Location Entry Codes

UNIVERSITY of CAMBRIDGE
International Examinations

As part of CIE's continual commitment to maintaining best practice in assessment, CIE has begun to use different variants of some question papers for our most popular assessments with extremely large and widespread candidature, The question papers are closely related and the relationships between them have been thoroughly established using our assessment expertise. All versions of the paper give assessment of equal standard.

The content assessed by the examination papers and the type of questions are unchanged.

This change means that for this component there are now two variant Question Papers, Mark Schemes and Principal Examiner's Reports where previously there was only one. For any individual country, it is intended that only one variant is used. This document contains both variants which will give all Centres access to even more past examination material than is usually the case.

The diagram shows the relationship between the Question Papers, Mark Schemes and Principal Examiner's Reports.

Question Paper	**Mark Scheme**	**Principal Examiner's Report**
Introduction	Introduction	Introduction
First variant Question Paper	First variant Mark Scheme	First variant Principal Examiner's Report
Second variant Question Paper	Second variant Mark Scheme	Second variant Principal Examiner's Report

Who can I contact for further information on these changes?
Please direct any questions about this to CIE's Customer Services team at: international@cie.org.uk

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the May/June 2009 question paper

for the guidance of teachers

9709 MATHEMATICS

9709/71 Paper 7, maximum raw mark 50

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

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CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

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SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1 $H_0: \mu = 18.5$ $H_1: \mu < 18.5$	B1	Both hypotheses correct
Test statistic $z = \dfrac{18.1 - 18.5}{(1.1/\sqrt{20})}$	M1	Standardising, must have $\sqrt{20}$
$= -1.626$	A1	For correct z
CV $z = \pm 1.96$	M1	Correct comparison with correct CV or finding area on LHS of –1.626 and comparing with 2.5 % (OR comparison with 2.241 oe if one-tail test set up)
Not enough evidence to support the claim that fingers are smaller.	A1ft **[5]**	Correct conclusion must ft their CV and their z. No contradictions
2 (i) $\hat{\mu} = 227.(1)$	B1 B1	Correct mean 2.17 seen
$5 = 2.17 \times \sqrt{\dfrac{\hat{\sigma}^2}{50}}$	M1	Solving an equation with 5 or 10 on the LHS and some z value $\times \dfrac{\hat{\sigma}}{\sqrt{n}}$ on the RHS
$\hat{\sigma}^2 = 265$ or 266	A1 **[4]**	Correct answer
(ii) $4 = 2.17 \times \dfrac{16.3}{\sqrt{n}}$	B1ft	Correct equation ft their wrong z if the same as in part **(i)** and their σ
	M1	Solving an equation with their z and σ, and width 4 or 8
$n = 78$	A1 **[3]**	Correct answer (whole number)
3 (i) $\lambda = 2$	B1	Correct mean (used)
$P(X > 3) = 1 - P(0, 1, 2, 3)$ $= 1 - e^{-2}\left(1 + 2 + \dfrac{2^2}{2} + \dfrac{2^3}{3!}\right)$	M1	Poisson 1 – P(0,1,2,3) or P(0,1,2) or P(1,2,3)
$= 1 - 0.857 \quad = 0.143$	A1 **[3]**	Correct answer
(ii) $\lambda = 16/3$	B1	Correct new mean
$P(7) = e^{-16/3}\left(\dfrac{(16/3)^7}{7!}\right)$	M1	P(7) using a different mean from **(i)**
$= 0.118$	A1 **[3]**	Correct final answer
(iii) $X \sim N(160, 160)$	B1 M1	Correct mean and variance Standardising attempt with or without cc must have sq rt
$P(X < 137) = P\left(z < \dfrac{136.5 - 160}{\sqrt{160}}\right)$	M1	Cc of 136.5 or 137.5 and area < 0.5
$= P(z < -1.858)$ $= 1 - 0.9684 = 0.0316$	A1 **[4]**	Correct answer

