$ or 1.83

Question 5

Part **(i)** presented many candidates with difficulty. The perimeter of R_1 was usually correct as $2r + r\theta$, but the arc length of R_2 was often written as $r(\pi - \theta)$ instead of $r(2\pi - \theta)$. Part **(ii)** was well done with most candidates obtaining $r^2 = \frac{60}{\pi - 1}$. Those realising that the area of R_2 was $\pi r^2 - 30$ were generally correct; however the majority used the fact that the area of R_2 was $\frac{1}{2}r^2(2\pi - \theta)$ and many experienced problems in simplifying $(2\pi - (\pi - 1))$ to $\pi + 1$.

Answers: **(i)** Proof; **(ii)** 58.0º or 57.9º

Question 6

(i) Many candidates scored well on this question without fully realising that **OA.OB** = −6 and that no further working was required. Virtually all candidates proceeded to calculate the angle and then to deduce that 95.1º was obtuse. Remarkably, many obtained an angle of 95.1º and then deduced that the angle was acute, and many assumed that leaving the answer as 95.1º was sufficient.

(ii) It was pleasing that most candidates correctly used **AX** = **OX** − **OA** and obtained a correct answer for **OX**. A large number of candidates still fail to realise that **OX** is the position vector not the unit vector and it was rare to see **OX** divided by the modulus 6.

Answers: **(i)** −6, obtuse; **(ii)** ⅓(2**i** − 2**j** + **k**)

Question 7

(i) This was generally well answered with most candidates realising that the common ratio was 0.5^2. The most common reason for loss of marks was through writing the answer as 0.666 or 0.67 or 0.66.

(ii) Many candidates failed to realise that the number of terms in the progression could be calculated directly from '200'. There were many who incorrectly used $n = 49$, or 50 or even 49.75 in the formula for the sum of n terms of an arithmetic progression.

Answers: **(a)** ⅔ or 0.667; **(b)** 5150

Question 8

There were a large number of correct solutions with virtually all candidates attempting to find the equation of BD and then solving the equations representing AC and BD simultaneously. A few candidates took the gradient of $2y = x + 4$ as 1 instead of ½ and several expanded $y + 3 = -2(x - 10)$ as $y + 3 = -2x + 10$ or as $-2x - 20$. It was pleasing that most candidates were able to write down the coordinates of C directly from B by considering vector moves or by applying the 'mid-point' in reverse.

Answer: B (6, 5), C (12, 8)

Question 9

This question proved to be a source of high marks. In part **(i)**, virtually all candidates realised the need to differentiate and the majority recognised the function as being composite. Apart from those who failed to multiply by the differential of the bracket, the only other main loss of marks was to lose the '−' in the working. In part **(ii)**, the majority attempted to integrate πy^2, though there were many who either omitted the π or integrated y. The integration was less successful than the differentiation in part **(i)**. Many candidates failed to divide by 3 and weaker candidates often expanded the denominator and then attempted to integrate three individual powers of x.

Answers: **(i)** $-\frac{9}{8}$; **(ii)** 9π

Question 10

This question presented nearly all candidates with problems and there were only a handful of perfectly correct solutions.

(i) Most solutions were correct, though errors over the '2' were common and the final answer often appeared as $2(x-3)^2-2.5$.

(ii) Only a few candidates realised that because $y = f(x)$ has a line of symmetry at $x = 3$, then $A = 6$.

(iii) Again there were relatively few correct solutions. Many candidates realised that $x \geq c$, but it was rare for candidates to realise that there was an upper limit of 13.

(iv) It was insufficient just to state that an inverse existed because the function was 1:1. Some reference to the domain of $x \geq 4$ being to the 'right' of the minimum point was required.

(v) It was pleasing that the majority of candidates realised the need to use the answer to the first part. There were however a significant number of candidates who still attempt vainly to make x the subject of $2x^2 - 12x + 13$ without rewriting the expression in 'completed square' form.

Answers: **(i)** $2(x-3)^2-5$; **(ii)** $A = 6$; **(iii)** $-5 \leq x \leq 13$; **(iv)** Proof; **(v)** $\sqrt{\frac{x+5}{2}}+3$

Question 11

It was a pity that many weaker candidates seemed rushed to complete this question because for most candidates it was a source of high marks.

(i) Many candidates attempted to find the coordinates of B by solving $x^3 - 6x^2 + 9x = 0$, without realising that the coordinates of both A and B could be obtained by calculus. Surprisingly, many solutions were seen in which the y-values of A and B were incorrectly stated following correct values for x.

(ii) The majority of solutions were correct, though a small proportion found either the equation of the tangent instead of the normal or failed to realise the need to use calculus.

(iii) This presented most candidates with some difficulty. The integration was of a high standard and many obtained the area between the curve and the x-axis correctly. Dealing with the area of the trapezium presented most difficulties with the y-coordinate of D often being incorrectly evaluated. Many candidates attempted to obtain the answer by considering the integral of $y_1 - y_2$, but common errors were to have sign errors in the expansion of $-(x^3 - 6x^2 + 9x)$ or to attempt to collect terms together prior to integration and to make an error in the coefficient of x i.e. $-\frac{26x}{3}$.

