UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the May/June 2010 question paper

for the guidance of teachers

9709 MATHEMATICS

9709/23 Paper 23, maximum raw mark 50

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the May/June 2010 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only - often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures - this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1		State or imply $y \log 2.8 = x \log 13$	B1	
		Rearrange into form $y = \dfrac{\log 13}{\log 2.8}x$ or equivalent	B1	
		Obtain answer $k = 2.49$	B1	[3]
2	**(i)**	State or imply correct ordinates 0.27067..., 0.20521..., 0.14936...	B1	
		Use correct formula, or equivalent, correctly with $h = 0.5$ and three ordinates	M1	
		Obtain answer 0.21 with no errors seen	A1	[3]
	(ii)	Justify statement that the trapezium rule gives an over-estimate	B1	[1]
3	**EITHER**	State or imply non-modular inequality $(2x-1)^2 < (x+4)^2$, or corresponding equation or pair of linear equations	M1	
		Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations	M1	
		Obtain critical values -1 and 5	A1	
		State correct answer $-1 < x < 5$	A1	[4]
	OR	Obtain one critical value, e.g. $x = 5$, by solving a linear equation (or inequality) or from a graphical method or by inspection	B1	
		Obtain the other critical value similarly	B2	
		State correct answer $-1 < x < 5$	B1	
4	**(a)**	Obtain integral $a \sin 2x$ with $a = \pm\left(1, 2 \text{ or } \dfrac{1}{2}\right)$	M1	
		Use limits and obtain $\dfrac{1}{2}$ (AG)	A1	[2]
	(b)	Use $\tan^2 x = \sec^2 x - 1$ and attempt to integrate both terms	M1	
		Obtain $3\tan x - 3x$	A1	
		Attempt to substitute limits, using exact values	M1	
		Obtain answer $2\sqrt{3} - \dfrac{\pi}{2}$	A1	[4]
5	**(i)**	Use product rule	M1	
		Obtain correct derivative in any form	A1	
		Show that derivative is equal to zero when $x = 3$	A1	[3]
	(ii)	Substitute $x = 1$ into gradient function, obtaining $2e^{-1}$ or equivalent	M1	
		State or imply required y-coordinate is e^{-1}	B1	
		Form equation of line through $(1, e^{-1})$ with gradient found (NOT the normal)	M1	
		Obtain equation in any correct form	A1	[4]
6	**(i)**	Make a recognisable sketch of a relevant graph, e.g. $y = \ln x$ or $y = 2 - x^2$	B1	
		Sketch a second relevant graph and justify the given statement	B1	[2]
	(ii)	Consider sign of $\ln x - (2 - x^2)$ at $x = 1.3$ and $x = 1.4$, or equivalent	M1	
		Complete the argument correctly with appropriate calculations	A1	[2]

	(iii)	Show that given equation is equivalent to $x = \sqrt{(2 - \ln x)}$ or *vice versa*	B1	[1]
	(iv)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 1.31	A1	
		Show sufficient iterations to justify its accuracy to 2 d.p. or show there is a sign change in the interval (1.305, 1.315)	B1	[3]
7	**(i)**	Substitute $x = 3$ and equate to 30	M1	
		Substitute $x = -1$ and equate to 18	M1	
		Obtain a correct equation in any form	A1	
		Solve a relevant pair of equations for a or for b	M1	
		Obtain $a = 1$ and $b = -13$	A1	[5]
	(ii)	Either show that f(2) = 0 or divide by $(x - 2)$, obtaining a remainder of zero	B1	
		Obtain quadratic factor $2x^2 + 5x - 3$	B1	
		Obtain linear factor $2x - 1$	B1	
		Obtain linear factor $x + 3$	B1	
		[Condone omission of repetition that $x - 2$ is a factor.]		
		[If linear factors $2x - 1$, $x + 3$ obtained by remainder theorem or inspection, award B2 + B1.]		[4]
8	**(i)**	Use correct $\sin(A - B)$ and $\cos(A - B)$ formulae	M1	
		Substitute exact values for sin 30° etc.	M1	
		Obtain given answer correctly	A1	[3]
	(ii)	State $\sqrt{3}\sin x = \frac{1}{2}\sec x$	B1	
		Rearrange to $\sin 2x = k$, where k is a non-zero constant	M1	
		Carry out evaluation of $\frac{1}{2}\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$	M1	
		Obtain answer 17.6°	A1	
		Carry out correct method for second answer	M1	
		Obtain remaining 3 answers from 17.6°, 72.4°, 197.6°, 252.4° and no others in the range	A1	[6]
		[Ignore answers outside the given range]		