CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary and Advanced Level

MARK SCHEME for the November 2004 question papers

9709 MATHEMATICS
8719 HIGHER MATHEMATICS

8719/07, 9709/07 – Paper 7 (Probability and Statistics 2)
maximum raw mark 50

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which Examiners were initially instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began. Any substantial changes to the mark scheme that arose from these discussions will be recorded in the published *Report on the Examination*.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the *Report on the Examination*.

- CIE will not enter into discussion or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the November 2004 question papers for most IGCSE and GCE Advanced Level syllabuses.

UNIVERSITY of CAMBRIDGE
International Examinations

Grade thresholds taken for Syllabus 8719 and 9709 (Mathematics and Higher Mathematics) in the November 2004 examination.

	maximum mark available	minimum mark required for grade:		
		A	B	E
Component 7	50	41	38	23

The thresholds (minimum marks) for Grades C and D are normally set by dividing the mark range between the B and the E thresholds into three. For example, if the difference between the B and the E threshold is 24 marks, the C threshold is set 8 marks below the B threshold and the D threshold is set another 8 marks down. If dividing the interval by three results in a fraction of a mark, then the threshold is normally rounded down.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

November 2004

GCE A AND AS LEVEL

MARK SCHEME

MAXIMUM MARK: 50

SYLLABUS/COMPONENT: 8719/07 AND 9709/07

MATHEMATICS AND HIGHER MATHEMATICS
Paper 7 (Probability and Statistics 2)

UNIVERSITY of CAMBRIDGE
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1	$\lambda = 2.3 \times 3 = 6.9$ $P(6, 7, 8) = e^{-6.9}\left(\frac{6.9^6}{6!} + \frac{6.9^7}{7!} + \frac{6.9^8}{8!}\right)$ $= e^{-6.9}(425.06)$ $= 0.428$	M1 A1 A1ft A1	**4**	For attempt at Poisson, any mean For correct mean For correct expression with their mean For correct answer
2(i)	$\overline{X} \sim N\left(6.7, \frac{3.1^2}{300}\right)$ $z_1 = \frac{6.8 - 6.7}{3.1/\sqrt{300}} = 0.5587$ $z_2 = \frac{6.5 - 6.7}{3.1/\sqrt{300}} = -1.117$ Prob = $\Phi(0.5587) - \{1 - \Phi(1.117)\}$ $= 0.7119 - (1 - 0.8679)$ $= 0.580$	M1 A1 M1 A1	**4**	For standardising, (with or without 300 in denom) For two correct expressions for z For subtracting 2 probabilities For correct answer
(ii)	300 is large, so $\overline{X}$ is approx normal even if X is not i.e. CLT application	B1	**1**	For reference to large n and/or CLT
3(i)	$\bar{x} = \frac{4080}{150} = 27.2$ $s^2 = \frac{1}{149}\left(159252 - \frac{4080^2}{150}\right) = 324$	B1 M1 A1	**3**	For 4080/150 For correct expression, (from formulae sheet or equiv.) For correct answer
(ii)	94% CI $= 27.2 \pm 1.882 \times \sqrt{\frac{324}{150}}$ = (24.4, 30.0)	M1 B1 A1ft A1	**4**	For one of correct form $\bar{x} + z \times \frac{s}{\sqrt{n}}$ or $\bar{x} - z \times \frac{s}{\sqrt{n}}$ For z = 1.881 or 1.882 only For correct expression with their $s/\sqrt{150}$, z and $\bar{x}$ Or equivalent statement (c.w.o.)
4(i)	5M + 2W ~ N(355 + 114, $7^2 \times 5$ + $5^2 \times 2$) ~ N(469, 295)	B1 B1	**2**	For mean = $5 \times 71 + 2 \times 57$ For variance = $7^2 \times 5 + 5^2 \times 2$
(ii)	Y ~ 4M + 3W ~ N(455, 271) $X - Y$ ~ (5M + 2W) – (4M + 3W) ~ N(14, 566) Mean = 14, s.d. = $\sqrt{566} = 23.8$ P ($X - Y > 22$) = $1 - \Phi\left(\frac{22 - 14}{\sqrt{566}}\right)$ = $1 - \Phi(0.3363)$ = 1 – 0.631 or 1 – 0.632 = 0.368 or 0.369	B1 M1 A1ft M1 A1	**5**	For correct mean and variance of 4M + 3W For adding their two variances and subtracting their two means For both correct (must be s.d.), ft on wrong mean and var of Y For standardising and using tables, either end, need the sq rt For correct answer

