MATHEMATICS

Paper 9709/11
Paper 11

General comments

Candidates generally found the paper accessible and there were many excellent responses. **Questions 5** and **6** caused considerable difficulty, probably because of their non-stereotyped nature. The algebra required in **Question 8** also presented problems, but it was pleasing that the majority of candidates, even those unable to complete the request in part **(i)**, used the answer correctly in the rest of the question. The standard of presentation was good and there was little evidence that candidates had been short of time.

Comments on specific questions

Question 1

Although the majority of candidates correctly equated $2x + 15$ with $\tan^{-1}\left(\frac{4}{3}\right)$, many weaker candidates automatically replaced $\tan(2x + 15)$ by $\tan 2x + \tan 15°$ or by $2 \tan x + \tan 15°$. A surprising number added 15º to 53.13º, rather than subtracting, and a sizeable minority of all candidates only considered angles in the first quadrant.

Answer: 19.1º, 109.1º.

Question 2

This question presented problems, with many candidates failing to realise that the curves 'flattened' at $x = 0$ and at $x = \pi$. Although most candidates realised that the graph lay between $y = -3$ and $y = 3$, it was very common to see graphs that were 'V-shaped' or in the shape of a parabola. The drawing of the straight line also caused difficulty with many candidates failing to realise that the line had a negative gradient and passed through $(\pi, 0)$.

Question 3

(i) This was very well answered with the majority of candidates correctly using binomial coefficients.

(ii) It was pleasing that most candidates realised that the product of $(1 + 2x + ax^2)$ with $(2 - x)^6$ gave three terms in x^2.

Answers: **(i)** $64 - 192x + 240x^2$; **(ii)** 3.

Question 4

This was a straightforward question that generally was answered well.

(i) The majority of candidates differentiated correctly and set the differential to zero. Surprisingly, the solution of the equation $4x^3 + 4 = 0$ was often quoted as $x = \pm 1$, or even as $x = 1$.

(ii) Nearly all candidates realised the need to integrate the equation of the curve. The standard of integration and use of limits was excellent.

Answers: **(i)** (−1, 6), Minimum; **(ii)** 11.2.

Question 5

This was found to be a difficult question with most candidates unable to find the radius of the sector *ABD*. Many attempts incorrectly assumed that the radius of the sector was 6 cm and it was rare for candidates to use trigonometry or Pythagoras to find *AB*. Even more surprising was the failure to realise that the angle of the sector was 45º. Many candidates attempted to use the formula $s = r\theta$ or $A = \frac{1}{2}r^2\theta$ with the angle in degrees.

Answers: **(i)** 6.66 cm; **(ii)** 10.3 cm².

Question 6

(i) This also proved to be a difficult question with a minority of candidates realising that the gradients of the tangents at $x = 2$ and $x = 3$ were $k - 4$ and $k - 6$ respectively and that consequently $(k-4)(k-6) = -1$. It was also common to see this equation replaced by $k^2 - 10k + 23 = 0$ instead of $k^2 - 10k + 25 = 0$.

(ii) Most candidates recognised the need to integrate and the majority included a constant of integration. Many weaker candidates however confused the equation of the curve with the equation of the tangent and attempted to use the equation $y = mx + c$.

Answers: **(i)** 5; **(ii)** $y = 5 + 5x - x^2$.

Question 7

(i) The differentiation was generally correct, though many candidates did not recognise that the function was composite and consequently omitted the '$\times 2x$'.

(ii) Surprisingly, some candidates confused 'normal' with 'tangent', and many others left the gradient of the normal as a function of *x*, thereby finishing with the equation of a curve rather than with a straight line.

(iii) Most candidates correctly used the link between the rate of change of *y* and the rate of change of *x* and obtained the available follow-through marks.

Answers: **(i)** $-24x(x^2+3)^{-2}$; **(ii)** $3y = 2x + 7$; **(iii)** −0.018 units per second.

Question 8

(i) Most candidates correctly identified the equations '$8 + 4d = 8r$' and '$8 + 7d = 8r^2$', but a minority were able to solve for *r* or *d*. Many candidates lost marks by assuming that $r = \frac{3}{4}$ to find the value of *d*.

(ii) The formula for the sum to infinity of a geometric progression was well known and this part of the question was usually correct.

(iii) Most candidates obtained the sum of the first 8 terms of the arithmetic progression as $4(16 + 7d)$ but poor arithmetic often led to an incorrect answer.

Answers: **(i)** $-\frac{1}{2}$; **(ii)** 32; **(iii)** 50.

Question 9

(i) Most candidates were confident in using the scalar product of two vectors. However two basic errors affected the accuracy mark. It was a common error to assume that the two vectors needed to find angle *AOB* were $\overrightarrow{AO}$ and $\overrightarrow{OB}$ instead of $\overrightarrow{OA}$ and $\overrightarrow{OB}$, and many candidates correctly used vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ but then assumed that the angle had to be acute.

(ii) This part of the question was poorly answered. Only a small minority of candidates realised the need to find the unit vector in the direction of $\overrightarrow{AC}$ and then to multiply this by 30.

(iii) Most candidates obtained the components of vector $\overrightarrow{OA} + p\overrightarrow{OB}$ in terms of *p* and then equated the scalar product of this vector with vector $\overrightarrow{OC}$ to zero.

Answers: **(i)** 160.5º; **(ii)** $\begin{pmatrix} -20 \\ 10 \\ 20 \end{pmatrix}$; **(iii)** $\frac{1}{2}$.

Question 10

(i) Candidates were confident in obtaining an expression for gf(x). Solving the equation gf(x) = x caused difficulty with many obtaining the quadratic equation as $2x^2 + 1 = 0$ instead of $2x^2 - 1 = 0$ and others offering the solutions as $\pm \frac{1}{2}\sqrt{2}$.

(ii) Candidates were confident in finding the inverse of a function and the inverse of f(x) was nearly always correctly found. Weaker candidates however often struggled with the algebra required to find the inverse of g(x).

(iii) Candidates realised the need to form a quadratic equation by equating $g^{-1}(x)$ with x. Unfortunately only about a half of all candidates realised that for the equation $g^{-1}(x) = x$ to have no solutions, then the discriminant ($b^2 - 4ac$) had to be negative.

(iv) Most candidates recognised that the lines representing $y = f(x)$ and $y = f^{-1}(x)$ were symmetrical in the line $y = x$. Unfortunately, only a small minority of all candidates realised that the graph of $y = f(x)$ started at the point (0, 1) and the graph of $y = f^{-1}(x)$ started at (1, 0).

Answers: **(i)** $x = \frac{1}{2}\sqrt{2}$; **(ii)** $\frac{1}{2}(x-1)$, $\frac{1+3x}{2-x}$.

MATHEMATICS

Paper 9709/12
Paper 12

General comments

Many of the candidates taking this paper showed a sound understanding of the subject, knowledge of the particular topics covered and ability to use appropriate methods for problem solving.

Candidates generally made good use of their time, enabling them to deal adequately with the longer, more difficult questions towards the end of the paper.

There were considerable differences in the standard of presentation between Centres. While in some it was uniformly high, in others it was less good, with work, sometimes whole pages, crossed out, untidily set out and carelessly written. This is a particular disadvantage in a subject calling for clarity and precision.

While the rubric was mostly observed, the instruction to give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles, was not. In **Question 6(ii)** the answer 53.0° was often written as 53° and in **Question 7(ii)** the exact answer $156\frac{1}{4}$ (or 156.25) was often rounded to 156.

Comments on specific questions

Question 1

This question was generally well answered. Candidates recognised the need to write $\frac{3}{\sqrt{x}}$ in index form in order to integrate. Some left their answer in the unsatisfactory from $\frac{3x^{\frac{1}{2}}}{\frac{1}{2}}$. Having integrated correctly many candidates found the correct value of the constant of integration. The commonest fault was failure to integrate and to attempt to use $\frac{dy}{dx}$ to find a gradient in the equation of a straight line.

Answer: $y = 6\sqrt{x} - \frac{1}{2}x^2 + 2$.

Question 2

(i) This was very well answered. Only a few candidates left the coefficients as $^8C_1$, $^8C_2$, etc.

(ii) This was also well answered. The commonest error was to mistake terms for coefficients, leading to $28k^6x^2 = 56k^5x^3$ and $k = 2x$.

Answers: **(i)** $k^8 + 8k^7x + 28k^6x^2 + 56k^5x^3$; **(ii)** 2.

Question 3

(i) The structure of an AP was well understood, with many candidates correctly writing $a + d = 96$, $a + 3d = 54$. They mostly managed the algebra, finding $d = -21$ and hence $a = 117$.

(ii) This part was equally well answered, with candidates correctly writing $ar = 96$ and $ar^3 = 54$. The algebra proved manageable, although some candidates did not reject the value $r = -\frac{3}{4}$. A few, having found *r*, omitted to find the value of *a*.

Answers: **(i)** 117; **(ii)** 128.

Question 4

(i) The fact that the range is a set of *y*-values was poorly understood. Moreover, candidates had difficulty in finding the end-values 2 and 8.

(ii) Sketching the graph caused a lot of difficulty, as candidates did not appreciate the three key features: one complete cycle starting and finishing at $y = 5$, convex down then convex up.

Answers: **(i)** $2 \leqslant f(x) \leqslant 8$; **(iii)** No inverse, not one-one.

Question 5

(i) This part was well answered. The majority of candidates multiplied out the left-hand side and correctly used $s^2 + c^2 = 1$ to obtain the right-hand side. Some found one or other of the alternatives $(s + c)\ (1 - sc) = (s + c)\ (s^2 + c^2 - sc)$, etc. or $s - sc^2 + c - cs^2 = s(1 - c^2) + c(1 - s^2) = ss^2 + cc^2$

(ii) Solving the equation caused quite a lot of difficulty. Common errors were $\frac{\cos^3 x}{\sin^3 x} = \tan^3 x = 8$, $\frac{8\sin^3 x}{\cos^3 x} = 0$. It was generally well understood that from an equation in tan x the second value of x in the range $0 \leqslant x \leqslant 360$ is $180 + x$

Answer: **(ii)** 26.6°, 206.6°.