Answers: **(i)** $A\,(1, 4)$, $B\,(3, 0)$; **(ii)** $3y = x + 4$; **(iii)** $\frac{17}{12}$

MATHEMATICS

Paper 9709/02
Paper 2

General comments

Candidates found the later questions to be relatively straightforward compared to the later questions in previous years' papers. There was a wide range of marks with performance varying from poor to excellent. Many candidates were obviously not prepared for this paper and displayed a very weak grasp of basic calculus procedures as well as making frequent numerical and sign errors. The Examiners recommend that future candidates be encouraged to work carefully through past question papers. While candidates struggled with **Question 5**, Question **6** was well answered by even the weakest candidates.

Candidates should be discouraged from displaying their solutions down two parallel columns on each page, and from squeezing their work into a single page, or even into only one side of a single page.

Comments on specific questions

Question 1

Most candidates used logarithms correctly, but a surprising number lost the final mark. A frequent error was to invert the correct ratio.

Answer: 4.11.

Question 2

This was answered well and almost everyone scored 3 marks by squaring each side and solving the resultant quadratic equation or inequality. In order to decide on the final inequality, candidates are reminded that $x = 0$ satisfying (or not satisfying) the initial modulus inequality determines whether this value lies in the final solution set.

Answer: $-1 < x < -\frac{1}{2}$.

Question 3

Better candidates scored at least 2 marks. Some rounded $\sqrt{2}$ to 1.41 or 1.4, but at least 3 decimal places were required to justify the final answser. Weaker candidates did not attempt the use of the trapezium rule and used various imaginative, but incorrect, direct integration methods. In part **(ii)**, instead of arguing from a graph or from the fact that the individual trapezia both exceed the corresponding exact areas, comments were made regarding the curve being convex (or concave) up or down. A convincing reason needs to relate the trapezia to the curve.

Answer: **(i)** 1.21.

Question 4

Candidates almost always obtained a correct $\frac{dx}{d\theta}$ but $\frac{dy}{d\theta}$ proved more difficult to obtain. Some candidates erroneously set $\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{dx}{d\theta}$ or set $\frac{dy}{dx} = \frac{dx}{d\theta} \div \frac{dy}{d\theta}$. Only very good candidates scored to final mark by noting that $\sin 2\theta \equiv 2 \sin \theta \cos \theta$

Answer: $2 \sin \theta$.

Question 5

This was done well by only the very best candidates. Many poor solutions featured incorrect substitutions such as $\sec x = \tan x$ or $1 + \tan x$ or $1 + \tan^2 x$. Instead, the substitution $\tan^2 x = \sec^2 x - 1$ was required, or alternatively the substitutions $\sec x = \frac{1}{\cos x}$ and $\tan^2 x = \frac{1 - \cos^2 x}{\cos^2 x}$.

Answer: 48.2°, 120°.

Question 6

Almost everyone scored at least the first 5 marks on what was a very standard question. Even those who incorrectly said that $p(2) = 4$ or $p(1) = 0$ scored 2 marks or more in part **(i)** and a method mark in part **(ii)** for attempting to factorise their cubic expression.

Answers: **(i)** -4, 1; **(ii)** $(x + 1)$, $(x - 3)$.

Question 7

(i) A sizeable number of candidates failed to score after falsely finding $\frac{dy}{dx}$ to consist of only one term, whereas use of the product rule $(uv)' = uv' + vu'$ was required. A minority of better candidates lost the factor 2 when differentiating $2x$.

(ii) This part presented no problems for good candidates, but others used incorrect verions involving, for example, $\ln\left(\frac{20}{x}\right) = \ln 20 \times \ln x$.

(iii) Most candidates scored 2 marks, but a majority gave their final answer to 4, rather than 2, decimal places or rounded to 1.34.

Answers: **(i)** $\left(-\frac{1}{2}, -\frac{1}{2e}\right)$; **(iii)** 1.35.

Question 8

(a) Many candidates believed that the derivative of $\ln(3x - 2)$ equalled $\frac{1}{3x-2}$, thus omitting the factor 3. Others found $\frac{dy}{dx}$ but used $m_1 m_2 = -1$ to obtain the equation of the normal instead of the equation of the tangent.

(b) (i) Approximately half of all solutions were incorrect, with many having A as a function of x. As an identity was involved, the simplest technique was to give x a numerical value, say $x = 0$.

(ii) Weaker candidates did not use the hint. Others lost the $\frac{1}{3}$ when integrating $\frac{4}{3x-2}$. Few used the form of the integrand given in part **(b)(i)**.

Answers: **(a)** $y = 3x - 3$; **(b)(i)** 4.

MATHEMATICS

Paper 9709/03

Paper 3

General comments

There was considerable variation in the standard of work on this paper and this resulted in a wide spread of marks. Well prepared candidates appeared to have sufficient time, found no question to be of undue difficulty, and were able to achieve high scores. By contrast some candidates seemed unprepared for the paper and could make little or no progress on any question. The questions or parts of questions that were generally done well were **Question 3** (trigonometry), **Question 4** (iteration) and **Question 8** (partial fractions). Those that were done least well were **Question 1** (logarithms), **Question 2** (trapezium rule), **Question 7** (complex numbers) and **Question 9** (vector geometry).

