UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level and GCE Advanced Subsidiary Level

MARK SCHEME for the November 2005 question paper

9709, MATHEMATICS

8719, HIGHER MATHEMATICS

9709/03 and 8719/03 Paper 3 maximum raw mark 75

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which Examiners were initially instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began. Any substantial changes to the mark scheme that arose from these discussions will be recorded in the published *Report on the Examination*.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the *Report on the Examination*.

The minimum marks in these components needed for various grades were previously published with these mark schemes, but are now instead included in the Report on the Examination for this session.

- CIE will not enter into discussion or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the November 2005 question papers for most IGCSE and GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

- Marks are of the following three types:

  M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

  A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

  B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

- The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

- MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

- PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1	*EITHER*:	State or imply non-modular inequality $(x-3a)^2 > (x-a)^2$, or corresponding equation	B1	
		Expand and solve the inequality, or equivalent	M1	
		Obtain critical value $2a$	A1	
		State correct answer $x < 2a$ only	A1	
	OR:	State a correct linear equation for the critical value, e.g. $x - 3a = -(x - a)$, or corresponding inequality	B1	
		Solve the linear equation for x, or equivalent	M1	
		Obtain critical value $2a$	A1	
		State correct answer $x < 2a$ only	A1	
	OR:	Make recognizable sketches of both $y = \lvert x-3a \rvert$ and $y = \lvert x-a \rvert$ on a single diagram	B1	
		Obtain a critical value from the intersection of the graphs	M1	
		Obtain critical value $2a$	A1	
		Obtain correct answer $x < 2a$ only	A1	**[4]**
2		State or imply that $\ln y = \ln A + n \ln x$	B1	
		Equate estimate of $\ln y$ -intercept to $\ln A$	M1	
		Obtain value A between 1.97 and 2.03	A1	
		Calculate the gradient of the line of data points	M1	
		Obtain value $n = 0.25$, or equivalent	A1	**[5]**
3		State correct derivative $1 - 2\sin 2x$	B1	
		Equate derivative to zero and solve for x	M1	
		Obtain answer $x = \frac{1}{12}\pi$	A1	
		Carry out an appropriate method for determining the nature of a stationary point	M1	
		Show that $x = \frac{1}{12}\pi$ is a maximum with no errors seen	A1	
		Obtain second answer $x = \frac{5}{12}\pi$ in range	A1$\sqrt{}$	
		Show this is a minimum point	A1	**[7]**
4	**(i)**	Consider sign of $x^3 - x - 3$, or equivalent	M1	
		Justify the given statement	A1	**[2]**
	(ii)	Apply an iterative formula correctly at least once, with initial value $x_1 = 1.5$	M1	
		Show that (A) fails to converge	A1	
		Show that (B) converges	A1	
		Obtain final answer 1.67	A1	
		Show sufficient iterations to justify its accuracy to 2 d.p., or show there is a sign change in the interval (1.665, 1.675)	A1	**[5]**
5		State or imply that $R = 10$ or $R = -10$	B1	
		Use trig formula to find α	M1	
		Obtain $\alpha = 36.9°$ if $R = 10$ or $\alpha = 216.9°$ if $R = -10$, with no errors seen	A1	
		Carry out evaluation of $\sin^{-1}(\frac{7}{10})$ $(\approx 44.427...°)$	M1	
		Obtain answer 81.3°	A1	
		Carry out correct method for second answer	M1	
		Obtain answer 172.4° and no others in the range	A1	**[7]**
		[Ignore answers outside the given range.]		

