CAMBRIDGE
INTERNATIONAL EXAMINATIONS

NOVEMBER 2002

GCE Advanced Level
GCE Advanced Subsidiary Level

MARK SCHEME

MAXIMUM MARK : 75

SYLLABUS/COMPONENT :9709 /3, 8719 /3

MATHEMATICS
(Pure 3)

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

1	*EITHER*:	State or imply non-modular inequality $(9-2x)^2 < 1$, or a correct pair of linear inequalities, combined or separate, e.g. $-1 < 9-2x < 1$	B1	
		Obtain both critical values 4 and 5	B1	
		State correct answer $4 < x < 5$; accept $x > 4, x < 5$	B1	
	OR:	State a correct equation or pair of equations for both critical values e.g. $9-2x = 1$ and $9-2x = -1$, or $9-2x = \pm 1$	B1	
		Obtain critical values 4 and 5	B1	
		State correct answer $4 < x < 5$; accept $x > 4, x < 5$	B1	
	OR:	State one critical value (probably $x = 4$) from a graphical method or by inspection or by solving a linear inequality or equation	B1	
		State the other critical value correctly	B1	
		State correct answer $4 < x < 5$; accept $x > 4, x < 5$	B1	3
		[Use of $\le$, throughout, or at the end, scores a maximum of B2.]		
2	*EITHER*:	State first step of the form $kx^2 \ln x \pm \int kx^2 . \frac{1}{x} \mathrm{d}x$	M1	
		Obtain correct first step i.e. $\frac{1}{2}x^2 \ln x - \int \frac{1}{2} x \,\mathrm{d}x$	A1	
		Complete a second integration and substitute both limits correctly	M1	
		Obtain correct answer $2 \ln 2 - \frac{3}{4}$, or exact two-term equivalent	A1	
	OR:	State first step of the form $I = x(x\ln x \pm x) \pm \int (x \ln x \pm x)\mathrm{d}x$	M1	
		Obtain correct first step i.e. $I = x(x\ln x - x) - I + \int x \,\mathrm{d}x$	A1	
		Complete a second integration and substitute both limits correctly	M1	
		Obtain correct answer $2 \ln 2 - \frac{3}{4}$, or exact two-term equivalent	A1	4
3	(i)	Use law for addition (or subtraction) of logarithms or indices	M1*	
		Use $\log_{10} 100 = 2$ or $10^2 = 100$	M1(dep*)	
		Obtain $x^2 + 5x = 100$, or equivalent, correctly	A1	3
	(ii)	Solve a three-term quadratic equation	M1	
		State answer 7.81(allow 7.80 or 7.8) or any exact form of the answer i.e. $\frac{\sqrt{425}-5}{2}$ or better	A1	2
4	(i)	Obtain derivative $e^x - 8e^{-2x}$ in any correct form	B1	
		Equate derivative to zero and simplify to an equation of the form $e^{kx} = a$, where $a \neq 0$	M1*	
		Carry out method for calculating x with $a > 0$	M1(dep*)	
		Obtain answer $x = \ln 2$, or an exact equivalent (also accept 0.693 or 0.69)	A1	4
		[Accept statements of the form '$u^k = a$, where $u = e^x$' for the first M1.]		
	(ii)	Carry out a method for determining the nature of the stationary point	M1	
		Show that the point is a minimum correctly, with no incorrect work seen	A1	2

5	(i) State or imply at any stage that $R = 5$	B1	
	Use trig formula to find α	M1	
	Obtain answer $\alpha = 36.87°$	A1	3
	(ii) *EITHER*: Carry out, or indicate need for, calculation of $\sin^{-1}(\frac{2}{5})$	M1	
	Obtain answer 60.4° (or 60.5°)	A1	
	Carry out correct method for second root i.e. $180° - 23.578° + 36.870°$	M1	
	Obtain answer 193.3° and no others in range	A1 ✓	
	OR: Obtain a three-term quadratic equation in $\sin\theta$ or $\cos\theta$	M1	
	Solve a two- or three- term quadratic and calculate an angle	M1	
	Obtain answer 60.4° (or 60.5°)	A1	
	Obtain answer 193.3° and no others in range	A1	4
	(iii) State greatest value is 1	B1 ✓	1
	[Treat work in radians as a misread, scoring a maximum of 7. The angles are 0.644, 1.06 and 3.37.]		

