CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the May/June 2014 series

9709 MATHEMATICS

9709/31 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2014 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1 **(i)** State $\sin 2\alpha = 2\sin\alpha\cos\alpha$ and $\sec\alpha = 1/\cos\alpha$ B1
Obtain $2\sin\alpha$ B1 [2]

(ii) Use $\cos 2\beta = 2\cos^2\beta - 1$ or equivalent to produce correct equation in $\cos\beta$ B1
Solve three-term quadratic equation for $\cos\beta$ M1
Obtain $\cos\beta = \frac{1}{3}$ only A1 [3]

2 State $\frac{du}{dx} = 3\sec^2 x$ or equivalent B1

Express integral in terms of u and du (accept unsimplified and without limits) M1

Obtain $\int \frac{1}{3} u^{\frac{1}{2}}\, du$ A1

Integrate $Cu^{\frac{1}{2}}$ to obtain $\frac{2C}{3}u^{\frac{3}{2}}$ M1

Obtain $\frac{14}{9}$ A1 [5]

3 Obtain $\frac{2}{2t+3}$ for derivative of x B1

Use quotient of product rule, or equivalent, for derivative of y M1

Obtain $\frac{5}{(2t+3)^2}$ or unsimplified equivalent A1

Obtain $t = -1$ B1

Use $\frac{dy}{dx} = \frac{dy}{dt} / \frac{dx}{dt}$ in algebraic or numerical form M1

Obtain gradient $\frac{5}{2}$ A1 [6]

4 Separate variables correctly and recognisable attempt at integration of at least one side M1

Obtain $\ln y$, or equivalent B1

Obtain $k\ln\left(2 + e^{3x}\right)$ B1

Use $y(0) = 36$ to find constant in $y = A\left(2 + e^{3x}\right)^k$ or $\ln y = k\ln\left(2 + e^{3x}\right) + c$ or equivalent M1*

Obtain equation correctly without logarithms from $\ln y = \ln\left(A\left(2 + e^{3x}\right)^k\right)$ *M1

Obtain $y = 4\left(2 + e^{3x}\right)^2$ A1 [6]

5 **(i)** Either Multiply numerator and denominator by $\sqrt{3}+\mathrm{i}$ and use $\mathrm{i}^2=-1$ M1

Obtain correct numerator $18+18\sqrt{3}\mathrm{i}$ or correct denominator 4 B1

Obtain $\frac{9}{2}+\frac{9}{2}\sqrt{3}\mathrm{i}$ or $\left(18+18\sqrt{3}\mathrm{i}\right)/4$ A1

Obtain modulus or argument M1

Obtain $9\mathrm{e}^{\frac{1}{3}\pi\mathrm{i}}$ A1 [5]

OR Obtain modulus and argument of numerator or denominator, or both moduli or both arguments M1

Obtain moduli and argument 18 and $\frac{1}{6}\pi$ or 2 and $-\frac{1}{6}\pi$

or moduli 18 and 2 or arguments $\frac{1}{6}\pi$ and $-\frac{1}{6}\pi$ (allow degrees) B1

Obtain $18\mathrm{e}^{\frac{1}{6}\pi\mathrm{i}}\div 2\mathrm{e}^{-\frac{1}{6}\pi\mathrm{i}}$ or equivalent A1

Divide moduli and subtract arguments M1

Obtain $9\mathrm{e}^{\frac{1}{3}\pi\mathrm{i}}$ A1 [5]

(ii) State $3\mathrm{e}^{\frac{1}{6}\pi\mathrm{i}}$, following through their answer to part (i) B1√

State $3\mathrm{e}^{\frac{1}{6}\pi\mathrm{i}\pm\frac{1}{2}\pi\mathrm{i}}$, following through their answer to part (i) B1√

Obtain $3\mathrm{e}^{-\frac{5}{6}\pi\mathrm{i}}$

B1 [3]

6 **(i)** Use law for the logarithm for a product or quotient or exponentiation AND for a power M1

Obtain $(4x-5)^2(x+1)=27$ B1

Obtain given equation correctly $16x^3-24x^2-15x-2=0$ A1 [3]

(ii) Obtain $x=2$ is root or $(x-2)$ is a factor, or likewise with $x=-\frac{1}{4}$ B1

Divide by $(x-2)$ to reach a quotient of the form $16x^2+kx$ M1

Obtain quotient $16x^2+8x+1$ A1

Obtain $(x-2)(4x+1)^2$ or $(x-2)$, $(4x+1)$, $(4x+1)$ A1 [4]

(iii) State $x=2$ only A1 [1]