In general the presentation of the work was good but there are still candidates who present their work in a double column format. This makes marking difficult for Examiners and it would be helpful if Centres could continue to discourage this practice. Secondly, though the rubric for the paper informs candidates of 'the need for clear presentation in your answers', there are some who do not show sufficient steps or make clear the reasoning that leads to their answers. This can occur when they are working towards answers or statements given in the question paper, for example as in **Questions 4(i)** and **10(ii)**, but also when candidates only state the final answer to a question without showing the reasoning or method by which they arrived at it, for example as in **Questions 1**, **4(iii)** and **10(iii)**. Examiners penalise the omission of essential working.

Where numerical and other answers are given after the comments on individual questions, it should be understood that alternative forms are often acceptable and that the form given is not necessarily the sole 'correct answer'.

Comments on specific questions

Question 1

This question was generally poorly answered. Many candidates incorrectly assumed that $\ln(2+e^{-x}) = \ln 2 + \ln(e^{-x})$ or that $\ln(2+e^{-x}) = \ln 2.\ln(e^{-x})$. Of those who removed logarithms correctly, reaching $2+e^{-x} = e^2$ for example, a significant number failed to solve correctly for *x*.

Answer: –1.68.

Question 2

This question was poorly answered. Though most candidates showed some familiarity with the trapezium rule, they often failed to apply it correctly, making errors such as (a) using the wrong interval width, (b) using abscissae instead of ordinates, (c) systematically calculating ordinates incorrectly, and (d) using three (or five) ordinates instead of four. In part **(ii)** the answer 'less' occurred more often than 'greater' and most candidates went on to earn the mark for this part by supporting 'less' with an adequate explanation.

Answer: **(i)** 0.98.

Question 3

This question was generally well answered. In part **(i)** the majority expressed the left hand side in terms of trigonometric functions of θ and tried to reduce it to cot θ. Those who used the double angle formula for tan 2θ tended to be less successful in completing the proof than those who used $\cot 2\theta \equiv \frac{\cos 2\theta}{\sin 2\theta}$. Some candidates believed that cosec 2θ was equivalent to $\frac{1}{\cos 2\theta}$. Most of the candidates who attempted part **(ii)** gave a correct solution.

Answer: **(ii)** 26.6°, 206.6°.

Question 4

This question was generally very well answered. In part **(i)** most candidates realised that an argument based on a change in sign is needed and that it is not enough to simply calculate the required function values (–3 and 2) and repeat the given statement that the root lies between 1 and 2.

Answer: **(iii)** 1.77.

Question 5

In part **(i)** most candidates were able to state the first two terms of the expansion of $(1+ax)^{\frac{2}{3}}$ and use them to obtain a value for *a*. Similarly there were many good answers to part **(ii)**. Common errors included algebraic or arithmetic slips in the terms in x^2 and x^3 of the expansion of $(1+ax)^{\frac{2}{3}}$, and the omission of the x^3 term in the expansion.

Answers: **(i)** –3; **(ii)** $-\frac{10}{3}x^3$.

Question 6

In part **(i)** the differentiation of the parametric expressions was disappointing. In part **(ii)** only a minority succeeded in deriving the given equation of the tangent correctly. Those who tried to use $y = mx + c$ often failed to reach an expression for *c* to substitute back into their equation. Part **(iii)** was often omitted even though it can be attempted independently of the previous two parts. Nevertheless some good solutions were presented.

Answer: **(i)** $-\tan t$.

Question 7

For some candidates this was a straightforward question. However in part **(i)** there were many candidates who seemed unable to use complex coefficients correctly in the quadratic formula and others who went on to spoil correct answers by incorrect simplification. In part **(iii)** the correct process for finding the modulus of a complex number was usually evident, but the work on the arguments was often incorrect because candidates tended to use trigonometry correctly but ended up with an angle in the wrong quadrant, or with several conflicting answers. The answers to part **(iv)** suggested that some candidates were unclear as to the difference between an isosceles triangle and an equilateral triangle.

Answers: **(i)** $1-\sqrt{3}\mathrm{i}$, $-1-\sqrt{3}\mathrm{i}$; **(iii)** 2, –60°; 2, –120°.

Question 8

Part **(i)** was generally well answered. Candidates usually started out with a form such as $\frac{A}{x}+\frac{B}{x^2}+\frac{C}{10-x}$ or $\frac{Ax+B}{x^2}+\frac{C}{10-x}$ and had a sound method for determining the constants. The most frequent error was the omission of one of the constants, usually A, from the initial form.

In part **(ii)** the separation of the variables and subsequent incorporation of the result of part **(i)** was not always successful. Indeed there were some candidates who did not see the link with part **(i)** at all. The most common error was in handling the factor of $\frac{1}{100}$ in the differential equation. Only the strongest candidates reached a correct expression for t in terms of x.

Answers: **(i)** $\frac{1}{x}+\frac{10}{x^2}+\frac{1}{10-x}$; **(ii)** $t=\ln\left(\frac{9x}{10-x}\right)-\frac{10}{x}+10$.

Question 9

Examiners found that this question discriminated well. There were many different approaches to part **(i)**. Most candidates could obtain one correct equation in b and c but far less saw how to find a second correct equation. Many candidates did not know how to answer part **(ii)**. However six sound methods were seen. The most popular one took a general point Q on l and used a scalar product to determine the value of the parameter t for which PQ is perpendicular to l.

Answers: **(i)** -2, 3.