6	(i) State or imply $f(x) \equiv \frac{A}{(2-x)} + \frac{Bx+C}{(x^2+1)}$	B1*	
	State or obtain $A = 4$	B1(dep*)	
	Use any relevant method to find B or C	M1	
	Obtain both $B = 4$ and $C = 1$	A1	4
	(ii) *EITHER*: Use correct method to obtain the first two terms of the expansion of $(1-\frac{1}{2}x)^{-1}$, or $(1+x^2)^{-1}$, or $(2-x)^{-1}$	M1*	
	Obtain unsimplified expansions of the fractions e.g. $\frac{4}{2}(1+\frac{1}{2}x+\frac{1}{4}x^2+\frac{1}{8}x^3)$; $(4x+1)(1-x^2)$	A1✓+ A1✓	
	Carry out multiplication of expansion of $(1+x^2)^{-1}$ by $(4x+1)$	M1(dep*)	
	Obtain given answer correctly	A1	
	[Binomial coefficients involving −1, such as $\begin{pmatrix}-1\\1\end{pmatrix}$, are not sufficient for the first M1.]		
	[f.t. is on A, B, C.]		
	[Apply this scheme to attempts to expand $(6+7x)(2-x)^{-1}(1-x^2)^{-1}$, giving M1A1A1 for the expansions, M1 for multiplying out fully, and A1 for reaching the given answer.]		
	OR: Differentiate and evaluate $f(0)$ and $f'(0)$	M1	
	Obtain $f(0) = 3$ and $f'(0) = 5$	A1 ✓	
	Differentiate and obtain $f''(0) = -1$	A1 ✓	
	Differentiate, evaluate $f'''(0)$ and form the Maclaurin expansion up to the term in x^3	M1	
	Simplify coefficients and obtain given answer correctly	A1	5
	[f.t. is on A, B, C.]		
	[SR: B or C omitted from the form of partial fractions. In part (i) give the first B1, and M1 for the use of a relevant method to obtain A, B, or C, but no further marks. In part (ii) only the first M1 and A1✓+ A1✓ are available if an attempt is based on this form of partial fractions.]		

