CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the May/June 2013 series

9709 MATHEMATICS

9709/32 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2013 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a "fortuitous" answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through $\sqrt{}$" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	*EITHER*:	State or imply non-modular equation $(x-2)^2 = \left(\frac{1}{3}x\right)^2$,		
		or pair of equations $x-2=\pm\frac{1}{3}x$	M1	
		Obtain answer $x = 3$	A1	
		Obtain answer $x = \frac{3}{2}$, or equivalent	A1	
	OR:	Obtain answer $x = 3$ by solving an equation or by inspection	B1	
		State or imply the equation $x - 2 = -\frac{1}{3}$, or equivalent	M1	
		Obtain answer $x = \frac{3}{2}$, or equivalent	A1	[3]
2	**(i)**	Use the iterative formula correctly at least once	M1	
		Obtain final answer 3.6840	A1	
		Show sufficient iterations to at least 6 d.p. to justify 3.6840, or show there is a sign change in the interval (3.68395, 3.68405)	A1	[3]
	(ii)	State a suitable equation, e.g. $x = \frac{x(x^3+100)}{2(x^3+25)}$	B1	
		State that the value of α is $3\sqrt{50}$, or exact equivalent	B1	[2]
3	*EITHER*:	State or imply $\ln y = \ln A - kx^2$	B1	
		Substitute values of $\ln y$ and x^2, and solve for k or $\ln A$	M1	
		Obtain $k = 0.42$ or $A = 2.80$	A1	
		Solve for $\ln A$ or k	M1	
		Obtain $A = 2.80$ or $k = 0.42$	A1	
	OR1:	State or imply $\ln y = \ln A - kx^2$	B1	
		Using values of $\ln y$ and x^2, equate gradient of line to $-k$ and solve for k	M1	
		Obtain $k = 0.42$	A1	
		Solve for $\ln A$	M1	
		Obtain $A = 2.80$	A1	
	OR2:	Obtain two correct equations in k and A and substituting y– and x^2 – values in $y = Ae^{-kx^2}$	B1	
		Solve for k	M1	
		Obtain $k = 0.42$	A1	
		Solve for A	M1	
		Obtain $A = 2.80$	A1	[5]
		[SR: If unsound substitutions are made, e.g. using x = 0.;64 and y = 0.76, give B1M0A0M1A0 in the *EITHER* and *OR1* schemes, and B0M1A0M1A0 in the *OR2* scheme.]		

4 **(i)** Substitute $x = -\frac{1}{3}$, or divide by $3x+1$, and obtain a correct equation,

e.g. $-\frac{1}{27}a - \frac{20}{9} - \frac{1}{3} + 3 = 0$ — B1

Solve for a an equation obtained by a valid method — M1

Obtain $a = 12$ — A1 [3]

(ii) Commence division by $3x+1$ reaching a partial quotient $\frac{1}{3}ax^2 + kx$ — M1

Obtain quadratic factor $4x^2 - 8x + 3$ — A1

Obtain factorisation $(3x+1)(2x-1)(2x-3)$ — A1 [3]

[The M1 is earned if inspection reaches an unknown factor $\frac{1}{3}ax^2 + Bx + C$ and an equation in B and/or C, or an unknown factor $Ax^2 + Bx + 3$ and an equation in A and/or B, or if two coefficients with the correct moduli are stated without working.]
[If linear factors are found by the factor theorem, give B1B1 for $(2x – 1)$ and $(2x – 3)$, and B1 for the complete factorisation.]
[Synthetic division giving $12x^2 – 24x + 9$ as quadratic factor earns M1A1, but the final factorisation needs $(x + \frac{1}{3})$, or equivalent, in order to earn the second A1.]

[SR: If $x = \frac{1}{3}$ is used in substitution or synthetic division, give the M1 in part **(i)** but give M0 in part **(ii)**.]

5 *EITHER*: State $2ay\frac{dy}{dx}$ as derivative of ay^2 — B1

State $y^2 + 2xy\frac{dy}{dx}$ as derivative of xy^2 — B1

Equate derivative of LHS to zero and set $\frac{dy}{dx}$ equal to zero — M1

Obtain $3x^2 + y^2 - 6ax = 0$, or horizontal equivalent — A1

Eliminate y and obtain an equation in x — M1

Solve for x and obtain answer $x = \sqrt{3}a$ — A1

OR1: Rearrange equation in the form $y^2 = \frac{3ax^2 - x^3}{x+a}$ and attempt differentiation of one side — B1

Use correct quotient or product rule to differentiate RHS — M1

Obtain correct derivative of RHS in any form — A1

Set $\frac{dy}{dx}$ equal to zero and obtain an equation in x — M1

Obtain a correct horizontal equation free of surds — A1

Solve for x and obtain answer $x = \sqrt{3a}$ — A1

OR2: Rearrange equation in the form $y = \left(\frac{3ax^2 - x^3}{x+a}\right)^{\frac{1}{2}}$ and differentiation of RHS — B1