Question 6

(i) Many candidates answered this part correctly, although some made it needlessly complicated by writing, for example $\overrightarrow{OQ} = \overrightarrow{OA} + \overrightarrow{AP} + \overrightarrow{PQ}$ rather than the direct $\frac{1}{2}\overrightarrow{OA} + \frac{1}{2}\overrightarrow{OC} + \overrightarrow{OD}$.

(ii) Candidates showed a very good understanding of the method for finding the angle between two vectors. Mistakes which did occur included using the wrong sides of triangle *OPQ*, reversing the vectors, and getting **i**, **j** and **k** in different orders, leading to mistakes in finding the scalar product. As mentioned in the general comments, there was a very common failure to give the answer correct to 1 decimal place, with 53.0° written as 53°.

Answers: **(i)** 3**i** + 3**j** + 6**k**, –3**i** + **j** + 6**k**; **(ii)** 53.0°.

Question 7

(i) Most candidates produced sound answers to this part, showing good knowledge of the formulae for arc length and sector area, and managing the algebra to produce the given result.

(ii) This was also well answered, as far as finding $\frac{dA}{dr} = 25 - r$, setting it to 0 and obtaining $r = 12.5$. However, many candidates omitted to find the corresponding value of *A*, and many of those who did rounded the exact answer $156\frac{1}{4}$ to 156. Most correctly found $\frac{d^2A}{dr^2} = -2$ and deduced that the value of *A* was a maximum.

Answers: **(ii)** $156\frac{1}{4}$, Maximum.

Question 8

(i) Many candidates managed to find $f'(x) = \frac{-6}{(2x+5)^2}$ but were unable to explain clearly why f is a decreasing function. Few said $(2x + 5)^2$ is always positive, so $f'(x)$ is always negative.

(ii) This was generally well answered. A few candidates confused $f'(x)$ and $f^{-1}(x)$.

(iii) This part posed some difficulty. Most candidates attempted to find $\pi\int_0^2 \frac{9}{(2x+5)^2}\,dx$ but some found the integration hard, while others assumed wrongly that the value of the integral evaluated at 0 was 0.

Answers: **(i)** $\frac{-6}{(2x+5)^2}$; **(ii)** $\frac{1}{2}\left(\frac{3}{x} - 5\right)$; **(iii)** 1.26.

Question 9

(i) The reason why the *y*-coordinate of *D* is 6 was not well explained, as relatively few candidates appreciated that the diagonals *AC* and *BD* bisect and that the *y*-coordinate of the mid-point of *AC* is $-2 + \frac{14-(-2)}{2} = 6$.

(ii) Candidates found this the easiest part of the question. Those giving their answers as $\frac{8}{h}$ and $\frac{8}{12-h}$ often made useful progress with part **(iii)**. Those who gave the gradients as $\frac{8}{h}$ and $-\frac{h}{8}$ rarely made any further progress.

(iii) Using the product of the gradients of *AD* and *CD* equals –1, many candidates correctly arrived at $h = 16, -4$ from the quadratic equation $h^2 - 12h - 64 = 0$. A few found a good alternative method by calculating the length of *AC* (= 20) and using this with the mid-point of *BD* to obtain 6 + 10 and 6 – 10 as the required *x*-coordinates.

(iv) Candidates generally found this very difficult. The most commonly used method was to find the lengths of *AD* and *CD* and calculate their product. Rarely used alternatives were: area = 2 × area of triangle *BCD* = $2 \times \frac{1}{2} \times 20 \times 8$ or using a determinant.

Answers: **(ii)** $\frac{8}{h}$, $\frac{8}{12-h}$ or $\frac{8}{h}$, $\frac{-h}{8}$; **(iii)** 16, –4; **(iv)** 160.

Question 10

(i) (a) Many candidates were able to solve the simultaneous quadratic and linear equations to find the *x*-coordinates of *A* and *B*, although a significant number went on to find the *y*-coordinates, which were not asked for. Those who chose to eliminate *x* to get a quadratic in *y* gave themselves unnecessary work.

(b) Many candidates correctly found the gradient of the tangent at *B* from $\frac{dy}{dx} = 2x - 4$ and went on to find its equation correctly. There was some confusion between tangent and normal, with candidates using $-\frac{1}{2}$ instead of 2 as the required gradient.

(c) This proved to be the least well answered part question of all. The most common method, finding the difference between $\tan^{-1}2$ and $\tan^{-1}\frac{1}{2}$, was beyond most candidates. A few attempted to use the formula tan(*A* – *B*) = $\frac{\tan A - \tan B}{1 + \tan A \tan B}$, but of these some either quoted it or used it wrongly. Even fewer tried the useful vector method for finding the angle between $\begin{pmatrix}1\\2\end{pmatrix}$ and $\begin{pmatrix}2\\1\end{pmatrix}$.

(ii) This was generally well answered, but some solutions were spoiled by errors in the inequality sign or the arithmetic.

Answers: **(i)(a)** 3, 1.5, **(b)** $y = 2x - 2$, **(c)** 36.9°; **(ii)** $k < 3.875$.

MATHEMATICS

Paper 9709/21
Paper 21

General comments

Three questions were found to be challenging: **Question 2** on logarithms, **Question 5** on basic knowledge of trigonometric formulae and integration and **Question 8** on implicit differentiation. Only a small number of candidates could cope with **Question 2** and very few were capable of successfully attempting **Question 5**.

Candidates of modest ability found **Questions 1, 3, 6** and **7** to be accessible to a greater or lesser extent. The level of algebraic competence displayed by many candidates was disappointing. Care with signs, accuracy when solving simple equations, appropriate use of brackets and knowledge of logarithm properties are expected at this level. Often careless mistakes or basic misunderstandings cost candidates dearly.

Mistakes equivalent to $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$ and $(a+b)^2 = a^2 \pm b^2$ were often noted.

A small minority of candidates scored heavily and displayed a wide-ranging grasp of the syllabus, deserving great praise for their efforts. Almost everyone set out their arguments thoughtfully and clearly, even if not always logically. There was no indication that lack of time was a problem for candidates.

Comments on specific questions

Question 1

Almost everyone opted to square each side of the given inequality to form a two-term quadratic equation or inequality $3x^2 + 18x \leqslant 0$. Some candidates then 'cancelled out' a factor of $3x$ to end with a linear inequality $x + 6 \leqslant 0$, not realising that x may be zero or negative. However, most realised that the critical values were $x = 0, -6$, even if many struggled to decide upon appropriate inequality signs.

Answer: $-6 < x < 0$.

Question 2

Many candidates fell at the first step, stating that $\ln(3 - x^2) = \ln 3 - \ln x^2$. All that was necessary was to note that $2\ln x = \ln x^2$ and hence $3 - x^2 = x^2$ as $\ln a = \ln b$ implies $a = b$.

Answer: 1.22.

Question 3

(i) This was well done by those that realised that $p(-\frac{1}{2}) = 0$, though sign errors in simplifying this equation were frequent.

(ii) Most candidates found the remaining factor as $(2x^2 - 5x - 3)$ and successfully factorised this. A minority found division of $p(x)$ by $(2x + 1)$ beyond them due to sign errors.

Answers: **(i)** -11; **(ii)** $(2x + 1)^2(x - 3)$.

Question 4

(i) A minority of candidates used an incorrect expansion for sin(a – b) and others never gave a value for sin 60°. A common error was to find correctly that ($\sqrt{3}$ cos x – sin x) = 4 sin x, but then to obtain the coefficient of sin x in the simplified equation to be 3.

(ii) This was well attempted by those who had a value for k from part **(i)**, though a significant number could not evaluate $\tan^{-1}\left(\frac{\sqrt{3}}{5}\right)$.

Answers: **(ii)** 19.1°, 199.1°.

Question 5

Though not as poorly attempted as **Question 2**, candidates found this question challenging. Almost everyone struggled with part **(i)** and many long, inaccurate forms were produced for $\cos^2 2x$ in terms of cos 4x, even by those who mentioned that $\cos 2\theta \equiv 2\cos^2\theta - 1$ was relevant. Most simply opted to state that $\cos^2 2x \equiv \cos 4x$.

In part **(ii)**, the cos 4x from part **(i)** was usually successfully integrated and evaluated at x = 0 and $x = \frac{1}{8}\pi$. Those who had some incorrect result from part **(i)** produced a wide variety of incorrect integrated functions.

Answers: **(i)** $\frac{1}{2}+\frac{1}{2}\cos 4x$; **(ii)** $\frac{1}{16}\pi+\frac{1}{8}$.

Question 6

(i) Differentiation was invariably excellent, with only a small minority of candidates obtaining a single-term derivative of y. Surprisingly, a few simplified $\frac{x}{x}$ to be 0, rather than 1. After correctly obtaining the stationary point given by $x = e^{-1}$, most candidates left the y-value, $y(e^{-1})$, as $e^{-1}\ln(e^{-1}) = -e^{-1}$, but others did not simplify the expression for y or did so wrongly.

(ii) Almost everyone noted that $\frac{d^2y}{dx^2} = x^{-1}$ and argued correctly that a minimum point was involved.

Answer: **(i)** $(\frac{1}{e}, -\frac{1}{e})$.

Question 7

(i) Many candidates realised that $\int_0^p e^{-x}\,dx$ was involved, though about half of the candidates incorrectly set the bottom limit as 1, rather than 0. Evaluating $-e^{-x}$ at x = 0 did not always prove simple. Those who correctly obtained the integral as $(1 - e^{-p})$ often did not realise that this area must be subtracted from that of the rectangle, namely p.

(ii) This was only accessible to a minority with a correct answer to part **(i)**.

(iii) This was well done by the vast majority, though many worked to only 2 or 3 decimal places for each iteration or failed to give their final answer correct to 2 decimal places.

Answers: **(i)** $p + e^{-p} - 1$; **(iii)** 1.84.

Question 8

(i) A surprising number of candidates set $x = 1$ and solved the quadratic $y^2 + 2y - 3 = 0$ correctly, but then 're-calculated' x to obtain values, one of which was not equal 1.

(ii) Many candidates could not differentiate $2xy$ correctly, often obtaining only a single term. Sign errors were common and only roughly one half of solutions featured $\frac{dy}{dx} = \frac{x - y}{x + y}$. Some candidates set this expression equal to zero and argued that $y = x$, but then gave the value 1 to x, without having earlier shown that (1, 1) was a point in part **(i)**.