Question 10

This question also discriminated well. In part **(i)** mistakes in applying the chain rule and algebraic errors spoiled otherwise sound approaches to the problem. In part **(ii)** the use of the given substitution was disappointing. Many candidates overlooked the need to consider the change in the values of the limits and sometimes not enough working was shown to adequately justify the given double angle form of the final answer. There were relatively few successful solutions to part **(iii)**. Most of the worthwhile attempts tried to apply the double angle formula to $\sin^2 2\theta$ but there were errors in the algebra, in the subsequent integration and in the substitution of limits. Occasionally integration by parts was used with success.

Answers: **(i)** $\frac{\sqrt{6}}{3}$; **(iii)** $\frac{1}{16}\pi$.

MATHEMATICS

Paper 9709/04
Paper 4

General comments

A significant minority of candidates was not suitably prepared for examination. This was reflected by the lack of understanding of the most basic principles, such as the balance of forces acting on a body in equilibrium and the work/energy balance of a body in motion.

There was a tendency for some candidates to expect to be able to use a ‘recipe’, usually in the form of a formula, to answer every question without reference to its relevance or an understanding of its use.

In a significant number of Centres the presentation of the work was poor, particularly where candidates ruled a mid-line down the page and worked in two columns, thus reducing the width of the working space to half the width of the page. Centres should discourage this practice. Some candidates appeared to attempt to fit all of their work onto one sheet of answer paper. Centres should ensure that an adequate supply of answer paper is available.

Questions 1, **2** and **5** were the worst attempted questions and in each of **Questions 1** and **2** many candidates scored no marks. It seems that a very large number of candidates reacted without careful consideration of the diagram in each of **Questions 1** and **2**, and with only superficial attention to the text.

The other four questions were significantly better attempted, with **Questions 3** and **7** being the best attempted.

Comments on specific questions

Question 1

Those candidates who demonstrated an understanding of equilibrium answered the question with an apparent economy of effort.

Unfortunately a very significant proportion of candidates reacted to superficial examination of the given diagram by writing $50 - T = 5a$ and $T - 40 = 4a$ to obtain $T = 44.4$.

Many others gave two answers for T : $T = 40$ and $T = 50$.

Answers: 40 N, 10 N.

Question 2

Many candidates were able to quote the formula $W = Fd\cos\alpha$ but a large proportion of such candidates sought to use it to calculate W rather than F. This is of course possible if F is found first by obtaining $mg\sin\theta = 21$ using PE = mgh, $R = \frac{900}{100}$ and $a = 0$, in applying Newton’s second law. Very few such candidates made useful progress.

It was expected that candidates would react to the instruction ‘write down’ by producing an answer from a linear combination of the 900 J and the 2100 J. Many candidates did so and obtained the correct answer of 3000 J, although 1200 J was also fairly common.

Answers: 3000 J, 31.1.

Question 3

This was the best attempted question.

(i) Many candidates scored full marks in this part, although some left their answers as $7 + 10\cos 50° - 15\cos 80°$ and $10\sin 50° + 15\sin 80°$ and did not proceed to give the corresponding numerical values. Such candidates did however usually give numerical answers in all other questions. Some candidates did not understand that the component of the resultant was required in each of the cases **(a)** and **(b)**, and gave their answers instead as the components of the individual three forces.

(ii) Many candidates did not attempt to find the required direction, but calculated the magnitude of the resultant instead.

Answers: **(i)(a)** 10.8 N, **(b)** 22.4 N; **(ii)** 64.2° anticlockwise from the x-axis.

Question 4

(i) This part of the question was reasonably well attempted, although some candidates took no account of the tension in the string.

(ii) In this part of the question a substantial number of candidates simply used the answers in part **(i)** to substitute into $\mu = \frac{F}{R}$, without taking account of the cutting of the string.

Answers: **(i)** 14.4 N, 75.2 N; **(ii)** 0.364.

Question 5

Premature approximation made a significant impact on the accuracy of answers in this question. Many candidates obtained the height of A above B as 11.3 m, which is not accurate enough to generate answers which are correct to three significant figures. Using this value leads to 9040 J (instead of 9070 J) and 5800 J (instead of 5830 J) in part **(i)**, 96.6 m (instead of 96.0 m) in part **(ii)**, and 0.773 ms^{-2} (instead of 0.771 ms^{-2}) in part **(iii)**.

Candidates should be aware that the use of intermediate values that are accurate to 3 significant figures does not guarantee this degree of accuracy in subsequently calculated values.

(i) Despite the reference to the work done against the resistance in the question, a substantial number of candidates calculated either the kinetic energy gain or the potential energy loss, and then gave the same value to the other, using (inappropriately) the principle of conservation of energy.

(ii) This part was not well attempted and many candidates made no attempt to find the value of the resistance common to both the section AB and section BC. A common error was to start by finding the acceleration of the cyclist while moving from A to B, and then to use this value as the deceleration on moving from B to C. Candidates then used $v^2 = u^2 + 2ad$ to find d.

(iii) Many candidates omitted the resistance when applying Newton's second law, and consequently 1.06 ms^{-2} was a very common incorrect answer. Some candidates had incorrect values for the resistance, of which 5830 (the value of the work done against resistance) was the most common.

Answers: **(i)** 3240 J, 9070 J, 5830 J; **(ii)** 96.0; **(iii)** 0.771 ms^{-2}.

Question 6

The first three parts were generally well attempted.