Use correct quotient or product rule and chain rule — M1

Obtain correct derivative in any form — A1

		Equate derivative to zero and obtain an equation in x	M1	
		Obtain a correct horizontal equation free of surds	A1	
		Solve for x and obtain answer $x = \sqrt{3}a$	A1	[6]
6	**(i)**	Use correct quotient or chain rule to differentiate sec x	M1	
		Obtain given derivative, sec x tan x, correctly	A1	
		Use chain rule to differentiate y	M1	
		Obtain the given answer	A1	[4]
	(ii)	Using $\mathrm{d}x\sqrt{3}\sec^2\theta\ \mathrm{d}\theta$, or equivalent, express integral in terms of θ and dθ	M1	
		Obtain $\int\sec\theta\ \mathrm{d}\theta$	A1	
		Use limits $\frac{1}{6}\pi$ and $\frac{1}{3}\pi$ correctly in an integral form of the form $k\ln(\sec\theta + \tan\theta)$	M1	
		Obtain a correct exact final answer in the given form, e.g. $\ln\left(\frac{2+\sqrt{3}}{\sqrt{3}}\right)$	A1	[4]
7	**(i)**	Use cos (A + B) formula to express the given expression in terms of cos x and sin x	M1	
		Collect terms and reach $\frac{\cos x}{\sqrt{2}} - \frac{3}{\sqrt{2}}\sin x$, or equivalent	A1	
		Obtain R = 2.236	A1	
		Use trig formula to find α	M1	
		Obtain α = 71.57° with no errors seen	A1	[5]
	(ii)	Evaluate $\cos^{-1}$ (2/2.236) to at least 1 d.p. (26.56° to 2 d.p., use of $R = \sqrt{5}$ gives 26.57°)	B1√	
		Carry out an appropriate method to find a value of x in the interval $0° < x < 360°$	M1	
		Obtain answer, e.g. x = 315° (315.0°)	A1	
		Obtain second answer, e.g. 261.9° and no others in the given interval	A1	[4]
		[Ignore answers outside the given range.]		
		[Treat answers in radians as a misread and deduct A1 from the answers for the angles.]		
		[SR: Conversion of the equation to a correct quadratic in sin x, cos x, or tan x earns B1, then M1 for solving a 3-term quadratic and obtaining a value of x in the given interval, and A1 + A1 for the two correct answers (candidates must reject spurious roots to earn the final A1).]		
8	**(i)**	Use any relevant method to determine a constant	M1	
		Obtain one of the values A = 1, B = –2, C = 4	A1	
		Obtain a second value	A1	
		Obtain the third value	A1	[4]
		[If A and C are found by the cover up rule, give B1 + B1 then M1A1 for finding B. If only one is found by the rule, give B1M1A1A1.]		
	(ii)	Separate variables and obtain one term by integrating $\frac{1}{y}$ or a partial fraction	M1	
		Obtain $\ln y = -\frac{1}{2} - 2\ln(2x+1) + c$, or equivalent	A3√	

		Evaluate a constant, or use limits $x = 1$, $y = 1$, in a solution containing at least three terms of the form $k \ln y$, l/x, $m \ln x$ and $n \ln (2x + 1)$, or equivalent	M1	
		Obtain solution $\ln y = -\dfrac{1}{2} - 2\ln x + 2\ln(2x+1) + c$, or equivalent	A1	
		Substitute $x = 2$ and obtain $y = \dfrac{25}{36}e^{\frac{1}{2}}$, or exact equivalent free of logarithms	A1	[7]
		(The f.t. is on A, B, C. Give A2$\checkmark$ if there is only one error or omission in the integration; A1$\checkmark$ if two.)		
9	**(a)**	Substitute $w = x + iy$ and state a correct equation in x and y	B1	
		Use $i^2 = -1$ and equate real parts	M1	
		Obtain $y = -2$	A1	
		Equate imaginary parts and solve for x	M1	
		Obtain $x = 2\sqrt{2}$, or equivalent, only	A1	[5]
	(b)	Show a circle with centre 2i	B1	
		Show a circle with radius 2	B1	
		Show half line from –2 at $\frac{1}{4}\pi$ to real axis	B1	
		Shade the correct region	B1	
		Carry out a complete method for calculating the greatest value of $\lvert z \rvert$	M1	
		Obtain answer 3.70	A1	[6]
10	**(i)**	Carry out a correct method for finding a vector equation for AB	M1	
		Obtain $\mathbf{r} = 2\mathbf{i} - 3\mathbf{j} + 2\mathbf{k} + \lambda(3\mathbf{i} + \mathbf{j} - \mathbf{k})$ or $\mathbf{r} = \mu(2\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}) + (1 - \mu)(5\mathbf{i} - 2\mathbf{j} + \mathbf{k})$, or equivalent	A1	
		Substitute components in equation of p and solve for λ or for μ	M1	
		Obtain $\lambda = \dfrac{3}{2}$ or $\mu = -\dfrac{1}{2}$ and final answer $\dfrac{13}{2}\mathbf{i} - \dfrac{3}{2}\mathbf{j} + \dfrac{1}{2}\mathbf{k}$, or equivalent	A1	[4]
	(ii)	Either equate scalar product of direction vector of AB and normal to q to zero or substitute for A and B in the equation of q and subtract expressions	M1*	
		Obtain $3 + b - c = 0$, or equivalent	A1	
		Using the correct method for the moduli, divide the scalar product of the normals to p and q by the product of their moduli and equate to $\pm\dfrac{1}{2}$, or form horizontal equivalent	M1*	
		Obtain correct equation in any form, e.g. $\dfrac{1+b}{\sqrt{(1+b^2+c^2)}\sqrt{(1+1)}} = \pm\dfrac{1}{2}$	A1	
		Solve simultaneous equations for b or for c	M1 (dep*)	
		Obtain $b = -4$ and $c = -1$	A1	
		Use a relevant point and obtain final answer $x - 4y - z = 12$, or equivalent	A1$\checkmark$	[7]
		(The f.t. is on b and c.)		