7	(i) State or obtain a relevant equation e.g. $2r\alpha = 100$	B1	
	State or obtain a second independent relevant equation e.g. $2r \sin \alpha = 99$	B1	
	Derive the given equation in x (or α) correctly	B1	3
	(ii) Calculate ordinates at $x = 0.1$ and $x = 0.5$ of a suitable function or pair of functions	M1	
	Justify the given statement correctly	A1	2
	[If calculations are not given but the given statement is justified using correct statements about the signs of a suitable function or the difference between a pair of suitable functions, award B1.]		
	(iii) State $x = 50\sin x - 48.5x$, or equivalent	B1	
	Rearrange this in the form given in part (i) (or *vice versa*)	B1	2
	(iv) Use the method of iteration at least once with $0.1 \le x_n \le 0.5$	M1	
	Obtain final answer 0.245, showing sufficient iterations to justify its accuracy to 3d.p., or showing a sign change in the interval (0.2445, 0.2455)	A1	2
	[SR: both the M marks are available if calculations are attempted in degree mode.]		
8	(a) *EITHER*: Square $x + iy$ and equate real and/or imaginary parts to -3 and/or 4 respectively	M1	
	Obtain $x^2 - y^2 = -3$ and $2xy = 4$	A1	
	Eliminate one variable and obtain an equation in the other variable	M1	
	Obtain $x^4 + 3x^2 - 4 = 0$, or $y^4 - 3y^2 - 4 = 0$, or 3-term equivalent	A1	
	Obtain final answers $\pm(1 + 2\text{i})$ and no others	A1	
	[Accept $\pm 1 \pm 2\text{i}$, or $x = 1, y = 2$ and $x = -1, y = -2$ as final answers, but not $x = \pm 1, y = \pm 2$.]		
	OR: Convert $-3 + 4\text{i}$ to polar form (R, θ)	M1	
	Use fact that a square root has polar form $(\sqrt{R}, \frac{1}{2}\theta)$	M1	
	Obtain one root in polar form e.g. $(\sqrt{5}, 63.4°)$ (allow 63.5°; argument is 1.11 radians)	A1	
	Obtain answer $1 + 2\text{i}$	A1	
	Obtain answer $-1 - 2\text{i}$ and no others	A1	5
	(b) (i) Carry out multiplication of numerator and denominator by $2 - \text{i}$	M1	
	Obtain answer $\frac{1}{5} + \frac{7}{5}\text{i}$ or $0.2 + 1.4\text{i}$	A1	2
	(ii) Show all three points on an Argand diagram in relatively correct positions	B1 ✓	1
	[Accept answers on separate diagrams.]		
	(iii) State that $OC = \dfrac{OA}{OB}$, or equivalent	B1	1
	[Accept the answer $OA.OC = 2OB$, or equivalent.]		
	[Accept answers with $\lvert OA \rvert$ for OA etc.]		
9	(i) State or imply that $\dfrac{da}{dt} = ka(10 - a)$	B1	
	Justify $k = 0.004$	B1	2
	(ii) Resolve $\dfrac{1}{a(10-a)}$ into partial fractions $\dfrac{A}{a} + \dfrac{B}{10-a}$ and obtain values $A = B = \frac{1}{10}$	B1	
	Separate variables obtaining $\displaystyle\int \frac{da}{a(10-a)} = \int k\, dt$ and attempt to integrate both sides	M1	
	Obtain $\frac{1}{10}\ln a - \frac{1}{10}\ln(10-a)$	A1 ✓	
	Obtain $0.004t$, or equivalent	A1	
	Evaluate a constant, or use limits $t = 0, a = 5$	M1	
	Obtain answer $t = 25\ln\left(\dfrac{a}{10-a}\right)$, or equivalent	A1	6
	(iii) Substitute $a = 9$ and calculate t	M1	
	Obtain answer $t = 54.9$ or 55	A1	2
	[Substitution of $a = 0.9$ scores M0.]		

