UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2009 question paper for the guidance of teachers

9709 MATHEMATICS

9709/11 Paper 11, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1 3 tan(2x + 15°) = 4 tan(2x + 15°) = 1⅓ Sets the bracket to $\tan^{-1}(1⅓)$ 2x + 15 = 53.13° or 233.13° → x = 19.1° or 109.1°	M1 M1 A1 A1√ [4]	Removes the "3" first by division. Looks up $\tan^{-1}1⅓$, then uses bracket co. √ for (90 + 1st answer) and no other answers in the range.
2	B1 B1 B1 B1 [4]	1 complete oscillation 0 → π Range from −3 to 3 All correct (V shape B0) Line correct.
3 **(i)** $(2 - x)^6$ $64 - 192x + 240x^2$	3 × B1 [3]	co Allow 2^6.
(ii) $(1 + 2x + ax^2)(2 - x)^6$ Coeff of x^2 = 240 − 384 + 64a Equates to 48 → a = 3	M1 M1 A1 [3]	Considers at least 2 terms in x^2. Considers exactly 3 terms + solution co
4 $y = x^4 + 4x + 9$ **(i)** Differential = $4x^3 + 4$ Sets to 0 + solution → (−1, 6) 2nd differential = $12x^2$ Positive, → Minimum	B1 M1 A1 B1 [4]	co Differentiates and sets to 0. co. Statement only.
(ii) $A = \left[\frac{x^5}{5} + 2x^2 + 9x\right]$ Limits from 0 to 1 → 11.2	B1 M1 A1 [3]	co. Value at "1" − value at "0" in integral of y.
5 r = 6 cm **(i)** $AB = \sqrt{(6^2 + 6^2)} = \sqrt{72}$ Angle BAD = ¼π or 45° Arc length = $\sqrt{72} \times ¼\pi = 6.66(7)$	B1 B1 M1 A1 [4]	Use of Pythagoras – or trig (8.5 ok) In degrees or radians Use of $s = r\theta$ with θ in rads only – or correct with degrees. Use of r = 6 M0.
(ii) Sector area = $½r^2\theta = ½ \times 72 \times ¼\pi$ Area of triangle = ½ × 6 × 6 Shaded area = 10.3 or 9π − 18.	M1 B1 A1 [3]	Use of $½r^2\theta$ with θ in rad, and $r \neq 6$. co co

6	$\frac{dy}{dx} = k - 2x$		
	(i) At $x = 2$, $m = (k - 4)$ $x = 3$ $m = (k - 6)$ $(k - 4)(k - 6) = -1$ $\rightarrow$ $k = 5$	M1 M1 DM1A1 [4]	Obtains either gradient as f(k). Uses $m_1m_2 = -1$ with gradients f(k) Soln of quadratic = 0. co (watch for fortuitous answers)
	(ii) $y = kx - x^2$ (+ c) Substitutes (4, 9) $\rightarrow$ $c = 5$	B1√ M1 A1 [3]	For integration without c Realises need to substitute for x and y co (nb If $k = 5$ is fortuitous, loses last A1)
7	$y = \frac{12}{x^2 + 3}$		
	(i) $\frac{dy}{dx} = -12(x^2 + 3)^{-2} \times 2x$	B1 B1 [2]	Without the "×2x". For "×2x". Accept unsimplified answer
	(ii) At $x = 1$, $m = -\frac{3}{2}$ m of normal = ⅔ Eqn of normal $y - 3 = \frac{2}{3}(x - 1)$	 M1 M1 A1 [3]	 Uses $m_1m_2 = -1$ ....algebraic ok. Correct form of equation. co unsimplified
	(iii) $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt} = -\frac{3}{2} \times 0.012$ $\rightarrow$ −0.018	M1 A1√ [2]	Correct link between differentials co to his $\frac{dy}{dx}$. (Omission of x in part **(i)** causes fortuitous results in **(ii)** and **(iii)**.)
8	**(i)** $8 + 4d = 8r$ $8 + 7d = 8r^2$ Eliminates one of the variables $\rightarrow 4r^2 - 7r + 3 = 0$ Solution $\rightarrow r = ¾ \rightarrow d = -½$	B1 B1 M1 DM1 A1 A1 [6]	co – but allow if a in place of 8. co – but allow if a in place of 8. Complete elimination of either r or d. Correct method of solution. nb answer for r given. co (assumes $r = ¾$, give B1B1 for equations, B1 for d)
	(ii) $S_\infty = \frac{a}{1 - r} \rightarrow 32$	M1 A1 [2]	Correct formula used.
	(iii) $S_8 = 4(16 + 7d)$ $= 50$	M1 A1 [2]	Correct formula used. $64 + 28d$ ok co