Answers: **(i)** (1, 1), (1, –3); **(ii)** $2x + y + 1 = 0$.

MATHEMATICS

Paper 9709/22
Paper 22

General comments

All questions proved accessible, although in **Question 3(i)** many candidates attempted exact integration and ignored the instruction to use the trapezium rule. Many excellent scripts were seen. Marks were sometimes lost due to elementary mistakes, e.g. sign errors, but the general level of competence was good. Lack of time did not appear to be a problem for candidates.

Comments on specific questions

Question 1

Most candidates opted to square each side of the given inequality to obtain the inequality $3x^2 - 6x - 9 < 0$. This was usually correctly solved to yield critical values $x = -1, 3$ though sometimes candidates struggled to obtain the correct inequality signs in their final answer.

Answer: $-1 < x < 3$.

Question 2

Only a small minority of candidates failed to note that $\ln(y + 5) - \ln y = \ln\left(\frac{y+5}{y}\right)$ and that $2 \ln x = \ln x^2$. A few candidates had $\frac{y+5}{y} = 2x$, but most found this a reasonably straightforward question.

Answer: $y = \frac{5}{x^2 - 1}$.

Question 3

(i) A larger number of candidates opted to attempt to find an exact function equalling $\int \sec x \, dx$ rather than an approximate value via the trapezium rule for the definite integral. Those using the correct method often failed to correctly evaluate $\sec x$ at $x = \frac{1}{6}\pi$ and $\frac{1}{3}\pi$, though $h = \frac{\pi}{6}$ was invariably correctly stated.

(ii) Almost no-one failed to argue correctly form a sketch.

Answers: **(i)** 1.39.

Question 4

(i) Although most candidates realised that $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$, some could not evaluate $\frac{dx}{dt}$ and/or $\frac{dy}{dt}$

(ii) Simplifying the equation $e^{2t} - 1 = 2$ proved beyond many candidates, and a correct exact corresponding value for *t* was not frequently seen.

Answer: **(ii)** $\frac{1}{2}\ln 3$.

Question 5

This question proved highly popular and was usually well attempted. In part **(i)**, virtually everyone correctly set p(–1) and p(2) equal to zero, though sign errors in simplifying the resultant linear equations for *a* and *b* were not uncommon. Part **(ii)** was almost always correctly attempted.

Answers: **(i)** 3, –4; **(ii)** $3x - 1$.

Question 6

(i) Almost all solutions featured correct values for *R* and *α*, though a few candidates had $\alpha = 36.87°$ instead of 53.13°.

(ii) Although the solution $x = 79°$ was usually obtained, not many candidates appreciated that $\cos(x - \alpha) = k$ implies $x = \alpha \pm \cos^{-1} k$ here. Many candidates who could not do part **(i)** struggled to solve part **(ii)**, though some who said that $(3\cos x)^2 = (4.5 - 4 \sin x)^2$ and $\cos^2 x + \sin^2 x = 1$ eventually found at least one correct value for *x*.

Answers: **(i)** $5\cos(x - 53.13°)$; **(ii)** 79.0°, 27.3°.

Question 7

(i) The differentiation was almost uniformly well done and the result was obtained. Very few solutions featured an incorrect single-term derivative.

(ii) Almost everyone set $f(x) = \tan x - 2(x)^{-1}$ and evaluated f(1.0) and f(1.2), then drew the correct conclusion from the feature $f(1.0) \times f(1.2) < 0$.

(iii) A few candidates confused x_n in radians with the (incorrect) *x* in degrees, but responses were almost all excellent. A few candidates used only 2 or 3 decimal places when iterating or failed to round their final iterate to 2 decimal places.

Answer: **(iii)** 1.08.

Question 8

(a) Some candidates struggled to integrate sin 2x correctly, giving a result $\frac{1}{2}\cos 2x$ or $-\cos 2x$. After integrating, many could not evaluate cos 2x or tan x at $x = \frac{1}{3}\pi$.

(b) Occasionally, the integral of $\frac{1}{2x}$ was given as ln(2x) instead of $\frac{1}{2}\ln x$, but almost everyone obtained a linear combination of ln x and ln(x + 1) on integrating. The error involving ln a – ln b = $\frac{\ln a}{\ln b}$ was rare.

Answer: **(a)** $\frac{3}{4} + \sqrt{3}$.

MATHEMATICS

Paper 9709/31

Paper 31

General comments

There was considerable variation in the standard of work on this paper and this resulted in a broad spread of marks. Though no question appeared to be of undue difficulty, most questions discriminated well and this resulted in fewer very high scores than in some past papers. The questions or parts of questions that were generally done well were **Question 3** (iteration), **Question 8** (partial fractions) and **Question 9** (calculus). Those that were done least well were **Question 1** (inequality), **Question 6** (vector geometry) and **Question 10 (i)** (differential equation).

In general the presentation of the work was fairly good but there are still candidates who present their work in a double column format. This makes marking difficult for Examiners and it would be helpful if Centres could discourage this practice.

There were many scripts in which candidates offered widely spaced lengthy solutions to a question that could have been presented far more economically. For example, in **Question 9 (ii)** it was not uncommon to find candidates taking two pages of working to find the x-coordinate of M. The advantages to candidates of compact working are that (a) the work is easier for them to scan for errors when checking, and (b) it uses up less of their time.

In **Questions 3**, **5** and **9**, a request was made for the 'exact value' of an answer. Some candidates appeared to either not know or be unsure of the meaning of this term. Thus they either gave only an approximate answer or else, having given an exact answer, went on to calculate an approximate value.

Where numerical and other answers are given after the comments on individual questions, it should be understood that alternative forms are often acceptable and that the form given is not necessarily the sole 'correct answer'.

Comments on specific questions

Question 1

Completely sound responses to the problem of solving $2 - 3x < |x - 3|$ were rare and usually came from candidates who made use of a sketch of $y = 2 - 3x$ and $y = |x - 3|$ on a single diagram.

Candidates who worked with the non-modular quadratic inequality obtained by squaring both sides seemed unaware of the limitations of the approach. In an inequality such as this, involving a linear expression and the modulus of a linear expression, the method cannot be safely relied on to do more than identify possible critical values of the original inequality. In this particular problem one of the two possibilities found by squaring turns out to be critical for the original inequality. A few candidates realised this and went on to solve the problem correctly. For a further illustration of the weakness of the method, consider the inequality $x > |2x + 1|$. This has no critical values and is false for all values of x. The corresponding quadratic inequality $x^2 > (2x+1)^2$ has two critical values and is true for $-1 < x < -\frac{1}{3}$.

Those who worked with non-modular linear inequalities almost always ignored the conditions under which these inequalities were defined. For example, the solution of 2 – 3*x* < 3 – *x*, the form of the inequality when *x* < 3, was taken to be $x > -\frac{1}{2}$ rather than $-\frac{1}{2} < x < 3$. Similarly, in the case when $x \geqslant 3$, the inequality is equivalent to 2 – 3*x* < *x* – 3. Here almost all candidates took the solution to be $x > \frac{5}{4}$ rather than $x \geqslant 3$.

Answer: $x > -\frac{1}{2}$.

Question 2

This drew a very mixed response. Some candidates began by taking logarithms of both sides, mistakenly took the logarithm of a sum to be the sum of the logarithms and scored nothing at all. On the other hand those with a sound grasp of indices found this an easy question. However a sizeable minority, having reached an equation of the form 9*y* – *y* – 8 = 0, where $y = 3^x$, attempted to obtain *y* using the quadratic formula with *a* = 9, *b* = –1 and *c* = –8, rather than treat the equation as linear in *y*.

Answer: 0.107.

Question 3

This question was well answered. In part **(i)** the majority gave their iterates to 4 decimal places and the final answer to 2 decimal places as requested. In part **(ii)** most candidates could write down an appropriate equation in α but some made slips in rearranging it. For example, $\sqrt[4]{15}$ was a common incorrect answer. Also some omitted to state the exact value of α and gave an approximate value instead.

Answers: **(i)** 2.78; **(ii)** $\sqrt[4]{60}$.

Question 4

This question was fairly well answered. Most candidates differentiated correctly. The task of reaching an equation in one trigonometric function was easiest for those who immediately expressed $\sec^2 x$ in terms of tan *x*. Those who went on to reach $3\sin x\cos x = 1$ were not always able to convert it into $\sin 2x = \frac{2}{3}$; a quite common erroneous conversion was to $\sin 3x = 1$.

Answer: 0.365, 1.206.

Question 5

In part **(i)** well prepared candidates produced a wide variety of correct, though often longwinded, solutions. However there were many candidates whose attempts failed because they could not state a correct alternative expression for $\cos 4\theta$. The phrase 'Using this result' was included in the wording of part **(ii)** to ensure that candidates would take advantage of the identity in part **(i)**. However Examiners found that a large number ignored the identity and made almost invariably unsuccessful attempts to integrate $\sin^4\theta$ from scratch. Those that used the identity sometimes forgot to include the factor of $\frac{1}{8}$, but there was much careful and accurate work to be seen.

Answer: **(ii)** $\frac{1}{32}(2\pi - \sqrt{3})$.

Question 6

This apparently straightforward exercise in plane vector geometry was surprisingly poorly answered. In part **(i)** the majority of candidates failed to find the position vector of *M*, the mid-point of *AB*, correctly, usually calling it $\frac{1}{2}\overrightarrow{AB}$. The fact that no component of this vector lies between the corresponding components of the position vectors of *A* and *B* did not appear to be of concern. Examiners found that candidates were more successful in finding the position vector of *N*, one of the points of trisection of *AC*, though again there were many candidates without a sound approach. In general candidates knew how to proceed in part **(ii)**, but the prevalence of errors made in part **(i)** meant that fully correct solutions were rare.

Answer: **(i)** $\mathbf{r} = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k} + \lambda(\mathbf{i} - 3\mathbf{j} + 3\mathbf{k})$; **(ii)** $5\mathbf{i} - 8\mathbf{j} + 7\mathbf{k}$.

Question 7

Some candidates lacked a sound method for part **(i)**, but those that had one usually made a good attempt. The unsound methods included treating the 3-term cubic as a quadratic. Part **(ii)** was generally correctly answered, the commonest incorrect answer being 2 – i. In part **(iii)** the modulus was usually correct but many candidates gave the incorrect value of –26.6° for the argument. Most candidates scored a mark or two in part **(iv)** but few made completely correct sketches.

