UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the May/June 2010 question paper

for the guidance of teachers

9709 MATHEMATICS

9709/32 Paper 32, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the May/June 2010 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	*EITHER*:	Attempt to solve for 2^x	M1	
		Obtain $2^x = 6/4$, or equivalent	A1	
		Use correct method for solving an equation of the form $2^x = a$, where $a > 0$	M1	
		Obtain answer $x = 0.585$	A1	
	OR:	State an appropriate iterative formula, e.g. $x_{n+1} = \ln((2^{x_n} + 6) / 5) / \ln 2$	B1	
		Use the iterative formula correctly at least once	M1	
		Obtain answer $x = 0.585$	A1	
		Show that the equation has no other root but 0.585	A1	[4]

[For the solution 0.585 with no relevant working, award B1 and a further B1 if 0.585 is shown to be the only root.]

2		Integrate by parts and reach $\pm x^2 \cos x \pm \int 2x \cos x \, dx$	M1	
		Obtain $-x^2 \cos x + \int 2x \cos x \, dx$, or equivalent	A1	
		Complete the integration, obtaining $-x^2 \cos x + 2x \sin x + 2 \cos x$, or equivalent	A1	
		Substitute limits correctly, having integrated twice	M1	
		Obtain the given answer correctly	A1	[5]
3	(i)	State or imply $\sin a = 4/5$	B1	
		Use $\sin(A - B)$ formula and substitute for $\cos a$ and $\sin a$	M1	
		Obtain answer $\frac{1}{10}(4\sqrt{3} - 3)$, or exact eqivalent	A1	[3]
	(ii)	Use $\tan 2A$ formula and substitute for $\tan a$, or use $\sin 2A$ and $\cos 2A$ formulae, substitute $\sin a$ and $\cos a$, and divide	M1	
		Obtain $\tan 2a = -\frac{24}{7}$, or equivalent	A1	
		Use $\tan(A + B)$ formula with $A = 2a$, $B = a$ and substitute for $\tan 2a$ and $\tan a$	M1	
		Obtain $\tan 3a = -\frac{44}{117}$	A1	[4]
4	(i)	Use correct quotient or product rule	M1	
		Obtain correct derivative in any form	A1	
		Equate derivative to zero and solve for x	M1	
		Obtain the given answer correctly	A1	[4]
	(ii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 4.49	A1	
		Show sufficient iterations to at least 4 d.p. to justify its accuracy to 2 d.p., or show that there is a sign change in the interval (4.485, 4.495)	A1	[3]
5	(i)	Substitute $x = -\frac{1}{2}$, equate to zero and obtain a correct equation, e.g. $-\frac{1}{4} + \frac{5}{4} - \frac{1}{2}a + b = 0$	B1	
		Substitute $x = -2$ and equate to 9	M1	
		Obtain a correct equation, e.g. $-16 + 20 - 2a + b = 9$	A1	
		Solve for a or for b	M1	
		Obtain $a = -4$ and $b = -3$	A1	[5]

(ii) Attempt division by $2x + 1$ reaching a partial quotient of $x^2 + kx$ M1
Obtain quadratic factor $x^2 + 2x - 3$ A1
Obtain factorisation $(2x+1)(x+3)(x-1)$ A1 [3]

[The M1 is earned if inspection has an unknown factor of $x^2 + ex + f$ and an equation in e and/or f, or if two coefficients with the correct moduli are stated without working.]
[If linear factors are found by the factor theorem, give B1 + B1 for $(x - 1)$ and $(x + 3)$, and then B1 for the complete factorisation.]

6 **(i)** *EITHER*: State or imply $\frac{1}{y}\frac{dy}{dx}$ as derivative of $\ln y$ B1
State correct derivative of LHS, e.g. $\ln y + \frac{x}{y}\frac{dy}{dx}$ B1
Differentiate RHS and obtain an expression for $\frac{dy}{dx}$ M1
Obtain given answer A1
OR 1: State $\ln y = \frac{2x+1}{x}$, or equivalent, and differentiate both sides M1
State correct derivative of LHS, e.g. $\frac{1}{y}\frac{dy}{dx}$ B1
State correct derivative of RHS, e.g. $-1/x^2$ B1
Rearrange and obtain given answer A1
OR 2: State $y = \exp(2 + 1/x)$, or equivalent, and attempt differentiation by chain rule M1
State correct derivative of RHS, e.g. $-\exp(2+1/x)/x^2$ B1 + B1
Obtain given answer A1 [4]
[The B marks are for the exponential term and its multiplier.]

