CAMBRIDGE INTERNATIONAL EXAMINATIONS

General Certificate of Education Advanced Subsidiary Level
General Certificate of Education Advanced Level

HIGHER MATHEMATICS **8719/3**

MATHEMATICS **9709/3**

PAPER 3 Pure Mathematics 3 **(P3)**

OCTOBER/NOVEMBER SESSION 2002

1 hour 45 minutes

Additional materials:
Answer paper
Graph paper
List of Formulae (MF9)

TIME 1 hour 45 minutes

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 75.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

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Local Examinations Syndicate

1 Solve the inequality $|9 - 2x| < 1$. [3]

2 Find the exact value of $\int_1^2 x \ln x \, dx$. [4]

3 (i) Show that the equation

$$\log_{10}(x + 5) = 2 - \log_{10} x$$

may be written as a quadratic equation in x. [3]

(ii) Hence find the value of x satisfying the equation

$$\log_{10}(x + 5) = 2 - \log_{10} x.$$ [2]

4 The curve $y = e^x + 4e^{-2x}$ has one stationary point.

(i) Find the x-coordinate of this point. [4]

(ii) Determine whether the stationary point is a maximum or a minimum point. [2]

5 (i) Express $4 \sin \theta - 3 \cos \theta$ in the form $R \sin(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, stating the value of α correct to 2 decimal places. [3]

Hence

(ii) solve the equation

$$4 \sin \theta - 3 \cos \theta = 2,$$

giving all values of θ such that $0° < \theta < 360°$, [4]

(iii) write down the greatest value of $\dfrac{1}{4 \sin \theta - 3 \cos \theta + 6}$. [1]

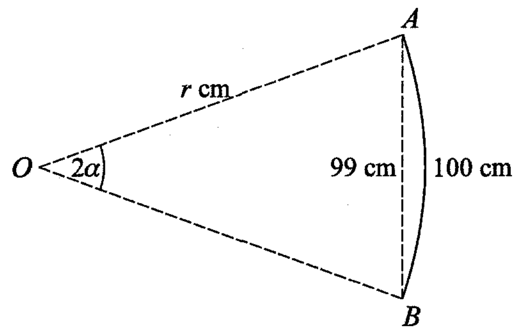
6 Let $f(x) = \dfrac{6 + 7x}{(2 - x)(1 + x^2)}$.

(i) Express $f(x)$ in partial fractions. [4]

(ii) Show that, when x is sufficiently small for x^4 and higher powers to be neglected,

$$f(x) = 3 + 5x - \tfrac{1}{2}x^2 - \tfrac{15}{4}x^3.$$ [5]

7



The diagram shows a curved rod AB of length 100 cm which forms an arc of a circle. The end points A and B of the rod are 99 cm apart. The circle has radius r cm and the arc AB subtends an angle of 2α radians at O, the centre of the circle.

(i) Show that α satisfies the equation $\frac{99}{100}x = \sin x$. [3]

(ii) Given that this equation has exactly one root in the interval $0 < x < \frac{1}{2}\pi$, verify by calculation that this root lies between 0.1 and 0.5. [2]

(iii) Show that if the sequence of values given by the iterative formula

$$x_{n+1} = 50 \sin x_n - 48.5 x_n$$

converges, then it converges to a root of the equation in part **(i)**. [2]

(iv) Use this iterative formula, with initial value $x_1 = 0.25$, to find α correct to 3 decimal places, showing the result of each iteration. [2]

8 **(a)** Find the two square roots of the complex number $-3 + 4\text{i}$, giving your answers in the form $x + \text{i}y$, where x and y are real. [5]

(b) The complex number z is given by

$$z = \frac{-1 + 3\text{i}}{2 + \text{i}}.$$

(i) Express z in the form $x + \text{i}y$, where x and y are real. [2]

(ii) Show on a sketch of an Argand diagram, with origin O, the points A, B and C representing the complex numbers $-1 + 3\text{i}$, $2 + \text{i}$ and z respectively. [1]

(iii) State an equation relating the lengths OA, OB and OC. [1]

9 In an experiment to study the spread of a soil disease, an area of $10\,\text{m}^2$ of soil was exposed to infection. In a simple model, it is assumed that the infected area grows at a rate which is proportional to the product of the infected area and the uninfected area. Initially, $5\,\text{m}^2$ was infected and the rate of growth of the infected area was $0.1\,\text{m}^2$ per day. At time t days after the start of the experiment, an area $a\,\text{m}^2$ is infected and an area $(10 - a)\,\text{m}^2$ is uninfected.

(i) Show that $\dfrac{\text{d}a}{\text{d}t} = 0.004a(10 - a)$. [2]

(ii) By first expressing $\dfrac{1}{a(10 - a)}$ in partial fractions, solve this differential equation, obtaining an expression for t in terms of a. [6]

(iii) Find the time taken for 90% of the soil area to become infected, according to this model. [2]

10 With respect to the origin O, the points A, B, C, D have position vectors given by

$$\overrightarrow{OA} = 4\mathbf{i} + \mathbf{k}, \quad \overrightarrow{OB} = 5\mathbf{i} - 2\mathbf{j} - 2\mathbf{k}, \quad \overrightarrow{OC} = \mathbf{i} + \mathbf{j}, \quad \overrightarrow{OD} = -\mathbf{i} - 4\mathbf{k}.$$

(i) Calculate the acute angle between the lines AB and CD. [4]

(ii) Prove that the lines AB and CD intersect. [4]

(iii) The point P has position vector $\mathbf{i} + 5\mathbf{j} + 6\mathbf{k}$. Show that the perpendicular distance from P to the line AB is equal to $\sqrt{3}$. [4]