5(i)	$\lambda = 1.8$ P(X > 2) = 1 – [P(0) + P(1) + P(2)] $= 1 - e^{-1.8}\left(1 + 1.8 + \frac{1.8^2}{2!}\right)$ = 1 – 0.7306 = 0.269	M1 A1 M1 A1 **4**	For attempt at Poisson, any mean For correct mean For finding 1 – P(0) – P(1) – P(2) or 1 – P(0) – P(1) For correct answer SR1 Normal scores B1 for $2.5 - 1.8/\sqrt{(1.7988)}$ SR2 Binomial scores M1 for complete method leading to final answer of 0.269 A1
(ii)	$\lambda = n/1500$ or P(0) < 0.01 i.e. $e^{\frac{-n}{1500}} < 0.01$ $\frac{-n}{1500} <$ ln 0.01 $n > 6907.7$ $n = 6908$ **OR** $(1499/1500)^n < 0.01$ $n = 6906$	B1 M1 A1 **3** (B1) (M1) (A1)	For correct Poisson mean For equation or inequality involving their P(0) and 0.01 For correct answer For correct Binomial p For correct equation/inequality involving their P(0) and 0.01 For correct answer
6(i)	$\int_{0.5}^{1} 3(1-x)^2 \, dx = \left[\frac{3(1-x)^3}{-3}\right]_{0.5}^{1}$ = [0] – [–1](0.5)^3 = 0.125	M1 A1 A1 **3**	For attempt at integrating and using limits Or equivalent correct integration (missing factors of 3 can still gain A1) For correct answer
(ii)	$E(X) = \int_0^1 3x(1-x)^2 \, dx = \int_0^1 3x - 6x^2 + 3x^3 \, dx$ $= \left[\frac{3x^2}{2} - \frac{6x^3}{3} + \frac{3x^4}{4}\right]_0^1$ $= \frac{3}{2} - 2 + \frac{3}{4} = 0.25$ Var (X) = $\int_0^1 3x^2(1-x)^2 \, dx - [E(X)]^2$ $= \int_0^1 3x^2 - 6x^3 + 3x^4 \, dx - (0.25)^2$ $= \left[\frac{3x^3}{3} - \frac{6x^4}{4} + \frac{3x^5}{5}\right]_0^1 - (0.25)^2$ = 0.0375	M1 A1 A1 M1 B1 A1 **6**	For attempt at $\int xf(x) \, dx$ with or without limits For 2 or 3 correct parts of the integral (missing factors of 3 can still gain A1) For correct answer For attempt at $\int x^2 f(x) \, dx - [E(X)]^2$ i.e. $-[E(X)]^2$ must be seen even if it is ignored in the next line For 2 or 3 correct parts of the integral (missing factors of 3 can still gain A1) For correct answer

7(i)	not random, could be more light etc.	B1	**1**	Any sensible reason
(ii)	One-tailed test	B1		For correct answer
	H_0: p = 0.35			
	H_1: p > 0.35	B1		For H_0 and H_1
	$P(8) = 0.35^8 = 0.000225$	M1*		For attempt at any Bin expression P(0) – P(8)
	$P(7) = 0.35^7 \times 0.65^1 \times {}_8C_7 = 0.0033456$ $P(6) = 0.35^6 \times 0.65^2 \times {}_8C_6 = 0.02174$ $P(5) = 0.35^5 \times 0.65^3 \times {}_8C_5 = 0.08077$	M1		For summing probabilities starting at P(8) and working backwards until > 0.05 (or equiv.)
	Crit region is 6, 7, 8 survive	A1		For correct answer
	4 is not in CR (**OR** $\Pr(\geq 4) = 0.294$ and comparison 0.5/or equiv.)	M1*dep		For deciding whether 4 is in their CR or not OR finding relevant prob and showing comparison
	$\Rightarrow$ no significant improvement in survival rate	A1ft	**7**	For correct conclusion (ft from their critical region)
(iii)	Saying no improvement when there is	B1	**1**	Or equivalent, relating to the question
(iv)	Need P(0, 1, 2, 3, 4, 5) or 1 – P(6, 7, 8)	M1		For identifying type II error
	$P(8) = 0.4^8$ (= 0.0006554)			
	$P(7) = 0.4^7 \times 0.6 \times {}_8C_7$ (= 0.007864)			
	$P(6) = 0.4^6 \times 0.6^2 \times {}_8C_6$ (= 0.04128)			
	$1 - (0.4^8 + 0.4^7 \times 0.6 \times {}_8C_7 + 0.4^6 \times 0.6^2 \times {}_8C_6)$ = 0.950	A1	**2**	For correct answer