Answers: **(i)** 20; **(ii)** –2 – i; **(iii)** $\sqrt{5}$, 153.4°.

Question 8

Examiners found that part **(i)** was very well answered. Candidates usually started out with an appropriate general form of fractions and had a sound method for determining the constants. The most frequent error was the omission of one of the constants from the initial form.

In part **(ii)** the expansions of the fractions were often done well; the expansion of $-3(3x+2)^{-1}$ seemed to cause the most difficulty.

Answers: **(i)** $\frac{1}{x+1} + \frac{2}{(x+1)^2} - \frac{3}{3x+2}$; **(ii)** $\frac{3}{2} - \frac{11}{4}x + \frac{29}{8}x^2$.

Question 9

This question was answered well. Part **(i)** was almost always correctly answered and most candidates had sound calculus methods for the remaining two parts. Algebraic and arithmetic slips when handling the fractional powers of *x* seemed to be the main reason why marks were lost. Examiners were pleased to see that in part **(iii)**, most candidates who had integrated substituted limits correctly and obtained $4\ln 4 - 4$, then showed sufficient working to justify the given answer $8\ln 2 - 4$.

Answers: **(i)** (1, 0); **(ii)** e^2.

Question 10

In part **(i)** only the strongest candidates could progress from $\frac{dA}{dt} = kV$ to the given differential equation in *r* and *t*. The fact that *k* = 0.48 and not 0.08 was a source of confusion for some. By contrast, part **(ii)** was quite well answered, though algebraic errors were often made when making *r* the subject of the solution. There were very few correct answers to part **(iii)**.

Answers: **(ii)** $r = \frac{5}{(1-0.4t)}$; **(iii)** $0 \leqslant t < 2.5$.

MATHEMATICS

Paper 9709/32

Paper 32

This paper proved to be accessible to all well-prepared candidates, the majority of whom were able to offer responses to all of the questions. Much of the work was clearly presented, with most candidates making their methods clear. Several questions included more challenging elements which differentiated between the candidates, resulting in a wide spread of marks. The parts of questions which candidates found most difficult were **Question 3(i)** (implicit differentiation), **Question 6(i)** (integration by substitution), **Question 6(ii)** (integration of $\cos^2 \theta$), **Question 7(iii)** (geometry on an Argand diagram), **Question 8(ii)** (binomial expansion), **Question 9(iii)** (manipulation of logarithms) and **Question 10(ii)** (distance between parallel planes).

Candidates should be reminded that it is not helpful to present their solution to a question in several parts scattered throughout their script, and there are still some candidates who make things very difficult for the Examiners by working in double column format.

Question 1

Many candidates scored full marks for this question. When errors were made this was usually due to poor understanding of logarithms or because the final answers were not given to 3 significant figures. Weaker candidates showed some confusion between decimal places and significant figures.

Answers: 1.38, 3.62.

Question 2

(i) Large numbers of candidates attempted to apply the quadratic formula to find the roots of this cubic equation. Relatively few took the simple expected route of substituting integer values to find a change in sign. A few candidates attempted a graphical approach, but most of these lacked sufficient detail to confirm the required interval.

(ii) The majority of answers were correct, with only a few candidates not observing the accuracy requirement.

Answers: **(i)** 3, 4; **(ii)** 3.43.

Question 3

(i) The implicit differentiation of y^3 was often correct, but applying the product rule to x^2y caused more problems, and many candidates made a sign error at this stage, with $-2xy + x^2\dfrac{dy}{dx}$ being a very common error. Some inserted an additional $\dfrac{dy}{dx}$ term and some did not differentiate the right-hand side of the original equation, but the method of rearranging to form an expression for $\dfrac{dy}{dx}$ was well understood.

(ii) Most candidates were able to find the value of their derivative at (2, 1) and form the equation of the tangent as required. Errors were usually due to attempting to find the normal or using (1, 2) in place of (2, 1).

Answers: **(i)** $\dfrac{3x^2 - 2xy}{x^2 + 3y^2}$; **(ii)** $8x - 7y - 9 = 0$.

Question 4

Most candidates made a good start in this question, with the majority arriving at a correct quadratic equation in $\tan\beta$. Most errors before this point were from candidates who thought that $\tan(\alpha + \beta) = \tan\alpha + \tan\beta$. In solving the equation, many candidates rejected $\tan\beta = -1$ believing it to have no solutions.

Answers: $\alpha = 45°$, $\beta = 26.6°$ and $\alpha = 116.6°$, $\beta = 135°$.

Question 5

(i) Candidates from some Centres did not appear to be aware of the factor theorem, choosing to approach this question by using algebraic division rather than by substituting $x = -2$ and equating the results to zero. Although it is a perfectly valid method, algebraic division proved to be a much longer method in this instance and offered many opportunities for algebraic or arithmetic slips. Errors in $p'(x)$ were common, both in differentiating $2x^3$ and with -4 remaining. The standard of presentation in solutions to this problem was disappointing, with many candidates forming correct expressions by substituting $x = -2$ but not immediately equating their expressions to zero. This part of the process was then implied through subsequent working.

(ii) Most candidates made correct attempts to express their cubic as the product of $(x + 2)$ and a quadratic factor. Some candidates with correct working to this point did not go on to factorise the quadratic. The incorrect answer $(x + 2)^2(x - \frac{1}{2})$ following no working was common.

Answers: **(i)** 7, 4; **(ii)** $(x + 2)^2 (2x - 1)$.

Question 6

(i) This part of the question proved to be particularly difficult for many candidates, either through algebraic errors or through errors in completing the substitution. Common errors included using $x^2 = 2\tan^2\theta$, and simply replacing dx by $d\theta$. Candidates who expanded $(4 + 4\tan^2\theta)^2$ rather than using $1 + \tan^2\theta = \sec^2\theta$ immediately often failed to derive the given answer. Some candidates simply ignored the change of limits from values of x to values of θ, and others only considered what happened when $x = 2$.

(ii) There were many correct answers to this part, but many candidates did not appreciate that the first part of the question assisted them with the second and produced false answers, often involving a logarithm. Although $\int \cos 2\theta \, d\theta$ is often asked for in this paper, many candidates were not familiar with the use of $\cos 2\theta$ to complete the integral. It is possible to use integration by parts but complete solutions by this method were rarely seen. Although the question asks for an exact answer, some candidates only gave their final answer rounded to 3 significant figures.

Answer: **(ii)** $\frac{1}{8}\pi + \frac{1}{4}$.

Question 7

(i) (a) This was frequently correct, but a surprising number of arithmetic slips were found.

(b) Most candidates multiplied numerator and denominator of the quotient by $3 - i$, but here again arithmetic errors often prevented them from obtaining the correct answer. The most common errors were in the signs in the numerator, or having $3^2 = 3$ in the denominator.

(ii) A majority of candidates stated $\arg\left(\frac{u}{v}\right) = -\frac{\pi}{4}$, placing $\frac{u}{v}$ in the wrong quadrant.

(iii) Only a few candidates appreciated the link between the angle *AOB* and arg *u* – arg *v*. Many resorted to the use of approximate decimal values for the sizes of the angles, which is not an acceptable method for reaching the exact given answer. Candidates who attempted to use the cosine rule often started correctly, but they did not always simplify their exact value for cos *AOB* sufficiently to reach a recognisable form for $\cos\left(\frac{3\pi}{4}\right)$.

(iv) Some candidates were able to pick up marks here through careful consideration of their Argand diagram, although many were looking for a more complicated relationship than required. Some gave only one part of the answer, and many incorrect answers involved a rhombus.

Answers: **(i)(a)** 1 + 2i, **(b)** $-\frac{1}{2}+\frac{1}{2}\text{i}$; **(ii)** $\frac{3}{4}\pi$; **(iv)** *OA* = *BC*, *OA* is parallel to *BC*.

Question 8

(i) Many candidates found a correct expression in partial fractions, using a variety of methods. Rather than starting with a statement of their proposed partial fractions, several candidates used the cover up rule to find a term of the form $\frac{A}{1-x}$ and then subtracted this from the given expression – when it worked this was very successful, but given the difficulty that many candidates have with the subtraction of algebraic fractions there were often errors in the working. Common errors included correct deduction that 3*A* = 2, but then concluding that $A=\frac{3}{2}$ or starting with an initial form omitting one of the terms in the linear numerator.

(ii) A small number of candidates chose to expand the original fraction rather than use their partial fractions. These candidates often gave up before multiplying out their product of three brackets, or omitted terms from their answer.

Candidates working from the partial fractions had mixed success. The expansion of $\frac{2}{3}(1-x)^{-1}$ was often correct in its unsimplified form, but there were errors in resolving $(-x)^2$. The term $\frac{2x-1}{3(2+x^2)}$ caused many more difficulties. Common errors included confusion over how to deal with the constants in the numerator resulting in a starting point of $\frac{2x-1}{9(2+x^2)}$, use of the incorrect form $(2+x^2)^{-1} = 2\left(14+\frac{x^2}{2}\right)^{-1}$, and errors in multiplying their expansion by (2*x* – 1).

Answers: **(i)** $\frac{2}{3(1-x)}+\frac{2x-1}{3(2+x^2)}$; **(ii)** $\frac{1}{2}+x+\frac{3}{4}x^2$.

Question 9

(i) Many correct solutions to the differential equation were seen. Common errors included incorrect or non-existent attempts to separate the variables in the given equation, or omission of a constant of integration. Those candidates who gave their answer to this part in the form $kt = \ln\left(\frac{3A}{\theta - A}\right)$ tended to be more successful with the rest of the question.

(ii) Some candidates were not able to deal with a negative term in their solution, usually resulting in $k = \ln\left(\frac{2}{3}\right)$.

(iii) There were many errors in the working to find an expression for θ, often due to false statements involving logarithms. The false statement $2\ln\left(\frac{3}{2}\right) = \ln 3$ was very common, and some candidates did not remove the logarithms completely to find θ in the simplest possible form.

Answers: **(i)** $\ln(\theta - A) = -kt + \ln 3A$; **(iii)** $\theta = \frac{7}{3}A$.

Question 10

(i) This was often correct, but the error $x + 4y + 2z = 2$ was quite common.