9	$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 3 \\ -6 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} 0 \\ -6 \\ 8 \end{pmatrix}$, $\overrightarrow{OC} = \begin{pmatrix} -2 \\ 5 \\ -2 \end{pmatrix}$.		
(i)	Scalar product = $-18 - 48$ $-66 = \lvert\mathbf{a}\rvert\,\lvert\mathbf{b}\rvert\cos\theta$ $\lvert\mathbf{a}\rvert = 7$ and $\lvert\mathbf{b}\rvert = 10$ $\rightarrow$ Angle $AOB = 160.5^\circ$	M1 M1 M1 A1 [4]	Use of $x_1x_2 + y_1y_2 + z_1z_2$ Linking everything correctly Correct modulus of either a or b. co
(ii)	$\overrightarrow{AC} = \mathbf{c} - \mathbf{a} = \begin{pmatrix} -4 \\ 2 \\ 4 \end{pmatrix}$ Modulus = 6 Vector = $5 \times \begin{pmatrix} -4 \\ 2 \\ 4 \end{pmatrix}$ or $\begin{pmatrix} -20 \\ 10 \\ 20 \end{pmatrix}$	B1 M1 A1 [3]	co. allow ±. For modulus and multiplying by "5" co
(iii)	$\begin{pmatrix} 2 \\ 3-6p \\ -6+8p \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 5 \\ -2 \end{pmatrix} = 0$ $\rightarrow p = \frac{1}{2}$	B1 M1 A1 [3]	For $\overrightarrow{OA} + p\overrightarrow{OB}$ as single vector. Scalar product = 0. Co (beware fortuitous answers)

10 $f: x \mapsto 2x + 1,\ x \in \mathbb{R},\ x > 0$

$g: x \mapsto \dfrac{2x-1}{x+3},\ x \in \mathbb{R},\ x \neq -3.$

(i) $gf(x) = \dfrac{2(2x+1)-1}{2x+1+3} = \dfrac{4x+1}{2x+4}$ — M1 — Must be gf , needs x replacing twice.

Equates to $x \rightarrow 2x^2 = 1$ — M1 — Forms quadratic + solution

$\rightarrow x = ½\sqrt{2}$ — A1 — Co. condone ±. [3]

(ii) $f^{-1}(x) = ½(x - 1)$ — B1 — Co

To find $g^{-1}(x)$, make x the subject — M1 — Attempt at x as the subject.

Order must be correct — M1 — Order correct. Allow for sign errors.

$\rightarrow g^{-1}(x) = \dfrac{-1-3x}{x-2}$ or $\dfrac{1+3x}{2-x}$ — A1 — Co – must be f(x). [4]

(iii) $\dfrac{1+3x}{2-x} = x \rightarrow x^2 + x + 1 = 0$ — M1 — Forms quadratic equation.

Looks at $b^2 - 4ac$ — M1 — Looks at discriminant or attempts to solve and

$\rightarrow$ negative $\rightarrow$ no roots. — A1 — finds √(negative). Co [3]

(iv) — B1 — Correct $y = 2x + 1$ on graph from (0, 1)

B1 — Correct $y = ½(x - 1)$ on graph from (1, 0)
(if −ve x plotted, B1 s.c. for both)

B1 — Shows or states or implies that f, f^{-1} are reflections in $y = x$. [3]