UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2006 question paper

9709 MATHEMATICS

9709/03 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

The grade thresholds for various grades are published in the report on the examination for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the October/November 2006 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only - often written by a ‘fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR -1	A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures - this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA -1	This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1	*EITHER*: State or imply non-modular inequality $-0.5 < 3^x - 8 < 0.5$, or $(3^x - 8)^2 < (0.5)^2$, or corresponding pair of linear equations or quadratic equation	B1	
	Use correct method for solving an equation of the form $3^x = a$, where $a > 0$	M1	
	Obtain critical values 1.83 and 1.95, or exact equivalents	A1	
	State correct answer $1.83 < x < 1.95$	A1	
	OR: Use correct method for solving an equation of the form $3^x = a$, where $a > 0$	M1	
	Obtain one critical value, e.g. 1.95, or exact equivalent	A1	
	Obtain the other critical value 1.83, or exact equivalent	A1	
	State correct answer $1.83 < x < 1.95$	A1	4
	[Do not condone ≤ for <. Allow final answer given in the form $1.83 < x$, (and) $x < 1.95$.]		
	[Exact equivalents must be in terms of ln or logarithms to base 10.]		
	[SR: Solutions given as logarithms to base 3 can only earn M1 and B1 of the first scheme.]		
2	*EITHER*: Use tan 2*A* formula and obtain a horizontal equation in tan *x*	M1	
	Simplify the equation to the form $3\tan^2 x = 1$, or equivalent	A1	
	Obtain answer 30°	A1	
	Obtain second answer 150° and no others in the range	A1	
	OR: Use sin 2*A* and cos 2*A* formulae and obtain a horizontal equation in sin *x* or cos *x*	M1	
	Simplify the equation to $4\sin^2 x = 1$, $4\cos^2 x = 3$, or equivalent	A1	
	Obtain answer 30°	A1	
	Obtain second answer 150° and no others in the range	A1	4
	[Ignore answers outside the given range.]		
	[Treat answers in radians as a MR and deduct one mark from the marks for the angles.]		
	[Methods leading to an equation in cos 3*x* or cos 2*x*, or to the equality of two tangents can also earn M1A1, and then A1 + A1 for 30° and 150° only.]		
	[SR: If the answer 30° is found by inspection or from a graph, and is exactly verified, award B2. If a second answer 150° is found and verified, and no others stated, award B2.]		
3	(i) State derivative is $6e^x - 3e^{3x}$	B1	
	EITHER: Equate derivative to zero and simplify to an equation of the form $e^{2x} = a$	M1*	
	Carry out method for calculating *x*, where $a > 0$	M1(dep*)	
	Obtain answer $x = \frac{1}{2}\ln 2$, or equivalent (0.347, or 0.346, or 0.35)	A1	
	OR: Equate terms of the derivative and obtain a linear equation in *x* by taking logs correctly	M1*	
	Solve the linear equation for *x*	M1(dep*)	
	Obtain answer $x = \frac{1}{2}\ln 2$, or equivalent (0.347, or 0.346, or 0.35)	A1	4
	(ii) Carry out a method for determining the nature of a stationary point	M1	
	Show that the point is a maximum with no errors seen	A1	2
4	Separate variables correctly and attempt to integrate one side	M1	
	Obtain terms $\frac{1}{2}\ln(1 + y^2)$ and *x*, or equivalent	A1 + A1	
	Evaluate a constant or use limits $x = 0$, $y = 2$ with a solution containing terms $k\ln(1 + y^2)$ and *x*, or equivalent	M1	
	Obtain any correct form of solution, e.g. $\frac{1}{2}\ln(1 + y^2) = x + \frac{1}{2}\ln 5$	A1	
	Rearrange and obtain $y^2 = 5e^{2x} - 1$, or equivalent	A1	6