(ii) Many candidates did a lot of work here but achieved very little, often demonstrating little idea of where to start. Those candidates familiar with the form $\mathbf{r}.\hat{\mathbf{n}}$ gave the correct answer very quickly. Many attempted to find the foot of the perpendicular from (1, 4, 2) to p, and a correct parameter value was usually obtained, but this was often then used to find the distance of the foot of the perpendicular from the origin rather than to find the distance between the two planes.

(iii) Many candidates used the vector product method here, and often gained full marks if they could avoid arithmetic and sign errors.

Candidates using the scalar product method usually demonstrated an understanding of the method to follow and made some progress, but often made slips; for example, interim equations such as $b + 2c = 0$ often became $b = 2c$.

Some candidates did more work than necessary by finding an equation for the line of intersection of the two planes but did not then go on to state the equation of a parallel line passing through the origin.

The final answer for l was not always expressed as an equation.

Answers: **(i)** $2x - 3y + 6z = 2$; **(ii)** 2; **(iii)** $\mathbf{r} = \lambda(-6\mathbf{i} - 2\mathbf{j} + \mathbf{k})$.

MATHEMATICS

Paper 9709/41

Paper 41

General comments

Candidates in general performed very well in **Question 1**, but performed much less well in **Questions 2** and **3**. Despite the level of careful thought needed in part **(iii)** of each of **Questions 6 and 7**, these last two questions were generally better attempted than **Questions 4 and 5**.

A feature of the work of a considerable minority of candidates was to omit required answers, even though work necessary to facilitate these answers had been satisfactorily undertaken.

Question 5(i) is in structured form and allowed weaker candidates to score some marks. In this structured form three specific answers are required, and three of the six marks available must necessarily be allocated one each to the required answers. However many candidates did not meet the requirement to provide values for the frictional and normal components to 3 significant figures.

Candidates for this examination are expected to have a clear understanding of Newton's second law. This is usually quoted in formula form as $F = ma$. In this formula F represents the resultant of the forces acting on a body of mass m, in the direction of its motion, and a is its acceleration in the same direction. In application candidates usually write a term or terms representing forces which in combination constitute the required resultant force on the left-hand side of an equation, with ma on the right-hand side. However in the case of a considerable minority of candidates F continued to appear on the left-hand side as a 'phantom' force. In some cases it was dropped at a later stage without being quantified, but in many other cases it was replaced by $+ma$ or by $-ma$, so that the effect was to use Newton's second law as $F = 0$ or as $F = 2ma$.

A number of candidates gave some answers correct to only 2 significant figures, particularly $Q = 8.9$ in **Question 3**, and 0.55 N and 5.7 N in **Question 5(i)**.

Comments on specific questions

Question 1

This question was very well attempted with a very high proportion of candidates scoring full marks. The most common incorrect answer was 50 000 J, from $\frac{1}{2}1000(30-20)^2$.

Answers: 20 ms^{-1}, 30 ms^{-1}; 250 000 J.

Question 2

Very many candidates recognised the need for the principle of conservation of energy to be applied in both parts. Most such candidates scored full marks, although some calculated the wrong height, usually 2.45 m, and some produced the answer of 6 ms^{-1} in part **(ii)**.

A disappointingly significant number of candidates failed to realise the need to invoke the principle of conservation of energy.

Answers: **(i)** 1.8 m; **(ii)** 7 ms^{-1}.

Question 3

The preferred method of candidates was to resolve forces in the 'x' and 'y' directions, although those who chose to use the sine rule in a triangle of forces, or Lami's theorem, were generally more successful.

The most common error in writing the equations obtained by resolving forces was to treat the resultant force as an equilibrant, thus having an incorrect sign attached to all occurrences of the '12' in the equations, and an answer for Q of –8.91.

Answer: 8.91.

Question 4

(i) Almost all candidates ignored the advice 'by resolving in the direction of PS' and used another method.

Many candidates who used the angles between PA and the upward vertical, and between PS and the upward vertical, used the incorrect value of 45° for both. In most such cases the candidate also assumed that the tension is the same in both strings, and was thus able to obtain a value for T, albeit erroneous, by resolving vertically and without recourse to resolving horizontally.

Some candidates who resolved vertically but not horizontally, using the correct angle(s), simply ignored the tension in the shorter string, or assumed it has the same value as in the longer string. Such candidates then obtained an incorrect value for T.

Candidates who used trigonometry in a triangle of forces acting on P were generally successful, but some of those who favoured the use of Lami's theorem were victims of the erroneous 45° assumption.

Also, in some cases of resolving forces the lengths in centimetres 30, 40 and 50 appeared as forces in the equations.

Some candidates launched into an answer based on the belief that the question is about the motion of connected particles.

(ii) In almost all cases candidates used $F = T\cos 36.9°$ in this part, even those who used 45° in part **(i)**.

(iii) A very considerable proportion of candidates based their answer on the wrong assumption that the weight of S is equal to the normal reaction acting upwards on S.

Answers: **(i)** 3 N; **(ii)** 2.4 N; **(iii)** 1.4.

Question 5

(i) Most candidates recognised the need to apply Newton's second law in the direction of motion, but some included $0.6 \times (-4)$ twice and errors of sign also occurred frequently. Almost all candidates obtained $R = 6\cos 18°$, although some used $R = 6$.

Most disturbingly in this question, a considerable minority of candidates failed to answer the specific demands to 'find the frictional and normal components' (to 3 significant figures), or to give the answer for the coefficient of friction to only 2 significant figures. Many candidates gave the answer for μ as 0.10 instead of 0.096.

(ii) Candidates who scored the method mark for finding the frictional component in part **(i)** were usually successful in obtaining at least the first mark in this part.

Answers: **(i)** 0.546 N, 5.71 N, 0.096; **(ii)** 2.18.

Question 6

(i) Almost all candidates answered this part correctly.

(ii) This was also well answered although some candidates used $a = g$ instead of $a = 1$ in this part, demonstrating a lack of understanding. Some candidates found the heights to be 2 m and 7 m instead of 3 m and 7 m.

(iii) This part was found to be difficult and, although some candidates were able to find the time taken by *P*, very few candidates were able to find the time taken by *Q*.

Answers: **(i)** 1 ms^{-2}; **(ii)(a)** 3 m, 7 m, **(b)** 2 ms^{-1}.

Question 7

(i) Most candidates found the speed when $t = 10$ to be 6 ms^{-1} and assumed implicitly and correctly that the velocity of *P* is continuous at $t = 10$. Thus $a = 0.6$ was obtained from $6 = 0 + 10a$.

However a significant minority of candidates assumed (again implicitly) that it is the acceleration of *P* that is continuous at $t = 10$, rather than *P*'s velocity.

(ii) Most candidates realised the need to differentiate $v(t)$ to obtain $a(t)$ for $10 < t < 20$, and did so correctly. However, very few candidates used a correct method to find the required value of *t*. The two main reasons for this are based on misunderstandings of the definition of '*a*' as given in the question. One reason is that candidates simply tried to solve $a(t) = -a(t)$, and the other is that candidates gave the answer 'all values of *t* between 10 and 20' because $a(t)$ is negative everywhere within this range.

(iii) Almost all candidates realised the need to integrate $v(t)$ and did so correctly. However, very few candidates saw a reason for finding the distance travelled in the first 10 s, and the constant of integration was more often found to be zero than the correct value of 130. Many candidates found an incorrect answer by integrating between 0 and 20 and many of those who integrated between 10 and 20 took no account of the motion during the first stage.

Answers: **(i)** 6 ms^{-1}, 0.6; **(ii)** 13.9; **(iii)** 50 m.

MATHEMATICS

Paper 9709/42
Paper 42

General comments

The work of the candidates was generally of sound quality and well presented, and it was encouraging to see good use of diagrams to support solutions. The questions provided good discrimination between candidates, including the more able candidates. **Question 7(ii)** was completed successfully by very few candidates. Candidates appeared to have sufficient time and usually adhered to the rubric, with the exception of the accuracy of answer given in **Question 1(i)**.

Comments on specific questions

Question 1

This question was usually answered by resolution of forces, which nearly all candidates did correctly in the case when the force *P* N is parallel to the plane. However, in the case when the force is horizontal some candidates resolved perpendicular to the plane and omitted to consider the normal reaction. Some candidates used a triangle of forces successfully; others who omitted to consider the reaction obtained an equation such as $\tan 40^\circ = \frac{12}{P}$. A surprising number of candidates gave answers of 7.7 and 10.1, correcting to 1 decimal place instead of to 3 significant figures.

Answers: **(i)** 7.71; **(ii)** 10.1.

Question 2

This was one of the least well answered questions.

In part **(i)**, while many candidates correctly calculated the potential energy loss, some quoted 'PE = KE', giving 1 470 000 J as their answer.

In part **(ii)** candidates used many of the variations $4.8 \times 10^6 \pm$ KE, $4.8 \times 10^6 \pm$ PE or $4.8 \times 10^6 \pm$ KE $\pm$ PE, with the most common incorrect answer 7 500 000 J obtained from adding the loss in potential energy to the work done by the resistive forces.

In part **(iii)** a significant number of candidates overlooked the instruction to consider energy and used a Newton's law approach instead. Although these candidates may have achieved a correct final answer, full marks were not available.

Answers: **(i)** 2.7×10^6 J; **(ii)** 2.1×10^6 J; **(iii)** 4.45×10^6 J.

Question 3

In the first part of this question candidates were given a result which they were asked to show. Many candidates stated '24000 = 600*v*, *v* = 40'. Some further explanation was expected indicating that increasing speed had been considered.

In part **(ii)** many candidates gained full marks. The two common erroneous answers seen were a = 1.28, from omitting to consider the resistance, and a = 1.76 from adding the resistive force thus $\frac{24000}{15} + 600 = 1250a$.

Answer: **(ii)** 0.8 ms^{-2}.

Question 4

Whilst many candidates knew how to attempt this question the majority did not achieve full marks. Many candidates calculated the mass correctly and went on to use Newton's second law, but often seemed to believe that the particle was moving down the plane with the frictional force acting up the plane leading to an answer of 0.4 ms^{-2}.

In part **(iii)** some candidates stated only that the particle would or would not move down the plane. A comparison between the frictional force and the component of weight down the plane was essential for a complete answer.

Answers: **(i)** 0.125 kg; **(ii)** 6 ms^{-2}; **(iii)** Remains at *X*.

