UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the May/June 2012 question paper

for the guidance of teachers

9709 MATHEMATICS

9709/33 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- Cambridge will not enter into discussions or correspondence in connection with these mark schemes.

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International Examinations

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1 *EITHER*: Obtain a correct unsimplified version of the x or x^2 term of the expansion of $(4+3x)^{-\frac{1}{2}}$ or $(1+\frac{3}{4}x)^{-\frac{1}{2}}$ M1

State correct first term $\frac{1}{2}$ B1

Obtain the next two terms $-\frac{3}{16}x+\frac{27}{256}x^2$ A1 + A1

OR: Differentiate and evaluate f(0) and f′(0), where $\mathrm{f}'(x)=k(4+3x)^{-\frac{3}{2}}$ M1

State correct first term $\frac{1}{2}$ B1

Obtain the next two terms $-\frac{3}{16}x+\frac{27}{256}x^2$ A1 + A1 [4]

[Symbolic coefficients, e.g. $\binom{-\frac{1}{2}}{2}$ are not sufficient for the M or B mark.]

2 Use law of the logarithm of a power and a product or quotient and remove logarithms M1

Obtain a correct equation in any form, e.g. $\frac{2x+3}{x^2}=3$ A1

Solve 3-term quadratic obtaining at least one root M1

Obtain final answer 1.39 only A1 [4]

3 Obtain $\frac{\mathrm{d}x}{\mathrm{d}\theta}=2\cos 2\theta-1$ or $\frac{\mathrm{d}y}{\mathrm{d}\theta}=-2\sin 2\theta+2\cos\theta$, or equivalent B1

Use $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}\theta}\div\frac{\mathrm{d}x}{\mathrm{d}\theta}$ M1

Obtain $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-2\sin 2\theta+2\cos\theta}{2\cos 2\theta-1}$, or equivalent A1

At any stage use correct double angle formulae throughout M1

Obtain the given answer following full and correct working A1 [5]

4 **(i)** Use correct quotient or product rule M1

Obtain correct derivative in any form, e.g. $\frac{2\mathrm{e}^{2x}}{x^3}-\frac{3\mathrm{e}^{2x}}{x^4}$ A1

Equate derivative to zero and solve a 2-term equation for non-zero x M1

Obtain $x=\frac{3}{2}$ correctly A1 [4]

(ii) Carry out a method for determining the nature of a stationary point, e.g. test derivative either side M1

Show point is a minimum with no errors seen A1 [2]

5 **(i)** Substitute for x, separate variables correctly and attempt integration of both sides — M1
Obtain term $\ln y$, or equivalent — A1
Obtain term e^{-3t}, or equivalent — A1
Evaluate a constant, or use $t = 0$, $y = 70$ as limits in a solution containing terms $a\ln y$ and be^{-3t} — M1
Obtain correct solution in any form, e.g. $\ln y - \ln 70 = e^{-3t} - 1$ — A1
Rearrange and obtain $y = 70\exp(e^{-3t} - 1)$, or equivalent — A1 [6]

(ii) Using answer to part (**i**), either express p in terms of t or use $e^{-3t} \to 0$ to find the limiting value of y — M1
Obtain answer $\frac{100}{e}$ from correct exact work — A1 [2]

6 **(i)** Use tan $(A + B)$ and tan $2A$ formulae to obtain an equation in tan x — M1
Obtain a correct equation in tan x in any form — A1
Obtain an expression of the form $a\tan^2 x = b$ — M1
Obtain the given answer — A1 [4]

(ii) Substitute $k = 4$ in the given expression and solve for x — M1
Obtain answer, e.g. $x = 16.8°$ — A1
Obtain second answer, e.g. $x = 163.2°$, and no others in the given interval — A1 [3]
[Ignore answers outside the given interval. Treat answers in radians as a misread and deduct A1 from the marks for the angles.]