10	(i) Find a direction vector for AB or CD e.g. $\overrightarrow{AB} = \mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$ or $\overrightarrow{CD} = -2\mathbf{i} - \mathbf{j} - 4\mathbf{k}$	B1	
	EITHER: Carry out the correct process for evaluating the scalar product of two relevant vectors in component form	M1	
	Evaluate $\cos^{-1}\left(\dfrac{\overrightarrow{AB}.\overrightarrow{CD}}{\lvert\overrightarrow{AB}\rvert\lvert\overrightarrow{CD}\rvert}\right)$ using the correct method for the moduli	M1	
	Obtain final answer 45.6°, or 0.796 radians, correctly	A1	
	OR: Calculate the sides of a relevant triangle using the correct method	M1	
	Use the cosine rule to calculate a relevant angle	M1	
	Obtain final answer 45.6°,or 0.796 radians, correctly	A1	4
	[SR: if a vector is incorrectly stated with all signs reversed and 45.6° is obtained, award B0M1M1A1.]		
	[SR: if 45.6° is followed by 44.4° as final answer, award A0.]		
	(ii) *EITHER*: State both line equations e.g. $4\mathbf{i} + \mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} - 3\mathbf{k})$ and $\mathbf{i} + \mathbf{j} + \mu(2\mathbf{i} + \mathbf{j} + 4\mathbf{k})$	B1✓	
	Equate components and solve for λ or for μ	M1	
	Obtain value $\lambda = -1$ or $\mu = 1$	A1	
	Verify that all equations are satisfied, so that the lines do intersect, or equivalent	A1	
	[SR: if both lines have the same parameter, award B1M1 if the equations are inconsistent and B1M1A1 if the equations are consistent and shown to be so.]		
	OR: State both line equations in Cartesian form	B1✓	
	Solve simultaneous equations for a pair of unknowns e.g. x and y	M1	
	Obtain a correct pair e.g. $x = 3, y = 2$	A1	
	Obtain the third unknown e.g. $z = 4$ and verify the lines intersect	A1	
	OR: Find one of $\overrightarrow{CA}, \overrightarrow{CB}, \overrightarrow{DA}, \overrightarrow{DB}$, ..., e.g. $\overrightarrow{CA} = 3\mathbf{i} - \mathbf{j} + \mathbf{k}$	B1	
	Carry out correct process for evaluating a relevant scalar triple product e.g. $\overrightarrow{CA}.(\overrightarrow{AB} \times \overrightarrow{CD})$	M1	
	Show the value is zero	A1	
	State that (a) this result implies the lines are coplanar, (b) the lines are not parallel, and thus the lines intersect (condone omission of one of (a) and (b))	A1	
	OR: Carry out correct method for finding a normal to the plane through three of the points	M1	
	Obtain a correct normal vector	A1	
	Obtain a correct equation e.g. $x + 2y - z = 3$ for the plane of A, B, C	A1	
	Verify that the fourth point lies in the plane and conclude that the lines intersect	A1	
	OR: State a relevant plane equation e.g. $\mathbf{r} = 4\mathbf{i} + \mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}) + \mu(-3\mathbf{i} + \mathbf{j} - \mathbf{k})$ for the plane of A, B, C	B1✓	
	Set up equations in λ and μ, using components of the fourth point, and solve for λ or μ	M1	
	Obtain value $\lambda = 1$ or $\mu = 2$	A1	
	Verify that all equations are satisfied and conclude that the lines intersect	A1	4

(continued)

10 (continued)

(iii) *EITHER*:	Find $\overrightarrow{PQ}$ for a general point Q on AB e.g. $3\mathbf{i} - 5\mathbf{j} - 5\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} - 3\mathbf{k})$	B1✓	
	Calculate $\overrightarrow{PQ}.\overrightarrow{AB}$ correctly and equate to zero	M1	
	Solve for λ obtaining $\lambda = -2$	A1	
	Show correctly that $PQ = \sqrt{3}$, the given answer	A1	
OR:	State $\overrightarrow{AP}$(or $\overrightarrow{BP}$) and $\overrightarrow{AB}$ in component form	B1✓	
	Carry out correct method for finding their vector product	M1	
	Obtain correct answer e.g. $\overrightarrow{AP} \times \overrightarrow{AB} = -5\mathbf{i} - 4\mathbf{j} + \mathbf{k}$	A1	
	Divide modulus by $\lvert\overrightarrow{AB}\rvert$ and obtain the given answer $\sqrt{3}$	A1	
OR:	State $\overrightarrow{AP}$(or $\overrightarrow{BP}$) and $\overrightarrow{AB}$ in component form	B1✓	
	Carry out correct method for finding the projection of AP(or BP) on AB i.e. $\dfrac{\lvert\overrightarrow{AP}.\overrightarrow{AB}\rvert}{\lvert\overrightarrow{AB}\rvert}$	M1	
	Obtain correct answer e.g. $AN = \dfrac{28}{\sqrt{14}}$ or $BN = \dfrac{42}{\sqrt{14}}$	A1	
	Show correctly that $PN = \sqrt{3}$, the given answer	A1	
OR:	State two of $\overrightarrow{AP}, \overrightarrow{BP}, \overrightarrow{AB}$ in component form	B1✓	
	Use the cosine rule in triangle ABP, or scalar product, to find the cosine of A, B, or P	M1	
	Obtain correct answer e.g. $\cos A = \dfrac{-28}{\sqrt{14}.\sqrt{59}}$	A1	
	Deduce the exact length of the perpendicular from P to AB is $\sqrt{3}$, the given answer	A1	4