Question 5

This question was well attempted with many candidates scoring full marks for part **(i)**, having resolved both horizontally and vertically and applied $\mu = \frac{F}{R}$.

In part **(ii)**, whilst candidates recognised the change from equilibrium to acceleration and attempted to apply Newton's second law, some omitted the frictional force and obtained the answer 10.8 ms^{-2}. An alternative common error was to use R = 12 and $\mu R = 12 \times 0.666$, leading to an acceleration of 4.17 ms^{-2}.

Answer: **(ii)** 8.33.

Question 6

Most candidates scored full marks for setting up equations of motion for particles *A* and *B* and solving them to obtain a = 4. However, candidates often failed to realise that the motion consists of two stages, with the string taut at first and then slack after *B* reaches the ground. Solutions to parts **(ii)** and **(iii)** were frequently incomplete with the time and distance calculated for only one stage.

Answers: **(i)** 4 ms^{-2}; **(ii)** 0.448 m; **(iii)** 0.56 s.

Question 7

Most candidates scored full marks in the first part of this question. A few candidates integrated instead of differentiating and a few others attempted to use constant acceleration formulae. In contrast, candidates rarely completed the second part of the question successfully, showing little or no understanding of average speed. Most candidates equated 27.5 to their expression for speed during the first part of the motion and solved the resulting quadratic equation.

Answers: **(i)** 30 ms^{-1}; **(ii)** 200.

MATHEMATICS

Paper 9709/51
Paper 51

General comments

The paper discriminated well. Unfortunately some candidates scored very low marks and were clearly not ready for the examination at this level.

The presentation of work for some candidates was often very poor and difficult to read.

It is pleasing to note that an increased number of candidates drew clear diagrams to help them with their solutions.

Only a few candidates used premature approximation and rounded to less than 3 significant figures.

The question paper clearly states that candidates should use $g = 10$ and most candidates complied with this rubric.

Comments on specific questions

Question 1

Very few candidates arrived at the correct distance between the plates. Most candidates used $T = 40$ N instead of 20 N and arrived at a compression of 0.1 m instead of 0.05 m. Some candidates treated the spring as being stretched. A few candidates considered the spring in two halves and arrived at the correct answer.

Answers: 0.05 m, 0.2 m.

Question 2

Most candidates found the KE gained and the elastic energy, but not the correct PE gained. Good candidates often scored full marks on this question. Some candidates had sign errors in the energy equation.

Answer: 0.9.

Question 3

The idea of taking moments was clearly understood. However, some candidates did not use $\frac{1}{3}$ of the height of the cone to find the centre of mass of the cone from its base.

Part **(ii)** was well done by most candidates.

Answer: **(ii)** 21.4.

Question 4

Some candidates took $\theta = \tan^{-1}\left(\frac{7}{24}\right)$ and as a result scored no marks for this question.

(i) $V\cos\theta = 24$ was often seen as the answer but on many occasions this came from incorrect working. The correct approach should have been $V\cos\theta = 25\cos(\tan^{-1}\left(\frac{7}{14}\right)) = 25 \times \frac{24}{25} = 24$.

(ii) More able candidates managed to obtain $V\sin\theta = 10$ and then went on to successfully find θ and V.

Answers: **(ii)** 10, 26, 22.6°.

Question 5

(i) Some candidates used $OG = 0.5\frac{\sin\left(\frac{\pi}{6}\right)}{\frac{\pi}{6}}$ instead of $OG = 2 \times 0.5\frac{\sin\left(\frac{\pi}{6}\right)}{3 \times \frac{\pi}{6}}$ where G is the centre of mass of the lamina. Attempts to take moments were often incorrect since $3 \times OG = F \times 0.5$ was seen instead of $3 \times \sin\left(\frac{\pi}{6}\right) \times OG = F \times 0.5$.

(ii) Many candidates attempted to find the horizontal and vertical components of the force acting at O but then failed to find the resultant.

Answers: **(i)** 0.955; **(ii)** 2.22 N.

Question 6

Parts **(i)** and **(ii)** were generally well done with many candidates scoring full marks. The only error that appeared was a sine/cosine interchange. Part **(iii)** was not particularly well done. Sometimes candidates used the tension found in part **(i)**.

Answers: **(i)** 1.94 N; **(ii)** 1.01 N; **(iii)** 4.32.

Question 7

(i) Many candidates managed to arrive at the given answer. The most common error was to use an incorrect sign when using Newton's second law.

(ii) The idea of separating the variables and integrating was used by most candidates. A few candidates failed to integrate correctly. Those candidates who successfully integrated correctly often went on to score full marks in this part.

(iii) This part was well done.

Answers: **(ii)** $v = 30e^{-t} - 10$; **(iii)** -30 ms^{-2}.

MATHEMATICS

Paper 9709/52
Paper 52

General comments

The paper discriminated well. Unfortunately some candidates scored very low marks and were clearly not ready for the examination at this level.

The work from some candidates was poorly presented and often difficult to read.

An increased number of candidates drew clear diagrams to help with their solutions.

Only a few candidates used premature approximation and rounded to less than 3 significant figures.

The question paper clearly states that candidates should use $g = 10$ and most candidates complied with this rubric.

Comments on specific questions

Question 1

Only a few candidates scored full marks on this question. Some candidates assumed that the triangle *ABC* was made up of three separate rods instead of *ABC* being the cross-section of a prism. The approach should have been to find the angle between *CA* and the median from *C* to the mid-point of *AB*. It was then necessary to say that 60° + this angle > 90°, so the prism falls on to the face containing *BC*.

Answer: Face containing *BC*.

Question 2

(i) The use of $\bar{y} = \frac{r \sin\alpha}{\alpha}$ was often attempted, but unfortunately some candidates failed to calculate the correct value for α.

(ii) This part of the question was generally well done. Some candidates used the wrong distance from the base for the body.

Answer: **(ii)** 22.2 cm.

Question 3

Many candidates scored full marks on this question.

Question 4

(i) Most candidates attempted to set up an energy equation but some had a sign error. The equation should have been Loss of PE = Gain in KE + EE. Most candidates scored full marks.

(ii) Acceleration = 20 ms^{-2} was often seen but the direction of the acceleration was not always stated.

Answers: **(ii)** 20 ms^{-2}, upwards.

Question 5

(i) Some candidates arrived at $\sin\theta = \frac{3}{8}$, but this often came from incorrect working or by making invalid assumptions.

(ii) This part was well done.

(iii) Most candidates used Newton's second law with $a = \omega^2 r$ and usually went on to find the correct answer. When the wrong answer appeared it was usually because an incorrect value of r was used.

Answers: **(ii)** 4.31 N; **(iii)** 2.25.

Question 6

(i) The principle of taking moments was clearly understood by most candidates and in many cases full marks were scored. The most common error was to take the distance of the weight from P to be $\frac{2}{3}PO$ instead of $\frac{3}{4}PO$.

(ii) This part was often well done with candidates finding two equations in T_P and θ and then going on to solve them.

Answers: **(i)** 39.9 N; **(ii)** 47.5, 18.5 N.

Question 7

(i) Most candidates differentiated correctly to find $\frac{dv}{dx}$ or $\frac{dx}{dv}$ and knew that $a = v\frac{dv}{dx}$. Some candidates then failed to reach the correct answer as a result of poor algebra.

(ii) Newton's second law was often used but on occasions only two terms were present or a sign error appeared.

(iii) The method of separating the variables and integrating was clearly understood. Sometimes, however, the integration was poorly done, but many candidates did manage to score full marks on this part.

Answers: **(ii)** $0.1v$ N; **(iii)** 1.53 s.

MATHEMATICS

Paper 9709/61

Paper 61

General comments

Again there was a wide range of marks. It was clear this year, however, that some candidates were answering questions without reading the question carefully. This was especially noticeable in **Questions 3** and **4**. Many candidates also found difficulty in answering **Question 5**, how many different numbers between 5000 and 6000 can be formed from the digits 1, 2, 3, 4, 5 and 6, both with and without repeats. There was no indication of candidates being short of time, and almost everybody attempted all the questions.

Comments on specific questions

Question 1

The majority of candidates tackled this question well. However, there were a significant number who failed to use the fact that the mean of a binomial distribution is np with, in this case, $n = 20$, to find the value of p. Instead they decided to use 1.6 for p and consequently -0.6 for q, fully oblivious to the fact that probabilities cannot be greater than 1 or negative. Another error was to read 'more than 2' as '2 or more' or even 'exactly 2'.

Answer: 0.212.

Question 2

Again this was usually answered well although a number of candidates did not differentiate carefully between $\sum x\mathrm{p}(x) = \mu$ and $\sum \mathrm{p}(x) = 1$. Many candidates did not appear to know that E(X) was the same as the mean. Often candidates used the incorrect statements $\sum x\mathrm{p}(x) = 1$ or $\sum x\mathrm{p}(x) = 0$ or even worse $\frac{\sum x\mathrm{p}(x)}{6} = 1.05$ to give the equations which they solved to find p and q , and even when their answers gave negative values or values greater than 1 for the probabilities they continued to use them to calculate the variance.

Answers: **(i)** 0.15, 0.27; **(ii)** 2.59.

Question 3

Part **(i)** involved finding a straightforward probability, although some candidates had difficulty realising that $\mathrm{P}(x < 100)$ was equal to 0.5. Some candidates used a continuity correction and scored no marks for standardising but could possibly have gained a method mark for finding the correct area. In the second part a diagram might have helped some realise they had to use the tables for the normal distribution backwards to find a z-value which gave the required 67% = 0.67 probability. Candidates also failed to read the question carefully and misinterpreted the information as being given in order 'short', 'standard', 'long', whereas the extremes were given first. This led to the candidates finding the z-value corresponding to 0.66 rather than 0.67. Some credit was given for this but clearly a few marks were lost. It would have helped greatly if candidates had used a diagram.

Answers: **(i)** 0.484; **(ii)** 96.9 minutes, 103 minutes.

Question 4

This question was very easy and straightforward. Again, however, many candidates failed to read the question carefully and were unable to distinguish between shelves and books. There were only relatively few candidates who scored full marks, mainly because of not labelling the axis in the box-and-whisker plot. Whilst not usually penalising omission of units, in this data representation question, units were considered an essential part of the answer. Although graph paper was not specified, candidates who drew a freehand box-and-whisker plot were usually penalised because their scale was not accurate enough. Candidates were expected to draw an interpretive conclusion about the number of books per shelf in part **(iii).**

Answer: **(i)** 67.