Part **(iv)** was poorly attempted with both 0.72 m and 0.432 m being very common incorrect answers. Candidates who obtained the former failed to take account of the further upward movement of *B* after *A* reaches the floor. Candidates who obtained the latter assumed that *B*'s upward movement with the string slack started at *B*'s initial position, taking no account of *B*'s rise as *A* was falling to the floor.

Many candidates recognised the link between 'maximum height' and '$v = 0$'. However most candidates who did so then used $v = 0$ in conjunction with $a = 2$, instead of with $a = -g$.

Answers: **(i)** 2 ms^{-2}; **(ii)** 3.6 N; **(iii)** 0.3 kg; **(iv)** 0.792 m.

Question 7

This question was generally very well attempted. The main source of error in the question was the assumption, usually implicit, that the given $v = 0.5t - 0.01t^2$ applies not only to the section *AB*, but also to one or both of the subsequent sections.

(i) This was very well attempted, although the erroneous $0.1 = \frac{v(t)}{t}$ was used to find t by a significant minority of candidates. So too was $0.5 - 0.02t = 0$, candidates taking no account of the given value of 0.1.

(ii) This part was also very well attempted, although some candidates used the $v(t)$ that applies only to the section *AB*, and attempted to solve the quadratic equation $14 = 0.5t - 0.01t^2$.

(iii) Almost all candidates were able to score both marks in this part. The most common incorrect answer was 4 ms^{-1} arising fortuitously. Candidates making this error used $0.3 = 0.5 - 0.02t \Rightarrow t = 10$, then $v = 0.5 \times 10 - 0.01 \times 10^2 = 4$.

(iv) Most candidates integrated $v(t)$ correctly and identified the resulting function of t as $s(t)$. Most used limits of 0 to 20 to find the distance *AB* correctly. However many candidates used different limits that implied that $v(t)$ applies beyond *B*, in some cases to include *BC* and others to include *BD*.

Candidates who did not fall into the latter category were usually successful in finding the distance *BC*.

Answers: **(i)** 20 s; **(ii)** 80 s; **(iii)** 4 ms^{-1}; **(iv)** 1170 m.

MATHEMATICS

Paper 9709/05
Paper 5

General comments

The paper was a good test of the candidates' abilities and the the majority of candidates had adequate time to attempt all the questions on the paper. Some candidates scored very low marks and were clearly not ready for the examination at this level.

It is necessary to report that the work from some candidates was poorly presented and sometimes difficult to read.

It is pleasing to see that many candidates attempted to draw diagrams to help with their solutions.

Premature approximation was not often seen and most candidates gave their answers to 3 significant figures or better.

The question paper clearly states that $g = 10$ should be used but on occasions $g = 9.8$ or 9.81 was seen.

Some candidates might have benefitted by consulting the list of formulae.

Comments on specific questions

Question 1

A minority of candidates used an incorrect formula for the centre of mass and others substituted an incorrect value for α in the formula: usually $\alpha = 1.5$ instead of 0.75. The correct formula was given on the list of formula. The majority of candidates used $v = r\omega$.

Question 2

Some candidates used an incorrect formula for the centre of mass. The correct formula appears in the list of formula. The principle of taking moments often caused a problem. Some candidates tried to solve the question by simply resolving and forgot that there would be a contact force at *A*.

Answer: 6.41 N

Question 3

(i) This part of the question was usually well done.

At times $\int \frac{1}{x+2}\,dx = \ln(\frac{1}{2}x + 1)$ was seen immediately. It would have been better to show an intermediate step: $\int \frac{1}{x+2}\,dx = \int \frac{1}{2(0.5x+1)}\,dx = \ln(\frac{1}{2}x + 1)$.

(ii) Most candidates scored full marks here.

Answer: **(ii)** 2.32.

Question 4

(i) Too many candidates found an angle by writing $\sin\theta = \frac{0.3}{0.5}$, followed by $\theta = 36.9°$, and used this value of θ in $R\sin\theta = 0.12g$ which does not give the exact value of R. However, $R\sin\theta = R \times \frac{0.3}{0.5} = 0.12g$ does lead to 2, the exact value of R.

(ii) Most candidates attempted to use Newton's second law with $a = \frac{v^2}{r}$, but sometimes $r = 0.5$ not 0.4 was used and/or $F = 2$ rather than $2\cos\theta$ was used.

(iii) This part was generally well done.

Answers: **(ii)** 2.31 ms^{-1}; **(iii)** 1.09 s.

Question 5

In part **(i)**, a number of candidates started from scratch to set up the equation of the trajectory and then went on to compare their result with $y = 0.75x - 0.02x^2$. The trajectory equation is quoted on the list of formula.

Parts **(ii)** and **(iii)** were usually well done, however, premature approximations did sometimes occur here.

Answers: **(i)** 36.9, 19.8; **(ii)** 37.5 m; **(iii)** 7.03 m.

Question 6

Many candidates scored full marks on the first three parts of this question.

Part **(iv)** was found to be the hardest part in the paper. Many candidates just equated the tensions. Other candidates did attempt to use elastic energy with very little success. The correct equation was very rarely seen and a good diagram would have helped candidates to see the correct extensions needed.

Answers: **(i)** 12 N, 24 N; **(iii)** 7.5 ms^{-2}; **(iv)** 0.5.

Question 7

(i) This part of the question was well done.

(ii)(a) Good candidates had no difficulty in explaining and proving that $\mu > \frac{5}{7}$. Some candidates simply quoted that for toppling $\mu > \tan\theta$ and so $\mu > \frac{5}{7}$, but this part is worth 5 marks and more explanation was required, along the following lines.

