MARK SCHEME for the May/June 2010 question paper

for the guidance of teachers

9709 MATHEMATICS

9709/71   Paper 71, maximum raw mark 50

UNIVERSITY *of* CAMBRIDGE
International Examinations

Mark Scheme Notes

Marks are of the following three types:

M   Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A   Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B   Mark for a correct result or statement independent of method marks.

• When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

• The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

• Note: B2 or A2 means that the candidate can earn 2 or 0.
            B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

• Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

• For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:
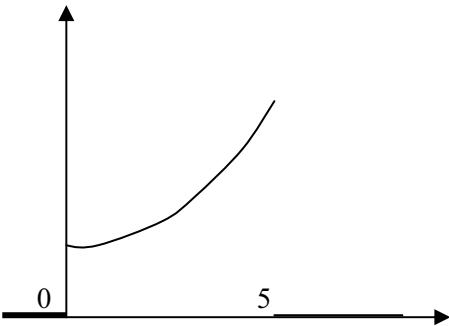
AEF   Any Equivalent Form (of answer is equally acceptable)

AG   Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD   Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO   Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO   Correct Working Only – often written by a 'fortuitous' answer

ISW   Ignore Subsequent Working

MR   Misread

PA   Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS   See Other Solution (the candidate makes a better attempt at the same question)

SR   Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1   A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through $\sqrt{}$" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1   This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	(i)	1/12	B1 [1]	Accept 0.0833
	(ii)	trains arrive every 12 minutes	B1 [1]	must have 'every 12 minutes'

2	(i)	0.145 $= 87 / n$ $n = 600$	B1 M1 A1 [3]	correct mid-point equating their mid-point with $87 / n$ correct answer
	(ii)	$0.0321 = z \times \sqrt{\dfrac{0.145(1-0.145)}{600}}$ $z = 2.233 \quad \Phi(z) = 0.9872$ width of CI is $1 - 2 \times (1 - 0.9872)$ $\alpha = 97.4\%$	B1 M1 M1 A1 [4]	0.0321 seen or implied Equating half-width with $z \times \sqrt{\dfrac{pq}{n}}$ Correct method to find width of CI Correct answer

3	(i)	$z = \dfrac{2.55 - 2.62}{0.3 / \sqrt{45}} = -1.565$ $P(z > -1.565) = 0.941$	M1 M1 A1 [3]	Standardising no cc Dividing 0.3 by $\sqrt{45}$ as denominator Correct answer (Accept equivalent method using totals)
	(ii)	rejection region is $m < a_1$ and $m > a_2$ where $\dfrac{a_1 - 2.62}{0.3 / \sqrt{30}} = -1.645$ and $\dfrac{a_2 - 2.62}{0.3 / \sqrt{30}} = 1.645$ $m < 2.53$ and $m > 2.71$	 B1 M1 M1 A1 [4]	±1.645 seen one correct unsimplified equation of correct form second unsimplified equation of correct form (or clear use of 1-tail test and ±1.282 used) correct answer

4	(i)	$Mr - 5Mrs \sim N(512 - 5\times89, 62^2 + 25\times7.4^2)$	B1	Correct unsimplified mean
		$\sim N(67, 5213)$	B1	Correct unsimplified variance
		$P(Mr > 5\,Mrs) = P(Mr - 5\,Mrs > 0)$	M1	Using distribution Mr – 5 Mrs
		$= P\left(z > \dfrac{0 - 67}{\sqrt{5213}}\right)$	M1	Standardising and using tables
		$= P(z > -0.9280)$		
		$= 0.823$	A1 [5]	Correct answer

	(ii)	$Mr + Mrs \sim N(601, 62^2 + 7.4^2)$	B1	Correct mean and variance
		$E[5/8(Mr + Mrs)] = 376$ miles	B1	Correct answer
				SR Two separate answers 320 and 55.6 B1
		$Var[5/8(Mr + Mrs)] = \dfrac{25}{64}\times3898.76$		
		$= 1520$		
		sd $= 39.0$ miles	B1 [3]	Correct answer

5	(i)	$\displaystyle\int_0^5 ke^{0.2t}\,dt = 1$	M1	Equating to 1 and attempting to integrate
		$\left[\dfrac{k}{0.2}e^{1.0}\right] - \left[\dfrac{k}{0.2}e^0\right] = 1$	A1	Correct integrand and limits
		$\dfrac{k}{0.2}(e-1) = 1$		
		$k = \dfrac{1}{5(e-1)}$ AG	A1 [3]	Correct answer legitimately obtained

	(ii)		B1	Correct curve shape
			B1 [2]	Correct horizontal lines (need to see a 5)

	(iii)	$\displaystyle\int_0^T ke^{0.2t}\,dt = 0.2$	M1	Equation relating T and 0.2 or 0.8
		$\left[5ke^{0.2T}\right] - \left[5k\right] = 0.2$	A1	Correct equation (can be in 'k')
		$e^{0.2T} = \dfrac{0.2}{5k} + 1 = 1.344$		
		$T = 1.48$ (seconds)	A1 [3]	Correct answer

6	**(i)**	$\lambda_A = np = 0.022 \times 55 = 1.21$ $\lambda_B = 0.058 \times 55 = 3.19$ total $\lambda = 4.4$ P(more than 2) = 1 – P(0, 1, 2) $= 1 - e^{-4.4}\left(1 + 4.4 + \dfrac{4.4^2}{2!}\right)$ $= 1 - 0.185$ $= 0.815$	M1 A1 M1 A1 [4]	Two different np (can be implied) Correct total 4.4 (or alt method: 6 correct combinations 0,0 1,0 etc stated and used) Finding 1 – P(0, 1, 2), Poisson, any mean, allow one end error. (Or combinations method – *use* at least 4 and find 1 – P($\leqslant$2)) Correct answer
	(ii)	$\lambda = 0.08n$ P(at least 1 stained tablecloth) = 1 – P(0) $1 - e^{-0.08n} > 0.99$ $0.01 > e^{-0.08n}$ $n > 57.6$ least value of $n = 58$	B1 M1 M1 A1 [4]	Correct λ Equation of correct form relating their λ and 0.99 Valid attempt to solve equation of correct form by logs or trial and error Correct answer (SR Accept use of Binomial leading to $n = 57$)
7	**(i)**	Type I error is made when we say the number of white blood cells has decreased when it hasn't. P(0) = $e^{-5.2}$ = 0.005516 P(1) = $e^{-5.2}$(5.2) = 0.02868 Σ < 0.10 P(2) = $e^{-5.2}$(5.2^2/2) = 0.07458 Σ > 0.10 P(Type I error) = 0.0342	B1 M1 M1* A1dep [4]	Correct and relating to question Evaluating at least 2 of P(X = 0, 1, 2) Comparing their Σ 3 probs with 10% (must be Σ probs) Correct answer, dep on previous M
	(ii)	H_0: $\lambda = 5.2$ H_1: $\lambda < 5.2$ P(0+1+2) = 0.1087 > 10% 2 not in C Region. Accept H_0. Not enough evidence to say the number of blood cells has decreased.	B1 M1 A1 [3]	Both hypotheses correct Stating 2 is not in the critical region from above, or evaluating P(0, 1, 2) and comparing with 10% again Correct conclusion no contradictions
	(iii)	P(Type II error) = 1 – P(0, 1) $= 1 - e^{-4.1}(1 + 4.1)$ $= 0.915$	B1 M1 A1 [3]	Identifying correct area (indep) Some form of (Poisson) expression with mean 4.1 Correct answer