Question 5

This question proved to be a good discriminator question between those candidates who could only do routine questions and those who could think through a question. Part **(a)** of this question proved a stumbling block to many candidates. Thinking the first digit could be a choice of two was a common error. To be a number between 5000 and 6000 the first digit needs to be 5. Also the idea of using the same digits again (i.e. allowing the digits to be repeated) confused some. An interesting observation was that candidates who realised what was required in part **(a)(i)** invariably got part **(a)(ii)** correct as well. Part **(b)(i)** was the only part of this question that was well done by almost every candidate, although even then some candidates missed out the fact that a team of five girls would have more girls than boys in it. In part **(b)(ii)** only the best candidates realised that the best way of choosing cousins in or cousins out would be ${}^{11}C_2 + {}^{11}C_5$. Candidates who listed options did not often manage to find all 7.

Answers: **(a)(i)** 60, **(ii)** 216; **(b)(i)** 1316, **(ii)** 517.

Question 6

This question caused very few problems to those candidates who knew the topic. There were more careless errors than one would have expected, e.g. missing out some of the options which could give the specified outcomes or not using the continuity correction in part **(iv)**. Again, a careful reading of the question could have avoided the mistake of not appreciating the difference between taking one pear from each of 121 boxes of fruit and looking for 39 pears from 121 boxes of fruit.

Answers: **(i)** 0.255; **(ii)** $\frac{7}{11}$; **(iii)** 0.4; **(iv)** 0.149.

MATHEMATICS

Paper 9709/62

Paper 62

General comments

Candidates found parts of this paper to be a fairly demanding, with **Questions 1** and **4** being particularly challenging. Nearly all candidates found **Question 3** straightforward, gaining most of the marks available. Very few candidates were unable to make an attempt at any particular question.

There appeared to be slightly more confusion than usual over which part of the syllabus was appropriate to answer a particular question. Many candidates fail to gain marks through not showing their working. Another common problem was the failure to work to an appropriate degree of accuracy, especially important when calculating a standard deviation, as in **Question 6(ii)**, but not uncommon in **Questions 5** and **7**.

There was no evidence of problems with the time allowed to answer the paper.

Comments on specific questions

Question 1

Many candidates were unable to see the connection between the box-and-whisker plot and the normal distribution.

(i) Candidates usually gave the correct mean. A few candidates stated that the median, rather than the mean, was equal to 51.

(ii) Quite often no attempt was made to answer this part of the question, candidates not realising that the probability of the wind speed being less than 63 km h^{-1} was 0.75. Some correct answers followed the use of simultaneous equations involving +0.674 and –0.674. Several candidates attempted to calculate the standard deviation of the integers 39, 51 and 63.

Answers: **(i)** 51; **(ii)** 17.8.

Question 2

This was usually answered well. Many good solutions involved a possibility space or a tree diagram. Errors stemmed from miscounting the number of possibilities or assuming that the dice had 6 faces. Sometimes the combination of 4 and 4 was overlooked or repeated. Several candidates proceeded to calculate $7\times\frac{2}{16}+8\times\frac{1}{16}$ and treat this as a probability, leading to an answer greater than 200. Sometimes answers were given to the nearest integer rather than 3 significant figures.

Answer: 37.5.

Question 3

(i) Fully labelled tree diagrams, especially those involving say *A* and *B*, or *X* and *Y*, should include a key. A small number of candidates wrote for example 0.68 rather than 0.8 on the second column of branches.

(ii) Most candidates showed a good understanding of conditional probability. Some candidates lost the final mark by giving their answer as 0.65.

Answer: **(ii)** $\frac{17}{26}$ or 0.654.

Question 4

(a) (i) Nearly every candidate gave the correct answer.

(ii) Candidates found this to be the most difficult part of the paper. Quite often the difference between three-digit numbers beginning with 5 or 6 (or ending with 5 as opposed to 1 or 3) was not appreciated. Many good answers involved a clear explanation, often accompanied by diagrams, but far too many candidates gave no explanation, merely writing down a list of sums and/or products, usually involving 3 or 4 but occasionally 7 or 8.

The most popular method was to consider three-digit numbers beginning with 5, three-digit numbers beginning with 6 and four-digit numbers. Several candidates chose to consider three-digit numbers ending with 1 or 3 or 5 and four-digit numbers ending with 1 or 3 or 5. A few considered all the possible numbers and subtracted those ending with 6. Another group listed the possible outcomes. Sometimes this was successful but usually some odd numbers were overlooked.

Candidates who confused odd/even or allowed digits to be repeated could gain half of the available marks.

(b) This part did not cause as many problems as part **(a)(ii)**. Several candidates overlooked the possibility of 4 and 5 being arranged in either order and an appreciable number of candidates left their answer as 480, ignoring the need for a probability to be given. A few candidates produced an alternative solution based on the number of digits between the 4 and 5. Most of these included a good diagram/explanation.

Answers: **(a)(i)** 24, **(ii)** 28; **(b)** $\frac{2}{3}$.

Question 5

Nearly all candidates gained some of the available marks on this question.

(i) Proofs required the involvement of 3. Generally proofs were not sufficiently convincing.

(ii) Most candidates produced a correct table. The most frequent error was to list the values of *r* rather than *X*. Some candidates failed to give their values as fractions or correct to three significant figures.

(iii) This was usually answered correctly. Occasionally the probability rather than the value of the mode was given. Some candidates calculated the mean or the median.

(iv) This was usually answered correctly. Sometimes P(X = 120) was included in the sum of probabilities.

Answers: **(ii)** 120, $\frac{1}{45}$; 60, $\frac{2}{45}$; 40, $\frac{3}{45}$; 30, $\frac{4}{45}$; 24, $\frac{5}{45}$; 20, $\frac{6}{45}$; 17.1, $\frac{7}{45}$; 15, $\frac{8}{45}$; 13.3, $\frac{9}{45}$;
(iii) $\frac{40}{3}$ or 13.3; **(iv)** $\frac{4}{9}$ or 0.444.

Question 6

Most candidates gained high marks on this question. Problems identifying the class boundaries led to incorrect frequency densities in part **(i)** and incorrect values for the mean and standard deviation in part **(ii)**. The better attempts contained a table which included frequency, frequency density, mid-interval values, xf and x^2f.

(i) The majority of candidates presented a good histogram. Sometimes the frequencies rather than the frequency densities were plotted. Marks were not gained through drawing the first boundary at 0 or 1 and the final boundary at 70, 75 or 80. Scales were nearly always sensible but occasionally the axes were not labelled.

(ii) Most candidates made good attempts to calculate the mean and standard deviation, the second of the formulae given for the standard deviation of grouped data on the List of Formulae being much more popular. The class width was sometimes used instead of the mid-interval value. A surprisingly high number of candidates obtained the incorrect value of 16.8 for the standard deviation following the use of 37.5 rather than 37.48 or better for the mean. It was not unusual to see $\sum x^2 f$ to eight significant figures in a calculation involving the mean to three significant figures.

Answers: **(ii)** 37.5, 16.9.

Question 7

Full marks were rarely gained on this question, although most candidates had some success. Greater use of sketches might have helped to answer parts **(i)** and **(ii)**.

(i) This part was usually answered correctly. Some candidates failed to subtract their 0.7623 from 1 and others included a continuity correction.

(ii) The majority of candidates found this use of the normal tables very demanding. Many candidates confused probabilities and z-values. Some otherwise correct solutions were marred by not working with probabilities to four significant figures, as per the normal tables.

(iii) Most candidates made a good attempt at this part, often realising that only the answer to part **(i)** was required. A few calculated P(2) only. Full marks were not gained very often, most candidates using 0.238 rather than 0.2377.

Answers: **(i)** 0.238; **(ii)** 116; **(iii)** 0.0909 or 0.0910.

MATHEMATICS

Paper 9709/71

Paper 71

General comments

On this paper, candidates were largely able to demonstrate and apply their knowledge in the situations presented. Whilst there were many good scripts, there were also candidates who appeared completely unprepared for the paper. In general, candidates scored well on **Questions 3, 5(i)** and **(iii)** and **7**, whilst **Question 4** proved particularly demanding.

Accuracy, as always, caused loss of marks for some candidates; there were the usual cases of candidates not adhering to the 3 significant figure accuracy required on non-exact answers (where the answers are not angles in degrees). On the whole, presentation was good and an adequate amount of working was shown by candidates, though there were some cases where Examiners had to withhold marks due to lack of essential working. This was particularly noted in **Question 4** (see further comments below). Lack of time did not appear to be a problem for candidates.

Detailed comments on individual questions follow. Whilst the comments indicate particular errors and misconceptions, it should be noted that there were also many very good and complete answers.

Comments on specific questions

Question 1

This question required a Poisson approximation to B(180, 0.02). Candidates who used the correct approximation were mostly successful in gaining full marks for the question. The most common error noted by Examiners was to use a normal approximation, or to calculate the probability directly from B(180, 0.02) without using an approximation.

Answer: 0.515.

Question 2

Many candidates made a good attempt at this question and gained full marks, whilst others only failed to score the last answer mark by giving n (which should have been found to be greater than 34.57) as 34 rather than 35. Other errors seen included using an incorrect z-value, and some candidates failed to half the confidence interval.

Answer: 35.

Question 3

This was a particularly well attempted question. Errors noted in part **(i)** included standard deviation/variance errors, and omission of $\sqrt{5}$ when standardising. In part **(ii)** a surprisingly common error was to successfully find $\sqrt{n} = 6$, but then conclude that $n = \sqrt{6}$ rather than 6^2. Algebraic errors were seen in rearranging a correct formula, and other errors included incorrect z-values as well as incorrect use of tables.

Answers: **(i)** 0.326; **(ii)** 36.

Question 4

This question was not well attempted. It was important in part **(i)** that all relevant working and comparisons were shown, particularly as the answer was given and candidates were asked to 'show that' the rejection region was $X > 4$. Many candidates used Po(1.8) and calculated P(0), P(1), P(2), P(3) and P(4). A particularly common error was to then compare individual probabilities with 0.05. Addition of these probabilities was necessary, and this needed to be clearly seen by the Examiner, whether comparison with 0.05 or 0.95 was being attempted. Many candidates showed lack of clarity of thought on this question. On occasions candidates incorrectly used a normal Distribution. Part **(ii)** was attempted with slightly more success, though omission of P(4) was often seen when calculating the probability of a Type II error.