(iii) Substitute $k = 2$, show $\tan^2 x < 0$ and justify given statement correctly — B1 [1]

7 **(i)** Substitute for x and dx throughout the integral — M1
Obtain $\int 2u\cos u \, du$ — A1
Integrate by parts and obtain answer of the form $au\sin u + b\cos u$, where $ab \neq 0$ — M1
Obtain $2u\sin u + 2\cos u$ — A1
Use limits $u = 0$, $u = p$ correctly and equate result to 1 — M1
Obtain the given answer — A1 [6]

(ii) Use the iterative formula correctly at least once — M1
Obtain final answer $p = 1.25$ — A1
Show sufficient iterations to 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (1.245, 1.255) — A1 [3]

8 **(i)** State or imply the form $A+\frac{B}{x+1}+\frac{C}{2x-3}$ B1

State or obtain $A = 2$ B1

Use a correct method for finding a constant M1

Obtain $B = -2$ A1

Obtain $C = -1$ A1 [5]

(ii) Obtain integral $2x-2\ln(x+1)-\frac{1}{2}\ln(2x-3)$ B3√

(Deduct B1√ for each error or omission. The f.t. is on A, B, C.)

Substitute limits correctly in an expression containing terms $a\ln(x+1)$ and $b\ln(2x-3)$ M1

Obtain the given answer following full and exact working A1 [5]

[SR: If A omitted from the form of fractions, give B0B0M1A0A0 in (**i**); B1√B1√M1A0 in (**ii**).]

[SR: For a solution starting with $\frac{B}{x+1}+\frac{Dx+E}{2x-3}$, give M1A1 for one of $B = -2$, $D = 4$, $E = -7$ and A1 for the other two constants; then give B1B1 for $A = 2$, $C = -1$.]

[SR: For a solution starting with $\frac{Fx+G}{x+1}+\frac{C}{2x-3}$ or with $\frac{Fx}{x+1}+\frac{C}{2x-3}$, give M1A1 for one of $C = -1$, $F = 2$, $G = 0$ and A1 for the other constants or constant; then give B1B1 for $A = 2$, $B = -2$.]

9 **(i)** Express general point of l or m in component form, i.e. $(3-\lambda,-2+2\lambda,1+\lambda)$ or $(4+a\mu,4+b\mu,2-\mu)$ B1

Equate components and eliminate either λ or μ from a pair of equations M1

Eliminate the other parameter and obtain an equation in a and b M1

Obtain the given answer A1 [4]

(ii) Using the correct process equate the scalar product of the direction vectors to zero M1*

Obtain $-a+2b-1=0$, or equivalent A1

Solve simultaneous equations for a or for b M1(dep*)

Obtain $a = 3$, $b = 2$ A1 [4]

(iii) Substitute found values in component equations and solve for λ or for μ M1

Obtain answer $\mathbf{i}+2\mathbf{j}+3\mathbf{k}$ from either $\lambda = 2$ or from $\mu = -1$ A1 [2]

10 (a) EITHER: Eliminate u or w and obtain an equation in w or in u — M1

Obtain a quadratic in u or w, e.g. $u^2 - 4iu - 5 = 0$ or $w^2 + 4iw - 5 = 0$ — A1

Solve a 3-term quadratic for u or for w — M1

OR1: Having squared the first equation, eliminate u or w and obtain an equation in w or u — M1

Obtain a 2-term quadratic in u or w, e.g. $u^2 = -3 + 4i$ — A1

Solve a 2-term quadratic for u or for w — M1

OR2: Using $u = a + ib$, $w = c + id$, equate real and imaginary parts and obtain 4 equations in a, b, c and d — M1

Obtain 4 correct equations — A1

Solve for a and b, or for c and d — M1

Obtain answer $u = 1 + 2i$, $w = 1 - 2i$ — A1

Obtain answer $u = -1 + 2i$, $w = -1 - 2i$ and no other — A1 [5]

(b) (i) Show point representing $2 - 2i$ in relatively correct position — B1

Show a circle with centre $2 - 2i$ and radius 2 — B1√

Show line for $\arg z = -\frac{1}{4}\pi$ — B1

Show line for $\mathrm{Re}\, z = 1$ — B1

Shade the relevant region — B1 [5]

(ii) State answer $2 + \sqrt{2}$, or equivalent (accept 3.41) — B1 [1]