4	**(i)**	$H_0 : p = 0.36$ $H_1 : p > 0.36$ $P(7) = {}^8C_7 \times (0.36)^7 (0.64)^1 = 0.00401$ $P(8) = (0.36)^8 = 0.000282$ $\Sigma P = 0.00429 < 0.05$ Accept driving instructor's claim	B1 M1 A1 M1 B1 **[5]**	Both hypotheses correct Evaluating P(7) or P(8) Correct answer for both Comparing their prob sum to 0.05 oe Correct conclusion cwo no contradictions
	(ii)	Type I error $P(6) = {}^8C_6 \times (0.36)^6 (0.64)^2 = 0.02496$ $P(5) = {}^8C_5 \times (0.36)^5 (0.64)^3 = 0.08876$, > 0.05 P(Type I error) = 0.0292 or 0.0293	B1 M1 B1 A1 **[4]**	Correct answer Evaluating P(6) Correct P(5) and showing this is not in the CR either by $\Sigma P > 0.05$ or $P(5) > 0.05$ Correct answer NB Marks for part **(ii)** may be awarded in part **(i)** but not vice versa.
5	**(i)**	$\int_3^6 k(6t - t^2)dt = 1$ $k\left[3t^2 - t^3/3\right]_3^6 = 1$ $k([108 - 216/3] - [27 - 9]) = 1$ $k = 1/18$ AG	M1 A1 A1 **[3]**	For equating to 1 and a sensible attempt to integrate Correct integration and correct limits Given answer correctly obtained
	(ii)	mean = $\int_3^6 k(6t^2 - t^3)dt$ $= \left[k(2t^3 - \frac{t^4}{4})\right]_3^6$ $= k(432 - 324) - k(54 - 81/4)$ $= \frac{33}{8}$ (4.13)	M1 A1 A1 **[3]**	Attempt to evaluate the integral of $tf(t)$ (t or x) Correct integral and correct limits (condone loss of k) Correct answer
	(iii)	$\int_5^6 k(6t - t^2)dt$ $= k\left[3t^2 - \frac{t^3}{3}\right]_5^6 = k(36 - \frac{100}{3})$ $= \frac{4}{27}$ (0.148)	M1 A1 **[2]**	Attempt to evaluate the integral between 5 and 6 oe Correct answer
	(iv)	the area on the left is > 0.75 or **(iii)** is < 0.25 UQ is less than 5	M1 A1ft **[2]**	sensible reason ft their **(iii)** SR B1ft correct but 0.25/0.75 implied

6 **(i)**	$T_1 + T_2 + T_4 - T_3 \sim N(-0.95, 4.345)$	M1	Attempt to find mean and var of $T_1 + T_2 + T_4 - T_3$ oe
		B1	Correct mean (3.75 + 3.1 + 3.2 – 11)
		A1	Correct variance
	$P[(T_1 + T_2 + T_4 - T_3) > 0]$	M1	Finding P their $[(T_1 + T_2 + T_4 - T_3) > 0]$ oe
	$= P\left(z > \frac{0 - -0.95}{\sqrt{4.345}}\right) = P(z > 0.4557)$	M1	Standardising (appropriate variance involving all 4) and area <0.5
	$= 1 - 0.0.6758$		
	$= 0.324$	A1 **[6]**	Correct answer
(ii)	$\overline{X} \sim N(3.1, 0.785/6)$	M1	Normal distribution mean 3.1, var 0.785/6, can be implied OR N(18.6, 4.71) if working with totals
	$P(\overline{X} < 4) = P\left(z < \frac{4 - 3.1}{\sqrt{0.785/6}}\right)$	M1	Standardising with sq rt OR (24 – 18.6)/√4.71 no mixed methods
	$= P(z < 2.488)$		
	$= 0.994$	A1 **[3]**	Correct answer

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1	$H_0: \mu = 1.746$ $H_1: \mu \neq 1.746$ Test statistic $z = \dfrac{1.765 - 1.746}{0.149/\sqrt{230})}$ $\pm -1.93(4)$ CV $z = \pm 1.645$ Evidence of a difference	B1 M1 A1 M1 A1ft **[5]**	Both hypotheses correct Standardising, must have $\sqrt{230}$ For correct z Correct comparison with correct CV or finding area on RHS of their z and comparing with 0.05 (must be 0.05) OR if one tail test comparison with 1.282 oe Correct conclusion must ft their CV and their z. No contradictions
2	**(i)** $\hat{\mu} = 227.(1)$ $5 = 2.17 \times \sqrt{\dfrac{\hat{\sigma}^2}{50}}$ $\hat{\sigma}^2 = 265$ or 266	B1 B1 M1 A1 **[4]**	Correct mean 2.17 seen Solving an equation with 5 or 10 on the LHS and some z value $\times \dfrac{\hat{\sigma}}{\sqrt{n}}$ on the RHS Correct answer
	(ii) $4 = 2.17 \times \dfrac{16.3}{\sqrt{n}}$ $n = 78$	B1ft M1 A1 **[3]**	Correct equation ft their wrong z if the same as in part **(i)** and their σ Solving an equation with their z and σ, and width 4 or 8 Correct answer (whole number)
3	**(i)** $\lambda = 4.5$ P(at most 2) = P(0, 1, 2) $= e^{-4.5}\left(1 + 4.5 + \dfrac{4.5^2}{2!}\right)$ $= 0.174$	B1 M1 A1 **[3]**	Correct mean (used) Poisson (0, 1, 2) or P(0, 1) or P(1, 2) Correct answer
	(ii) $\lambda = 7.5$ $P(6) = e^{-7.5}\left(\dfrac{7.5^6}{6!}\right)$ $= 0.137$	B1 M1 A1 **[3]**	New mean (1.5 + 6) used P(6) using a different mean from **(i)** Correct answer
	(iii) $X \sim N(90, 90)$ $P(X > 100) = P\left(z > \dfrac{100.5 - 90}{\sqrt{90}}\right)$ $= P(z > 1.107)$ $= 1 - 0.8657 = 0.134$	B1 M1 M1 A1 **[4]**	Correct mean and variance Standardising attempt with or without cc must have sq rt Cc of 100.5 or 99.5 and area < 0.5 Correct answer