Answer: **(ii)** 0.916.

Question 5

Questions on continuous random variables are usually well attempted. This question was no exception, other than for part **(iv)**. In part **(i)** most candidates were able to show convincingly that $k = \sqrt{2}$, though some candidates failed to show this 'exactly' and resorted to use of decimal values. Part **(ii)** was reasonably well attempted, though some candidates found $P(X < 0.4)$ rather than $P(X > 0.4)$. Part **(iii)** was particularly well attempted, with even the weakest candidate able to apply a correct method to find the upper quartile. A common error on both parts **(ii)** and **(iii)** was to use the degree mode on the calculator rather than radians. Part **(iv)** required candidates to calculate a binomial probability. Whilst some candidates recognised this, not many used the correct parameters. The answer obtained in part **(iii)** for the *value* of the upper quartile was all too often used as a probability, rather than 0.25. Some candidates, unnecessarily, calculated the probability of X having values greater than 0.559, causing time and possible accuracy penalties.

Answers: **(ii)** 0.449; **(iii)** 0.559; **(iv)** 0.0879.

Question 6

Part **(i)** of this question was well attempted, with even the weakest of candidates able to score. Common errors included accuracy errors (some candidates approximated their value for the mean prematurely in their variance formula), confusion between the two formulas for the unbiased estimate of the population variance, and omission (or even double inclusion) of the factor $\frac{n}{n-1}$. Part **(ii)** was not well attempted. It should be noted that 'in the context of the question' means that candidates cannot merely quote 'text book' definitions, but need to interpret these within the given situation. Some candidates attempted to calculate the probability of a Type I error, not appreciating the fact that the question clearly used the phrase 'state that'. Carrying out the test in part **(iii)** was reasonably well attempted, though Examiners noted the usual failure to show a valid comparison in order to reach a final conclusion, thus illustrating another area where candidates lost marks for lack of essential working.

Answers: **(i)** 14.8, 9.80; **(ii)** $H_0: \mu = 15.2$, $H_1: \mu < 15.2$, 0.1.

Question 7

This question was well attempted. Marks were lost due to incorrect values for the variance (particularly in part **(ii)**), but a high score, even if not completely full marks, was usually gained on this question. In part **(i)**, a correct mean for '$3C - 2W$' was usually found, but errors were made in calculating the variance of $3C$ and $2W$. However, even with these errors, most candidates used the correct method and thus marks were gained. Similarly, in part **(ii)**, correct methods were used, but incorrect values for the variance for the new drink were seen, with many candidates calculating $2 \times 5.2^2 + 0.5 \times 7.1^2$ or $2^2 \times 5.2^2 + 0.5^2 \times 7.1^2$ rather than $2 \times 5.2^2 + 0.5^2 \times 7.1^2$.

Answers: **(i)** 0.771; **(ii)** 0.890.

MATHEMATICS

Paper 9709/72
Paper 72

General comments

Many candidates showed a reasonable understanding of the required content and statistical techniques. Overall the work was well presented with methods clearly shown. Many candidates did give the required 3 significant figure accuracy for non-exact answers (where the answers are not angles in degrees). However sometimes candidates did not select the appropriate distribution for a situation. For instance, some candidates incorrectly used normal distributions in **Questions 3** and **4** where Poisson and binomial distributions were relevant.

In general candidates scored well on **Question 2** and **6(i), (ii)** and quite well on **Questions 1(ii)** and **5(ii)**, whilst **Questions 3**, **4**, **6(iii)** and **7** proved more demanding. Most candidates attempted all of the questions in the available time.

Sometimes there was a lack of essential working, especially when comparisons of probabilities or critical values were required, for example in **Questions 4** and **5**.

Comments on specific questions

Question 1

(i) It was necessary to state that class members with odd numbers were not included. Giving an example of this, for example the person numbered one was excluded, was accepted. The statement that not all values were equally likely did need to be supported by reference to the context of the question.

(ii) Many candidates knew how to find the confidence interval, and many calculated it correctly. Errors included using the wrong *z*-value (e.g. 2.054), using the variance instead of $\sqrt{0.0052}$, and even using the wrong mean (8.1 instead of 1.62). Most candidates included $\sqrt{5}$, but a few candidates wrongly used this twice. A few candidates found the sample variance (biased or unbiased), when the population variance was given and should be used.

Answer: **(ii)** (1.54, 1.70).

Question 2

This question was well answered by many candidates.

(i) Most candidates realised that a Poisson distribution was appropriate, and converted to the 5-hour period ($\lambda = 6.35$), and found 1 – P(0, 1), although sometimes P(1) was omitted.

(ii) Most candidates correctly approximated the Poisson distribution for 700 hours ($\lambda = 889$) by the normal distribution N(889, 889). In this question the majority of candidates did realise that the large value of λ indicated the use of the normal distribution, and explained this. Often the necessary continuity correction was applied and the correct area was used. Errors included omitting the continuity correction, or using an incorrect correction. A minority of candidates found the wrong area (corresponding to the wrong tail of the distribution).

Answers: **(i)** 0.987; **(ii)** 0.902.

Question 3

This question was found to be very demanding by candidates.

(i) There was much confusion with distributions involving the number of seats on the plane, the number of ticket buyers, and the number of people who did not arrive for the flight. The question did supply the hint to consider the number of people who did not arrive for the flight, but frequently this was not applied. Also candidates had to observe that the 'large *n*' and 'small *p*' with $np = 4.26 < 5$ indicated the use of the Poisson distribution ($\lambda = 4.26$) for these 'non-arrivers'. Those candidates who wrote this check down were more successful than those who did not. This also established that a normal distribution was not appropriate. Then candidates had to realise that 'overbooking' required that the probability of the small numbers of 'non-arrivers' (0, 1, 2) was to be found, not '1 – this probability', which was a frequent error. Some candidates set up the correct distribution and process but found the wrong Poisson terms (e.g. omitting P(0) and/or including P(3)).

(ii) The most efficient solution method was to note the new Poisson distribution for the second flight and combine the two Poisson distributions for the total number of 'non-arrivers' ($\lambda = 6.06$) to find the probability that the total equalled 5. Some candidates did find P(5) in a Poisson distribution, but used the wrong mean. Checking that $np = 1.8 < 5$ for the second flight established the use of a Poisson and not normal distribution. Some candidates did attempt the longer methods, using the two Poisson distributions or the two binomial distributions.

Answers: **(i)** 0.202; **(ii)** 0.159.

Question 4

This question was not well answered.

(i) Many candidates showed some understanding of the process for finding an acceptance region for a binomial distribution situation, but frequently there was a lack of essential working. Although many candidates wrote down individual binomial probability values they did not sum these probabilities (or did not write down these sums), and did not compare these successive sums to 0.05. Also many candidates used a one-tail test and compared to 0.1. For this small sample size (10) the normal distribution was not appropriate.

(ii) Many candidates did show a correct understanding of a Type II error, and gained some credit when following through from their wrong acceptance region in part **(i)**. B(10, 0.7) had to be used. A fairly common error was to find 1 – P(acceptance region) instead of P(acceptance region).

Answers: **(i)** (2, 3, 4, 5, 6, 7, 8); **(ii)** 0.851.

Question 5

(i) For this significance test, candidates first had to realise that H_0 was related to the population mean of 750 g, together with H_1 that $\mu < 750$ g. The hypotheses needed to be stated. Many candidates did used the normal distribution of means correctly and found $z = -1.626$ or the corresponding probability 0.0520. However many candidates did not write down a clear comparison between the test statistic and the corresponding critical value (–1.751 or 0.04), although fewer inconsistent sign comparisons and fewer *p*/*z* false comparisons were seen this session. The method of finding the critical value 745.7 from –1.751 was not well done. Here also the necessary comparison with the sample mean was frequently omitted.

(ii) A many candidates understood the method for finding *n*, using $\frac{121}{n}$ and –1.881. However some candidates lost the minus sign, or rounded 17.12 to 17, instead of to the least possible size 18. Errors also included using the wrong *z* (e.g. 2.17), or using a 'continuity correction', or incorrectly solving the inequality.

Answer: **(ii)** 18.

Question 6

The first two parts were well answered, but the third part was found to be more demanding by candidates.

(i) The majority of candidates knew to integrate between limits 1 and 2 and to equate the result to 1. This was usually carried out correctly, with complete working shown as required.

(ii) The majority of candidates found E(X) correctly, though in some cases the 'extra x' was omitted. When candidates made errors they were not alerted to check when their value was outside the [1, 2] interval.

(iii) The error of integrating from 0 to m instead of from 1 to m was seen quite often. Those candidates attempting this method of finding the median itself found themselves required to solve a cubic equation. Some candidates were able to use calculators to find the value (it had to be selected from the three roots, not left as a set of three). But many candidates stopped at the cubic equation stage. Those candidates who attempted the simpler method of integrating from 1 to 1.5 to find the probability area 0.477 were more successful. They still needed to compare this to 0.5 and deduce the conclusion clearly, which not all of them did. A diagram would have helped many to do this, as is often the case.

Answers: **(ii)** 1.52; **(iii)** Median > 1.5.

Question 7

(a) Very few candidates were able to deal correctly with the information given in this part. The essential link between Var(Y) and Var(X), namely b^2, from which $b^2 = 2^2$ followed, was not deduced.

(b) Candidates had more success with this part, though various errors appeared.

(i) Many candidates did find N(3μ, 13), although errors appeared with the use of 13 instead of $\sqrt{13}$ when standardising and with the use of an incorrect z-value.

(ii) A pleasing number of candidates realised that it was necessary to consider $S - R$ and found the probability that $S - R > 0$ in N(μ, 13), although some of these candidates subtracted variances instead of adding, and some candidates found the wrong area (corresponding to the wrong tail of the distribution). Candidates could gain credit here when following through from their wrong mean in part **(i)**, although some confusion was seen when their wrong mean was negative.

Answers: **(a)** –6.32, 2; **(b)(i)** 1.87, **(ii)** 0.699.