(ii) State or imply $x = -\frac{1}{2}$ when $y = 1$ B1
Substitute and obtain gradient of -4 B1√
Correctly form equation of tangent M1
Obtain final answer $y + 4x + 1 = 0$, or equivalent A1 [4]

7 **(i)** Separate variables correctly and attempt integration of both sides B1
Obtain term $\tan x$ B1
Obtain term $-\frac{1}{2}e^{-2t}$ B1
Evaluate a constant or use limits $x = 0$, $t = 0$ in a solution containing terms $a \tan x$ and be^{-2t} M1
Obtain correct solution in any form, e.g. $\tan x = \frac{1}{2} - \frac{1}{2}e^{-2t}$ A1
Rearrange as $x = \tan^{-1}(\frac{1}{2} - \frac{1}{2}e^{-2t})$, or equivalent A1 [6]

(ii) State that x approaches $\tan^{-1}(\frac{1}{2})$ B1 [1]

(iii) State that $1 - e^{-2t}$ increases and so does the inverse tangent, or state that $e^{-2t}\cos^2 x$ is positive B1 [1]

8	**(i)**	*EITHER*: State a correct expression for $\lvert z \rvert$ or $\lvert z \rvert^2$, e.g. $(1+\cos 2\theta)^2 + (\sin 2\theta)^2$	B1	
		Use double angle formulae throughout or Pythagoras	M1	
		Obtain given answer 2cos θ correctly	A1	
		State a correct expression for tangent of argument, e.g. $(\sin 2\theta/(1+\cos 2\theta)$	B1	
		Use double angle formulae to express it in terms of cos θ and sin θ	M1	
		Obtain tan θ and state that the argument is θ	A1	
		OR: Use double angle formulae to express z in terms of cos θ and sin θ	M1	
		Obtain a correct expression, e.g. $1+\cos^2\theta - \sin^2\theta + 2\mathrm{i}\sin\theta\cos\theta$	A1	
		Convert the expression to polar form	M1	
		Obtain $2\cos\theta(\cos\theta + \mathrm{i}\sin\theta)$	A1	
		State that the modulus is 2 cosθ	A1	
		State that the argument is θ	A1	[6]
	(ii)	Substitute for z and multiply numerator and denominator by the conjugate of z, or equivalent	M1	
		Obtain correct real denominator in any form	A1	
		Identify and obtain real part equal to $\frac{1}{2}$	A1	[3]
9	**(i)**	State or imply a correct normal vector to either plane, e.g. 3**i** + 2**j** + 4**k** or a**i + j** + **k**	B1	
		Equate scalar product of normals to zero and obtain an equation in a, e.g. $3a + 2 + 4 = 0$	M1	
		Obtain $a = -2$	A1	[3]
	(ii)	Express general point of the line in component form, e.g. $(\lambda, 1 + 2\lambda, -1 + 2\lambda)$	B1	
		Either substitute components in the equation of p and solve for λ, or substitute components and the value of a in the equation of q and solve for λ	M1*	
		Obtain $\lambda = 1$ for point A	A1	
		Obtain $\lambda = 2$ for point B	A1	
		Carry out correct process for finding the length of AB	M1(dep*)	
		Obtain answer $AB = 3$	A1	[6]
		[The second M mark is dependent on both values of λ being found by correct methods.]		
10	**(i)**	*EITHER*: Divide by denominator and obtain quadratic remainder	M1	
		Obtain $A = 1$	A1	
		Use any relevant method to obtain B, C or D	M1	
		Obtain one correct answer	A1	
		Obtain $B = 2$, $C = 1$ and $D = -3$	A1	
		OR: Reduce RHS to a single fraction and equate numerators, or equivalent	M1	
		Obtain $A = 1$	A1	
		Use any relevant method to obtain B, C or D	M1	
		Obtain one correct answer	A1	
		Obtain $B = 2$, $C = 1$ and $D = -3$	A1	[5]
		[SR: If $A = 1$ stated without working give B1.]		
	(ii)	Integrate and obtain $x + 2\ln x - \dfrac{1}{x} - \dfrac{3}{2}\ln(2x-1)$, or equivalent	B3√	
		(The f.t. is on A, B, C, D. Give B2√ if only one error in integration; B1√ if two.)		
		Substitute limits correctly in the complete integral	M1	
		Obtain given answer correctly following full and exact working	A1	[5]