CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Level

MARK SCHEME for the May/June 2015 series

9709 MATHEMATICS

9709/33 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol $\sqrt{}$ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a "fortuitous" answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through $\sqrt{}$" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	Use law for the logarithm of a product, quotient or power		M1	
	Obtain a correct equation free of logarithms, e.g. $\frac{x+4}{x^2}=4$		A1	
	Solve a 3-term quadratic obtaining at least one root		M1	
	Obtain final answer $x = 1.13$ only		A1	**4**
2	*EITHER*:	State or imply non-modular inequality $(x-2)^2 > (2x-3)^2$, or corresponding equation	B1	
		Solve a 3-term quadratic, as in Q1.	M1	
		Obtain critical value $x = \frac{5}{3}$	A1	
		State final answer $x < \frac{5}{3}$ only	A1	
	*OR*1:	State the relevant critical linear inequality $(2-x) > (2x-3)$, or corresponding equation	B1	
		Solve inequality or equation for x	M1	
		Obtain critical value $x = \frac{5}{3}$	A1	
		State final answer $x < \frac{5}{3}$ only	A1	
	*OR*2:	Make recognisable sketches of $y = 2x - 3$ and $y = \lvert x-2 \rvert$ on a single diagram	B1	
		Find x-coordinate of the intersection	M1	
		Obtain $x = \frac{5}{3}$	A1	
		State final answer $x < \frac{5}{3}$ only	A1	**4**
3	Use correct tan $2A$ and cot A formulae to form an equation in tan x		M1	
	Obtain a correct equation in any form		A1	
	Reduce equation to the form $\tan^2 x + 6\tan x - 3 = 0$, or equivalent		A1	
	Solve a three term quadratic in tan x for x, **as in Q1.**		M1	
	Obtain answer, e.g. 24.9° (24.896)		A1	
	Obtain second answer, e.g. 98.8 (98.794) and no others in the given interval		A1	**6**
	[Ignore outside the given interval. Treat answers in radians as a misread.]			
	Radian answers 0.43452, 1.7243			
4	Use correct quotient or product rule		M1	
	Obtain correct derivative in any form		A1	
	Equate derivative to zero and obtain a horizontal equation		M1	
	Carry out complete method for solving an equation of the form $ae^{3x} = b$, or $ae^{5x} = be^{2x}$		M1	
	Obtain $x = \ln 2$, or exact equivalent		A1	
	Obtain $y = \frac{1}{3}$, or exact equivalent		A1	**6**

5	**(i)**	State $\frac{dx}{dt} = -4a\cos^3 t\sin t$, or $\frac{dy}{dt} = 4a\sin^3 t\cos t$	B1	
		Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1	
		Obtain correct expression for $\frac{dy}{dx}$ in a simplified form	A1	**3**
	(ii)	Form the equation of the tangent	M1	
		Obtain a correct equation in any form	A1	
		Obtain the given answer	A1	**3**
	(iii)	State the x-coordinate of P or the y-coordinate of Q in any form	B1	
		Obtain the given result correctly	B1	**2**
6	**(i)**	Integrate and reach $\pm x\sin x \mp \int \sin x\, dx$	M1*	
		Obtain integral $x\sin x + \cos x$	A1	
		Substitute limits correctly, must be seen since AG, and equate result to 0.5	M1(dep*)	
		Obtain the given form of the equation	A1	**4**
	(ii)	*EITHER*: Consider the sign of a relevant expression at $a = 1$ and at another relevant value, e.g. $a = 1.5 \leqslant \frac{\pi}{2}$	M1	
		OR: Using limits correctly, consider the sign of $\left[x\sin x + \cos x\right]_0^a - 0.5$, or compare the value of $\left[x\sin x + \cos x\right]_0^a$ with 0.5, for a =1 AND for another relevant value, e.g $a = 1.5 \leqslant \frac{\pi}{2}$.	M1	
		Complete the argument, so change of sign, or above and below stated, both with correct calculated values	A1	**2**
	(iii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 1.2461	A1	
		Show sufficient iterations to 6 d.p. to justify 1.2461 to 4 d.p., or show there is a sign change in the interval (1.24605, 1.24615)	A1	**3**
7	**(i)**	Separate variables correctly and integrate one side	B1	
		Obtain term $2\sqrt{M}$, or equivalent	B1	
		Obtain term $50k\sin(0.02t)$, or equivalent	B1	
		Evaluate a constant of integration, or use limits M = 100, t =0 in a solution with terms of the form $a\sqrt{M}$ and $b\sin(0.02t)$	M1*	
		Obtain correct solution in any form, e.g. $2\sqrt{M} = 50k\sin(0.02t) + 20$	A1	**5**
	(ii)	Use values M =196, t =50 and calculate k	M1(dep*)	
		Obtain answer k = 0.190	A1	**2**
	(iii)	State an expression for M in terms of t, e.g. $M = (4.75\sin(0.02t) + 10)^2$	M1(dep*)	
		State that the least possible number of micro-organisms is 28 or 27.5 or 27.6 (27.5625)	A1	**2**