5	(i)	Simplify product and obtain $(1 + x) - (1 - x)$	B1	
		Complete the proof of the given result with no errors seen	B1	2
	(ii)	Use correct method to obtain the first two terms of the expansion of $\sqrt{1+x}$ or $\sqrt{1-x}$	M1	
		EITHER: Obtain any correct unsimplified expansion of the numerator of the RHS of the identity up to the terms in x^3	A1	
		Obtain final answer with constant term $\frac{1}{2}$	A1	
		Obtain term $\frac{1}{16}x^2$ and no term in x	A1	
		OR: Obtain any correct unsimplified expansion of the denominator of the LHS of the identity up to the terms in x^2	A1	
		Obtain final answer with constant term $\frac{1}{2}$	A1	
		Obtain term $\frac{1}{16}x^2$ and no term in x	A1	4
		[Symbolic binomial coefficients are not sufficient for the M1. Allow two correct separate expansions to earn the first A1 if the context is clear and appropriate.]		
		[Allow the use of Maclaurin, giving M1A1 for $f(0) = \frac{1}{2}$ and $f'(0) = 0$, A1 for $f''(0) = \frac{1}{8}$, and A1 for obtaining the correct final answer.]		

6	(i)	State $2(3y^2)\frac{dy}{dx}$ as derivative of $2y^3$, or equivalent	B1	
		State $3x\frac{dy}{dx} + 3y$ as derivative of $3xy$, or equivalent	B1	
		Solve for $\frac{dy}{dx}$	M1	
		Obtain given answer correctly	A1	4
		[The M1 is dependent on at least one of the B marks being obtained.]		
	(ii)	State or imply that the coordinates satisfy $y - x^2 = 0$	B1	
		Obtain an equation in x (or in y)	M1	
		Solve and obtain $x = 1$ only (or $y = 1$ only)	A1	
		Substitute x- (or y-)value in $y - x^2 = 0$ or in the equation of the curve	M1	
		Obtain $y = 1$ only (or $x = 1$ only)	A1	5
		[SR: If B1 is earned and (1, 1) stated to be the only solution with no other evidence, award B2. If the point is also shown to lie on the curve award a further B2.]		

7 (i)	*EITHER*:	State or imply general point of l has coordinates $(s, 1-2s, 1+s)$, or equivalent	B1	
		Substitute in LHS of plane equation	M1	
		Verify that the equation is satisfied	A1	
	OR:	State or imply the plane has equation $\mathbf{r}.(\mathbf{i}+2\mathbf{j}+3\mathbf{k}) = 5$, or equivalent	B1	
		Substitute for $\mathbf{r}$ in LHS and expand the scalar product	M1	
		Verify that the equation is satisfied	A1	
	OR:	Verify that a point of l lies on the plane	B1	
		Find a second point on l and substitute its coordinates in the equation of p	M1	
		Verify second point, e.g. $(1, -1, 2)$ lies on the plane	A1	
	OR:	Verify that a point of l lies on the plane	B1	
		Form scalar product of a direction vector of l with a vector normal to p	M1	
		Verify scalar product is zero and l is parallel to p	A1	3
(ii)	*EITHER*:	Use scalar product of relevant vectors to form an equation in a, b, c, e.g. $a-2b+c=0$ or $a+2b+3c=0$	M1*	
		State two correct equations in a, b, c	A1	
		Solve simultaneous equations and find one ratio, e.g. $a:b$	M1(dep)*	
		Obtain $a:b:c = 4:1:-2$, or equivalent	A1	
		Substitute correctly in $4x+y-2z=d$ to find d	M1	
		Obtain equation $4x+y-2z=1$, or equivalent	A1	
	OR:	Attempt to calculate vector product of relevant vectors, e.g. $(\mathbf{i}-2\mathbf{j}+\mathbf{k})\times(\mathbf{i}+2\mathbf{j}+3\mathbf{k})$	M2	
		Obtain 2 correct components of the product	A1	
		Obtain correct product, e.g. $-8\mathbf{i}-2\mathbf{j}+4\mathbf{k}$	A1	
		Substitute correctly in $4x+y-2z=d$ to find d	M1	
		Obtain equation $4x+y-2z=1$, or equivalent	A1	
		[SR: If the outcome of the vector product is the negative of the correct answer allow the final mark to be available, i.e. M2A0A0M1A1is possible.]		
	OR:	Attempt to form 2-parameter equation for the plane with relevant vectors	M2	
		State a correct equation, e.g. $\mathbf{r} = 2\mathbf{i}+\mathbf{j}+4\mathbf{k}+\lambda(\mathbf{i}-2\mathbf{j}+\mathbf{k})+\mu(\mathbf{i}+2\mathbf{j}+3\mathbf{k})$	A1	
		State 3 equations in x, y, z, λ, μ	A1	
		Eliminate λ and μ	M1	
		Obtain equation $4x+y-2z=1$, or equivalent	A1	6