Body on point of toppling implies G is vertically above O, so $\tan\theta = \frac{2.5}{3.5}$.

Before sliding $F < \mu R$ and $F = W\sin\theta$, $R = W\cos\theta$.

This leads to $\mu > \frac{W\sin\theta}{W\cos\theta} = \tan\theta$.

Hence $\mu > \frac{5}{7}$.

(b) Candidates often scored full marks here.

Answer: **(iii)** $\mu < \frac{7}{5}$.

MATHEMATICS

Paper 9709/06

Paper 6

General comments

The paper proved accessible to most candidates apart from **Question 1(ii)**. There was no problem with shortage of time apart from candidates who had two or three attempts at some questions, and almost everybody attempted questions in numerical order. It was pleasing to see that the majority of candidates worked with more than the correct number of significant figures and therefore did not lose any accuracy marks. Those who worked with 3 significant figures i.e. premature rounding, invariably gave an answer which was not accurate to 3 significant figures. Statistics requires good presentation and labelling of illustrations with appropriate units, and so this aspect is looked for when assessing scripts.

Comments on specific questions

Question 1

(i) Some candidates failed to obtain $z = 0.674$ through not realising that the z-value corresponding to an area of 0.75 is given at the foot of the normal distribution tables, or not understanding how to use the normal distribution tables in reverse (0.67, 0.671, 0.675 and 0.678 were all seen many times).

(ii) Only a small proportion of candidates appreciated that choosing 3 cartons from 900 meant choosing without replacement - thus use of the binomial distribution was not appropriate. Many candidates assumed a binomial distribution and failed to score.

Answers: **(i)** 997; **(ii)** 0.140.

Question 2

(i) It was surprising how many candidates were unable to show how the given answer was obtained because they had not understood the question. Options and possibility spaces were the usual ways of proceeding, and in fact use of $\frac{1}{4}+\frac{1}{4}\times\frac{1}{4}$ was sufficient to score two marks here.

(ii) Finding E(X) and Var(X) from the given table usually presented no problems for candidates who had covered the syllabus. There were some candidates who forgot to subtract $[E(X)]^2$ and also a number who misread their own poorly written 16 as 10 or 6.

Answers: **(ii)** 3.75, 2.19.

Question 3

(i) ‘Fewer than 3’ was often interpreted to mean ‘exactly 3’ or ‘fewer than or equal to 3’. Candidates do need to know the precise meaning of these words as similar phrases are likely to occur in this paper. Some candidates attempted the normal approximation in this part and failed to score. Some candidates worked to 3 significant figures and thus their final answer was not accurate enough.

(ii) This was found to be a straightforward question and was well done by the majority of candidates.

Answers: **(i)** 0.748; **(ii)** 0.887.

Question 4

This routine question was well done by a large number of candidates. Of those who lost marks, many added the four combinations instead of multiplying them in part **(i)**, and in part **(iii)** many omitted to multiply by 2.

Answers: **(i)** 33 033 000; **(ii)** 86400; **(iii)** 288.

Question 5

There were surprisingly few fully correct answers to part **(i)**, the most common being $\frac{1}{4}, \frac{1}{4}, \frac{1}{4}$ or $\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$. Some candidates did not read the question sufficiently clearly and failed to give the probabilities of going on the relevant rides, just giving probabilities of two heads, etc. Others multiplied the three probabilities together to find the probability of going on all three rides, despite the question asking for probabilities for each of the three rides. There were however, many good solutions to this question, and those who thought the probabilities were each $\frac{1}{3}$ gained appropriate credit.

Answers: **(i)** $\frac{1}{4}, \frac{1}{4}, \frac{1}{2}$; **(ii)** $\frac{29}{40}$; **(iii)** $\frac{3}{11}$.

Question 6

Scales were nearly always correct although they did not always start from zero, affecting the shape of the graph. Most candidates drew curves. There were some polygons but also some cumulative frequency step polygons which were inappropriate. Mistakes involved plotting the curve at mid-points, using 10.5, 20.5 etc and not labelling axes. Part **(iii)** stated that candidates should 'use your graph to estimate the median time…'. For this part, candidates were expected to show a median line at 270 or 270.5 and read off their answer, to show that they had used their graph and not linear interpolation. Failure to show the line resulted in the loss of a mark. In part **(v)** a few candidates did not use the definition of *s* given in part **(iv)**.

Answers: **(i)** 494, 46; **(iii)** 13.5 to 14.6 minutes; **(iv)** 18.2 minutes, 14.2 minutes; **(v)** 155 to 170 people.

Location Entry Codes

UNIVERSITY of CAMBRIDGE
International Examinations

As part of CIE's continual commitment to maintaining best practice in assessment, CIE has begun to use different variants of some question papers for our most popular assessments with extremely large and widespread candidature, The question papers are closely related and the relationships between them have been thoroughly established using our assessment expertise. All versions of the paper give assessment of equal standard.

The content assessed by the examination papers and the type of questions are unchanged.

This change means that for this component there are now two variant Question Papers, Mark Schemes and Principal Examiner's Reports where previously there was only one. For any individual country, it is intended that only one variant is used. This document contains both variants which will give all Centres access to even more past examination material than is usually the case.