8	**(i)**	*EITHER*: Substitute for u in $\frac{\mathrm{i}}{u}$ and multiply numerator and denominator by $1 + \mathrm{i}$	M1	
		Obtain final answer $-\frac{1}{2}+\frac{1}{2}\mathrm{i}$, or equivalent	A1	
		OR: Substitute for u, obtain two equations in x and y and solve for x or for y	M1	
		Obtain final answer $-\frac{1}{2}+\frac{1}{2}\mathrm{i}$, or equivalent	A1	**2**
	(ii)	Show a point representing u in a relatively correct position	B1	
		Show the bisector of the line segment joining u to the origin	B1	
		Show a circle with centre at the point representing i	B1	
		Show a circle with radius 2	B1	**4**
	(iii)	State argument $-\frac{1}{2}\pi$, or equivalent, e.g. 270°	B1	
		State or imply the intersection in the first quadrant represents 2 + i	B1	
		State argument 0.464, (0.4636)or equivalent, e.g. 26.6° (26.5625)	B1	**3**
9	**(i)**	State or imply a correct normal vector to either plane, e.g. $\mathbf{i}+3\mathbf{j}-2\mathbf{k}$, or $2\mathbf{i}+\mathbf{j}+3\mathbf{k}$	B1	
		Carry out correct process for evaluating the scalar product of two normal vectors	M1	
		Using the correct process for the moduli, divide the scalar product of the two normals by the product of their moduli and evaluate the inverse cosine of the result	M1	
		Obtain answer 85.9° or 1.50 radians	A1	**4**

(ii) *EITHER*: Carry out a complete strategy for finding a point on l — M1

Obtain such a point, e.g. (0, 2, 1) — A1

EITHER: State two equations for a direction vector $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ for l,

e.g. $a + 3b - 2c = 0$

and $2a + b + 3c = 0$ — B1

Solve for one ratio, e.g. $a : b$ — M1

Obtain $a : b : c = 11 : -7 : -5$ — A1

State a correct answer, e.g. $\mathbf{r} = 2\mathbf{j} + \mathbf{k} + \lambda(11\mathbf{i} - 7\mathbf{j} - 5\mathbf{k})$ — A1√

*OR*1: Obtain a second point on l, e.g. $\left(\frac{22}{7}, 0, -\frac{3}{7}\right)$ — B1

Subtract position vectors and obtain a direction vector for l — M1

Obtain $22\mathbf{i} - 14\mathbf{j} - 10\mathbf{k}$, or equivalent — A1

State a correct answer, e.g. $\mathbf{r} = 2\mathbf{j} + \mathbf{k} + \lambda(22\mathbf{i} - 14\mathbf{j} - 10\mathbf{k})$ — A1√

*OR*2: Attempt to find the vector product of the two normal vectors — M1

Obtain two correct components — A1

Obtain $11\mathbf{i} - 7\mathbf{j} - 5\mathbf{k}$, or equivalent — A1

State a correct answer, e.g. $\mathbf{r} = 2\mathbf{j} + \mathbf{k} + \lambda(11\mathbf{i} - 7\mathbf{j} - 5\mathbf{k})$ — A1√

*OR*3: Express one variable in terms of a second — M1

Obtain a correct simplified expression, e.g. $x = (22 - 11y)/7$ — A1

Express the same variable in terms of the third — M1

Obtain a correct simplified expression, e.g. $x = (11 - 11z)/5$ — A1

Form a vector equation for the line M1

State a correct answer, e.g. $\mathbf{r} = 2\mathbf{j} + \mathbf{k} + \lambda\left(\mathbf{i} - \frac{7}{11}\mathbf{j} - \frac{5}{11}\mathbf{k}\right)$ — A1√

*OR*4: Express one variable in terms of a second — M1

Obtain a correct simplified expression, e.g. $y = (22 - 7x)/11$ — A1

Express the third variable in terms of the second — M1

Obtain a correct simplified expression, e.g. $z = (11 - 5x)/11$ — A1

Form a vector equation for the line — M1

State a correct answer, e.g. $\mathbf{r} = 2\mathbf{j} + \mathbf{k} + \lambda\left(\mathbf{i} - \frac{7}{11}\mathbf{j} - \frac{5}{11}\mathbf{k}\right)$ — A1√ — **6**

[The √ marks are dependent on all M marks being earned.]

10 **(i)** State or imply $\mathrm{f}(x) \equiv \frac{A}{2x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$ — B1

Use a relevant method to determine a constant — M1

Obtain one of the values $A = 2$, $B = -1$, $C = 3$ — A1

Obtain the remaining values A1 + — A1 — **5**

[Apply an analogous scheme to the form $\frac{A}{2x-1} + \frac{Dx+E}{(x+2)^2}$; the values being $A = 2$, $D = -1$, $E = 1$.]

(ii) Integrate and obtain terms $\frac{1}{2}\cdot 2\ln(2x-1)-\ln(x+2)-\frac{3}{x+2}$ B1√ + B1√ + B1√

Use limits correctly, namely substitution must be seen in at least two of the partial fractions to obtain M1 Integrate all 3 partial fractions and substitute in all three partial fractions for A1 since AG. M1

Obtain the given answer following full and exact working A1 **5**

[The t marks are dependent on *A*, *B*, *C* etc.]

[SR: If *B*, *C* or *E* omitted, give B1M1 in part (**i**) and B1√B1√M1 in part (**ii**).]

[NB: Candidates who follow the *A*, *D*, *E* scheme in part (**i**) and then integrate $\frac{-x+1}{(x+2)^2}$ by parts should obtain $\frac{1}{2}\cdot 2\ln(2x-1)-\ln(x+2)+\frac{x-1}{x+2}$ (the third term is equivalent to $-\frac{3}{x+2}+1$).]