7 **(i)** Obtain $2x-3y+6z$ for LHS of equation B1

Obtain $2x-3y+6z=23$ B1 [2]

(ii) Either Use correct formula to find perpendicular distance M1

Obtain unsimplified value $\frac{\pm 23}{\sqrt{2^2+(-3)^2+6^2}}$, following answer to (i) A1√

Obtain $\frac{23}{7}$ or equivalent A1 [3]

OR 1 Use scalar product of (4, –1, 2) and a vector normal to the plane M1

Use unit normal to plane to obtain $\pm\frac{(8+3+12)}{\sqrt{49}}$ A1

Obtain $\frac{23}{7}$ or equivalent A1 [3]

OR 2 Find parameter intersection of p and $\mathbf{r} = \mu(2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k})$ M1

Obtain $\mu = \frac{23}{49}$ [and $\left(\frac{46}{49}, -\frac{69}{49}, \frac{138}{49}\right)$ as foot of perpendicular] A1

Obtain distance $\frac{23}{7}$ or equivalent A1 [3]

(iii) Either Recognise that plane is $2x - 3y + 6z = k$ and attempt use of formula for perpendicular distance to plane at least once M1

Obtain $\frac{|23-k|}{7} = 14$ or equivalent A1

Obtain $2x - 3y + 6z = 121$ and $2x - 3y + 6z = -75$ A1 [3]

OR Recognise that plane is $2x - 3y + 6z = k$ and attempt to find at least one point on q using l with $\lambda = \pm 2$ M1

Obtain $2x - 3y + 6z = 121$ A1

Obtain $2x - 3y + 6z = -75$ A1 [3]

8 (i) Sketch $y = \text{cosec}\, x$ for at least $0, x, \pi$ B1

Sketch $y = x(\pi - x)$ for at least $0, x, \pi$ B1

Justify statement concerning two roots, with evidence of 1 and $\frac{1}{4}\pi^2$ for y-values on graph via scales B1 [3]

(ii) Use $\text{cosec}\, x = \frac{1}{\sin x}$ and commence rearrangement M1

Obtain given equation correctly, showing sufficient detail A1 [2]

(iii) (a) Use the iterative formula correctly at least once M1

Obtain final answer 0.66 A1

Show sufficient iterations to 4 decimal places to justify answer or show a sign change in the interval $(0.655, 0.665)$ A1 [3]

(b) Obtain 2.48 B1 [1]

9 **(i)** Either State or imply partial fractions are of form $\frac{A}{3-x}+\frac{B}{1+2x}+\frac{C}{(1+2x)^2}$ B1

Use any relevant method to obtain a constant M1

Obtain $A = 1$ A1

Obtain $B=\frac{3}{2}$ A1

Obtain $C=-\frac{1}{2}$ A1 [5]

Or State or imply partial fractions are of form $\frac{A}{3-x}+\frac{Dx+E}{(1+2x)^2}$ B1

Use any relevant method to obtain a constant M1

Obtain $A = 1$ A1

Obtain $D = 3$ A1

Obtain $E = 1$ A1 [5]

(ii) Obtain the first two terms of one of the expansion of $(3-x)^{-1}, \left(1-\frac{1}{3}x\right)^{-1}$

$(1+2x)^{-1}$ and $(1+2x)^{-2}$ M1

Obtain correct unsimplified expansion up to the term in x^2 of each partial fraction, following in each case the value of A, B, C A1√ A1√ A1√

Obtain answer $\frac{4}{3}-\frac{8}{9}x+\frac{1}{27}x^2$ A1 [5]

[If A, D, E approach used in part (i), give M1A1√A1√ for the expansions, M1 for multiplying out fully and A1 for final answer]

10 **(i)** Use of product or quotient rule M1

Obtain $-5e^{-\frac{1}{2}x}\sin 4x + 40e^{-\frac{1}{2}x}\cos 4x$ A1

Equate $\frac{dy}{dx}$ to zero and obtain $\tan 4z = k$ or $R\cos(4x \pm \alpha)$ M1

Obtain $\tan 4x = 8$ or $\sqrt{65}\cos\left(4x \pm \tan^{-1}\frac{1}{8}\right)$ A1

Obtain 0.362 or 20.7° A1

Obtain 1.147 or 65.7° A1 [6]

(ii) State or imply that x-coordinates of T_n are increasing by $\frac{1}{4}\pi$ or 45° B1

Attempt solution of inequality (or equation) of form $x_1 + (n-1)k\pi \,.\, 25$ M1

Obtain $n > \frac{4}{\pi}(25-0.362)+1$, following through on their value of x_1 A1√

n = 33 A1 [4]