4	(i)	$H_0 : p = 0.36$ $H_1 : p > 0.36$ $P(7) = {}^8C_7 \times (0.36)^7 (0.64)^1 = 0.00401$ $P(8) = (0.36)^8 = 0.000282$ $\Sigma P = 0.00429 < 0.05$ Accept driving instructor's claim	B1 M1 A1 M1 A1 **[5]**	Both hypotheses correct Evaluating P(7) or P(8) Correct answer for both P(7) and P(8) Comparing their prob sum to 0.05 oe Correct conclusion cwo. No contradictions
	(ii)	Type I error $P(6) = {}^8C_6 \times (0.36)^6 (0.64)^2 = 0.02496$ $P(5) = {}^8C_5 \times (0.36)^5 (0.64)^3 = 0.08876$, > 0.05 P(Type I error) = 0.0292 or 0.0293	B1 M1 B1 A1 **[4]**	Correct answer Evaluating P(6) Correct expression for P(5) and showing this is not in the CR either by $\Sigma P > 0.05$ or $P(5) > 0.05$ Correct answer NB Marks for part **(ii)** may be awarded in part **(i)** but not vice versa.
5	(i)	$\int_0^2 kx^2(2-x)dx = 1$ $\left[\frac{2kx^3}{3} - \frac{kx^4}{4}\right]_0^2 = 1$ $\frac{16k}{3} - \frac{16k}{4} = 1$ $k = 3/4$ AG	M1 A1 A1 **[3]**	For equating to 1 and a sensible attempt to integrate Correct integration and correct limits Given answer correctly obtained
	(ii)	mean = $\int_0^2 2kx^3 - kx^4 dx$ $= \left[\frac{2kx^4}{4} - \frac{kx^5}{5}\right]_0^2$ $= \frac{32k}{4} - \frac{32k}{5}$ $= 1.2$ m	M1 A1 A1 **[3]**	Attempt to evaluate the integral of xf(x) Correct integral and correct limits (condone loss of k) Correct answer
	(iii)	$\int_{1.3}^2 kx^2(2-x)dx$ $= \left[\frac{2kx^3}{3} - \frac{kx^4}{4}\right]_{1.3}^2$ $= 1 - 0.563$ $= 0.437$	M1 A1 **[2]**	Attempt to evaluate the integral between 1.3 and 2 or equivalent Correct answer
	(iv)	the area on the right is < 0.5 oe median is less than 1.3 m	M1 A1ft **[2]**	Sensible reason ft their **(iii)** SR B1ft if correct but 0.5 implied

6 **(i)**	$T_1 + T_2 + T_4 - T_3 \sim N(-0.95, 4.345)$	M1 B1 A1	Correct method to find mean and var of $T_1 + T_2 + T_4 - T_3$ oe Correct mean (3.75 + 3.1 + 3.2 – 11) Correct variance
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	$= P\left(z > \frac{0 - -0.95}{\sqrt{4.345}}\right) = P(z > 0.4557)$	M1	Standardising (appropriate variance involving all 4) and area < 0.5
	= 1 – 0.0.6758 = 0.324	A1 **[6]**	Correct answer
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	$P(\overline{X} < 4) = P\left(z < \frac{4 - 3.1}{\sqrt{0.785/6}}\right)$ $= P(z < 2.488)$	M1	Standardising with sq rt OR (24 – 18.6)/√4.71 no mixed methods
	= 0.994	A1 **[3]**	Correct answer