6	(i)	State $\frac{dx}{d\theta} = 2\sin\theta\cos\theta$, or $dx = 2\sin\theta\cos\theta\, d\theta$	B1	
		Substitute for x and dx throughout	M1	
		Obtain any correct form in terms of θ	A1	
		Reduce to the given form correctly	A1	[4]
	(ii)	Use cos $2A$ formula, replacing integrand by $a + b\cos 2\theta$, where $ab \neq 0$	M1*	
		Integrate and obtain $\theta - \frac{1}{2}\sin 2\theta$	A1√	
		Use limits $\theta = 0$ and $\theta = \frac{1}{6}\pi$	M1(dep*)	
		Obtain exact answer $\frac{1}{6}\pi - \frac{1}{4}\sqrt{3}$, or equivalent	A1	[4]
7	(i)	Substitute $x = 1 + 2i$ and attempt expansions	M1	
		Use $i^2 = -1$ correctly at least once	M1	
		Complete the verification correctly	A1	[3]
	(ii)	State that the other complex root is $1 - 2i$	B1	[1]
	(iii)	Show $1 + 2i$ in relatively correct position	B1	
		Sketch a locus which		
		(a) is a straight line	B1	
		(b) relative to the point representing $1 + 2i$ (call it A), passes through the mid-point of OA	B1	
		(c) intersects OA at right angles	B1	[4]
8	(i)	Separate variables correctly and attempt to integrate both sides	M1	
		Obtain term $\ln x$, or equivalent	A1	
		Obtain term $-\frac{1}{2}kt^2$, or equivalent	A1	
		Use $t = 0$, $x = 100$ to evaluate a constant, or as limits	M1	
		Obtain solution in any correct form, e.g. $\ln x = -\frac{1}{2}kt^2 + \ln 100$	A1	[5]
	(ii)	Use $t = 20$, $x = 90$ to obtain an equation in k	M1*	
		Substitute $x = 50$ and attempt to obtain an unsimplified numerical expression for t^2, such as $t^2 = 400(\ln 100 - \ln 50)/(\ln 100 - \ln 90)$	M1(dep*)	
		Obtain answer $t = 51.3$	A1	[3]
9	(i)	State or imply partial fractions are of the form $\frac{A}{x+2} + \frac{Bx+C}{x^2+1}$	B1	
		Use any relevant method to obtain a constant	M1	
		Obtain $A = 2$	A1	
		Obtain $B = 1$	A1	
		Obtain $C = -1$	A1	[5]
	(ii)	Use correct method to obtain the first two terms of the expansion of $(2+x)^{-1}$, or $(1+\frac{1}{2}x)^{-1}$, or $(1+x^2)^{-1}$	M1*	
		Obtain complete unsimplified expansions of the fractions, e.g. $2.\frac{1}{2}(1-\frac{1}{2}x+\frac{1}{4}x^2-\frac{1}{8}x^3)$; $(x-1)(1-x^2)$	A1√ + A1√	
		Carry out multiplication of expansion of $(1+x^2)^{-1}$ by $(x-1)$	M1(dep*)	
		Obtain answer $\frac{1}{2}x + \frac{5}{4}x^2 - \frac{9}{8}x^3$	A1	[5]
		[Binomial coefficients involving –1, such as $\binom{-1}{1}$, are not sufficient for the first M1.]		
		[f.t. is on A, B, C.]		
		[Apply this scheme to attempts to expand $(3x^2+x)(x+2)^{-1}(1+x^2)^{-1}$, giving M1A1A1 for the expansions, M1 for multiplying out fully, and A1 for the final answer.]		

10	**(i)**	State or imply a direction vector of AB is $-\mathbf{i} + 2\mathbf{j} + \mathbf{k}$, or equivalent		B1	
		State equation of AB is $\mathbf{r} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(-\mathbf{i} + 2\mathbf{j} + \mathbf{k})$, or equivalent		B1√	
		Substitute in equation of p and solve for λ		M1	
		Obtain $4\mathbf{i} - 2\mathbf{j} - \mathbf{k}$ as position vector of C		A1	**[4]**
	(ii)	State or imply a normal vector of p is $\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$, or equivalent		B1	
		Carry out correct process for evaluating the scalar product of two relevant vectors, e.g. $(-\mathbf{i} + 2\mathbf{j} + \mathbf{k}).(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$		M1	
		Using the correct process for calculating the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine or inverse sine of the result		M1	
		Obtain answer 24.1°		A1	**[4]**
	(iii)	*EITHER*:	Obtain AC ($= \sqrt{24}$) in any correct form	B1√	
			Use trig to obtain length of perpendicular from A to p	M1	
			Obtain given answer correctly	A1	
		OR:	State or imply $\overrightarrow{AC}$ is $2\mathbf{i} - 4\mathbf{j} - 2\mathbf{k}$, or equivalent	B1√	
			Use scalar product of $\overrightarrow{AC}$ and a unit normal of p to calculate the perpendicular	M1	
			Obtain given answer correctly	A1	
		OR:	Use plane perpendicular formula to find perpendicular from A to p	M1	
			Obtain a correct unsimplified numerical expression, e.g. $\dfrac{\lvert 2 - 2(2) + 2(1) - 6 \rvert}{\sqrt{(1^2 + (-2)^2 + 2^2)}}$	A1	
			Obtain given answer correctly	A1	**[3]**