8 (i)	*EITHER*:	State or imply $f(x) \equiv \frac{A}{2x+1}+\frac{B}{x+1}+\frac{C}{(x+1)^2}$	B1	
		Use any relevant method to obtain a constant	M1	
		Obtain one of the values $A=2$, $B=-1$, $C=3$	A1	
		Obtain the remaining two values	A1 + A1	
		[A correct solution starting with third term $\frac{Cx}{(x+1)^2}$ or $\frac{Cx+D}{(x+1)^2}$ is also possible.]		
	OR:	State or imply $f(x) \equiv \frac{A}{2x+1}+\frac{Dx+E}{(x+1)^2}$	B1	
		Use any relevant method to obtain a constant	M1	
		Obtain one of the values $A=2$, $D=-1$, $E=2$	A1	
		Obtain the remaining two values	A1 + A1	5
(ii)		Integrate and obtain terms $\frac{1}{2}.2\ln(2x+1)-\ln(x+1)-\frac{3}{x+1}$, or equivalent	B1√ + B1√ + B1√	
		Use limits correctly, having integrated all the partial fractions	M1	
		Obtain given answer following full and exact working	A1	5
		[The f.t. is on A, B, C etc.]		
		[SR: If B, C, or E are omitted, give B1M1in part (i) and B1√B1√M1 in part (ii): max 5/10.]		

9	(i) *EITHER*: Multiply numerator and denominator by 2 + i , or equivalent	M1	
	Simplify numerator to 5 + 5i or denominator to 5	A1	
	Obtain answer 1 + i	A1	
	OR: Obtain two equations in x and y, and solve for x or for y	M1	
	Obtain $x = 1$	A1	
	Obtain $y = 1$	A1	
	OR: Using correct processes express u in polar form	M1	
	Obtain $u = \sqrt{2}$ (cos 45° + i sin 45°), or equivalent	A1	
	Obtain answer 1 + i	A1	3
	(ii) State that the modulus is $\sqrt{2}$ or 1.41	B1√	
	State that the argument is 45° or $\frac{1}{4}\pi$ (or 0.785)	B1√	2
	(iii) Show the point representing u in a relatively correct position	B1√	
	Show a circle with centre at the point representing u	B1√	
	Indicate or imply the radius is 1	B1	3
	[NB: If the Argand diagram has unequal scales the locus is not circular in appearance, but an ellipse with centre u and equal axes parallel to the axes of the diagram earns B1√, and B1 if both semi-axes are indicated or implied to be equal to 1. In such a situation only award B1√ for a circle with centre u and a horizontal or vertical radius indicated or implied to be 1.]		
	(iv) Carry out complete strategy for calculating min$\lvert z \rvert$ for the locus	M1	
	Obtain answer $\sqrt{2} - 1$ (or 0.414)	A1√	2
	[The f.t. is on the value of u.]		
10	(i) Use product rule	M1	
	Obtain correct derivative cos $2x - 2x$ sin $2x$	A1	
	Equate derivative to zero and obtain given answer correctly	A1	3
	(ii) Use the iterative formula correctly at least once	M1	
	Obtain final answer 0.43	A1	
	Show sufficient iterations to at least 3d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (0.425, 0.435)	A1	3
	(iii) Attempt integration by parts and obtain $\pm kx \sin 2x \pm \int l \sin 2x \, dx$, where $k, l = \frac{1}{2}$, 1, or 2	M1*	
	Obtain $\frac{1}{2} x \sin 2x - \int \frac{1}{2} \sin 2x \, dx$	A1	
	Obtain indefinite integral $\frac{1}{2} x \sin 2x + \frac{1}{4} \cos 2x$	A1	
	Use limits $x = 0$ and $x = \frac{1}{4}\pi$ having integrated twice	M1(dep)*	
	Obtain answer $\frac{1}{8}\pi - \frac{1}{4}$, or exact equivalent	A1	5