The diagram shows the relationship between the Question Papers, Mark Schemes and Principal Examiner's Reports.

Question Paper	**Mark Scheme**	**Principal Examiner's Report**
Introduction	Introduction	Introduction
First variant Question Paper	First variant Mark Scheme	First variant Principal Examiner's Report
Second variant Question Paper	Second variant Mark Scheme	Second variant Principal Examiner's Report

Who can I contact for further information on these changes?
Please direct any questions about this to CIE's Customer Services team at: international@cie.org.uk

MATHEMATICS

Paper 9709/71

Paper 71

General comments

On this paper, the majority of candidates were able to demonstrate and apply their knowledge in the situations presented. Whilst there were many good scripts, there were also candidates who appeared completely unprepared for the paper. In general, candidates scored well on **Questions 3, 5(i)**, **(ii)** and **(iii)** and **6(i)**, whilst **Question 4** proved more demanding.

Accuracy, as always, caused loss of marks for some candidates. There were a few cases of candidates not adhering to the required level of accuracy; either by rounding too early in the question or by giving a final answer to only 2, or even 1, significant figures. This was particularly noted in **Question 3(ii)**. There were also cases noted where candidates truncated intermediate answers leading to a lack of required accuracy in their final answer. Careless errors were also made by candidates miscopying their own figures; for example in **Question 2**, 2.17 became 2.71 and in **Question 6** 10.05 became 10.5. Timing did not seem to be a problem for candidates.

Detailed comments on individual questions follow. Whilst the comments indicate particular errors and misconceptions, it should be noted that there were also many very good and complete answers.

Comments on specific questions

Question 1

This was a reasonably well attempted question. It was pleasing to note that the majority of candidates started the question by stating their Null and Alternative hypotheses. Common errors included using a two-tailed test consistently (though in some cases not consistently) throughout the question. Many candidates successfully found the correct test statistic and made a valid comparison with the correct critical value. However, it should be noted that this comparison must be clearly made before a conclusion can be drawn. Some candidates lost marks here by not clearly stating (or showing on a diagram) the comparison they were making in order to reach their conclusion of accepting or rejecting the null hypothesis. There were many invalid comparisons made including the common one of comparing a z value with an area.

Answer: Not enough evidence to support the claim that fingers are smaller.

Question 2

In **part (i)** many candidates found correct values for μ and for σ^2 but many used a particularly lengthy, though correct, method solving simultaneous equations. The value of μ could be found very quickly using the fact that the confidence interval is symmetrical about the mean. This was missed by many candidates, causing time penalties with a lengthy method as well as the possibility of algebraic errors. Other errors included use of incorrect z values, leaving the answer as 16.29 (σ) rather than σ^2, as requested in the question.

In **part (ii)** common errors included factor of 2 errors, and failure to give the final answer for n as a whole number.

Answer: **(i)** 227.1 265
(ii) 78

Question 3

In **part (i)** Examiners noted common errors of using an incorrect mean, and calculating the probability of 1-P(0,1,2) rather than 1-P(0,1,2,3). In **part (ii)** errors were caused when candidates used a premature approximation for λ as 5.3 rather that 16/3, and in **part (iii)** many candidates used a correct Normal approximation but either omitted, or used a wrong, continuity correction.

Answers: **(i)** 0.143
(ii) 0.118
(iii) 0.0316

Question 4

This was not a well attempted question. Candidates often failed to set up a correct null and alternative hypothesis, and many did not use a Binomial distribution. Some candidates merely compared P(7) or P(8) with 0.05 rather than comparing the sum of P(7) and P(8) with 0.05. An incorrect comparison was often seen (including comparison with *z* values) and some candidates failed to show all necessary working in reaching their conclusion.

The majority of candidates did not realise what was required in **part (ii)**, with incorrect answers of 1- their answer to **part (i)**, or an answer of 0.05.

Answers: **(i)** Accept the driving instructor's claim
(ii) 0.0293

Question 5

Questions which test knowledge of probability density functions are usually well attempted by candidates; this question was no exception for **parts (i)**, **(ii)**, and **(iii)**. Part **(iv)**, however, proved to be more challenging. There were few errors in **part (i)** with candidates confident of the method to use, errors in **part (ii)** included confusion between the mean and the median, and in **part (iii)** the most common error noted was to find the probability of less than 5 rather than more than 5. In **part (iv)** a large proportion of candidates attempted to actually find a value for the upper quartile. This method resulted in a cubic equation which could have been solved by trial and improvement, but many elementary algebraic errors were made by candidates in their attempts to solve this equation. The simplest method expected in answer to this question, which some good candidates spotted, was to consider their answer to **part (iii)** and compare this with 0.25 (the area above the upper quartile). The majority of candidates were unable to gain the two marks here as they did not make this connection between **parts (iii)** and **(iv)**.

Answers: **(ii)** 33/8
(iii) 4/27
(iv) UQ is less than 5

Question 6

Many candidates successfully combined the independent normal distributions and found the mean and variance of $T_1+T_2+T_4–T_3$. Common errors were usually made in the calculation for the variance. Weaker candidates did not combine the distributions correctly and attempted to standardise using the wrong distribution, for example using $T_1+T_2+T_4$ or even using T_3. However, on the whole, **part (i)** was successfully attempted. There were, in contrast, many errors noted in **part (ii)**, usually from candidates inadvertently confusing two different methods. N(3.1,0.785/6) could be used to find P(<4), or equally N(18.6,4.71) could be used to find P(<24).

Answer: **(i)** 0.324
(ii) 0.994

MATHEMATICS

Paper 9709/72

Paper 72

General comments

On this paper, candidates were on the whole able to demonstrate and apply their knowledge in the situations presented. Whilst there were many good scripts, there were also candidates who appeared completely unprepared for the paper. In general, candidates scored well on **Questions 3, 5(i)**, **(ii)** and **(iii**) and **6(i)**, whilst **Question 4** proved more demanding.

Accuracy, as always, caused loss of marks for some candidates; there were a few cases of candidates not adhering to the accuracy required, either by rounding too early in the question or by giving a final answer to only 2, or even 1, significant figures. There were also cases noted where candidates truncated intermediate answers leading to a lack of required accuracy in their final answer. Examiners also noted careless errors made by candidates miscopying their own figures; for example in **Question 2**, 2.17 became 2.71 and in **Question 6** 10.05 became 10.5. Timing did not appear to be a problem for candidates.

Detailed comments on individual questions follow. Whilst the comments indicate particular errors and misconceptions, it should be noted that there were also many very good and complete answers.

Comments on specific questions

Question 1

This was a reasonably well attempted question. Some candidates started the question by stating their Null and Alternative hypotheses; it was surprising that this was completely omitted by others. Errors included use of a one-tailed test consistently (though in some cases not consistently) throughout the question. Many candidates successfully found the correct test statistic and made a valid comparison with the correct critical value. However, it should be noted that this comparison must be clearly made before a conclusion can be drawn. Some candidates lost marks here by not clearly stating (or showing on a diagram) the comparison they were making in order to reach their conclusion of accepting or rejecting the null hypothesis. There were many invalid comparisons made, including the common one of comparing a z value with an area.

Answer: Evidence of a difference.

Question 2

In **part (i)** many candidates found correct values for μ and for σ^2 but many used a particularly lengthy, though correct, method solving simultaneous equations. The value of μ could be found very quickly, using the fact that the confidence interval is symmetrical about the mean. This was missed by many candidates, causing time penalties with a lengthy method as well as the possibility of algebraic errors. Other errors included use of incorrect z values, leaving the answer as 16.29 (σ) rather than σ^2, as requested in the question. Other candidates, after obtaining the correct value of σ^2 went onto multiply it by 50/49 or similar.

In **part (ii)** common errors included factor of 2 errors, and failure to give the final answer for n as a whole number.

Answers: **(i)** 227.1 265
(ii) 78

Question 3

In **part (i)** Examiners noted common errors of using an incorrect mean, and calculating the probability of P(0,1) or P(1,2) rather than P(0,1,2). In **part (ii)** some candidates used 1.5 and 6 and combined separate probabilities; this could have led to the correct answer if all 7 combinations were considered and correctly combined. Some candidates successfully used this method. In **part (iii)** many candidates used a correct Normal approximation but either omitted, or used a wrong continuity correction.

Answers: **(i)** 0.174
(ii) 0.137
(iii) 0.134

Question 4

This was not a well attempted question. Candidates often failed to set up a correct null and alternative hypothesis, and many did not use a Binomial distribution. Some candidates merely compared P(7) or P(8) with 0.05 rather than comparing the sum of P(7) and P(8) with 0.05. An incorrect comparison was often seen (including comparison with *z* values) and some candidates failed to show all necessary working in reaching their conclusion.

The majority of candidates did not realise what was required in **part (ii)**, with incorrect answers of 1- their answer to **part (i)**, or an answer of 0.05.

Answers: **(i)** Accept the driving instructor's claim
(ii) 0.0293

Question 5

Questions which test knowledge of probability density functions are usually well attempted by candidates, this question was no exception for **parts (i)**, **(ii)**, and **(iii)**. **Part (iv)**, however, proved to be more challenging. There were few errors in **part (i)** with candidates confident of the method to use, errors in **part (ii)** included confusion between the mean and the median, and in **part (iii)** the most common error noted was to find the probability of less than 1.3 rather than more than 1.3, or to use limits of 1.4 and 2. In **part (iv)** a large proportion of candidates attempted to actually find a value for the median. This method resulted in an equation in x^4 , which could have been solved by trial and improvement, but many elementary algebraic errors were seen in the various attempts made by candidates to solve this equation. The simplest method expected in answer to this question, which some good candidates spotted, was to consider their answer to **part (iii)** and compare this with 0.5 (the area above the median). A large number of candidates were unable to gain the two marks here as they did not make this connection between **parts (iii)** and **(iv)**.

Answers: **(ii)** 1.2
(iii) 0.437
(iv) Median is less than 1.3

Question 6

Many candidates successfully combined the independent normal distributions and found the mean and variance of $T_1+T_2+T_4 - T_3$. Common errors were usually in the calculation for the variance. Weaker candidates did not combine the distributions correctly and attempted to standardise using the wrong distribution, for example using $T_1+T_2+T_4$ or even using T_3. However, on the whole, **part (i)** was successfully attempted. There were, in contrast, many errors noted in **part (ii)**, usually from candidates inadvertently confusing two different methods. N(3.1,0.785/6) could be used to find P(<4), or equally N(18.6,4.71) could be used to find P(<24).

Answers: **(i)** 0.324
(ii) 0.994