UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2009 question paper for the guidance of teachers

9709 MATHEMATICS

9709/12 Paper 12, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	$\frac{dy}{dx} = \frac{3}{\sqrt{x}} - x$ $(y) = 6\sqrt{x} - \frac{x^2}{2}(+c)$ (4, 6) fits $6 = 12 - 8 + c$ $\rightarrow c = 2$	B1, B1 M1 A1 [4]	B1 for each term Uses (4, 6) in an integration with $+c$ co
2	$(x + k)^8$		
	(i) $k^8 + 8k^7x + 28k^6x^2 + 56k^5x^3$	B3, 2, 1 [3]	Loses 1 for each error. He can gain these marks if appropriate in **(ii)**.
	(ii) $28k^6 = 56k^5$ $\rightarrow k = 2$	M1 A1 [2]	Correct method of solving. co. nb $k = 2x$ gets M1 A0.
3	**(i)** $a + d = 96$ and $a + 3d = 54$ $\rightarrow d = -21$ $a = 117$	B1 M1A1 [3]	For both expressions. Correct method of solution. co (nb no working, d correct, a wrong 0/3)
	(ii) $ar = 96$ and $ar^3 = 54$ $\rightarrow r^2 = \frac{54}{96} \rightarrow r = \frac{3}{4}$ $\rightarrow a = 128$	B1 M1 A1 [3]	For both expressions. Correct method of solution. co. $r = \pm ¾$, no penalty.
4			
	(i) $2 \leqslant f(x) \leqslant 8$	B1, B1 [2]	B1 for 2, B1 for 8. Must be stated, not on graph.
	(ii) $x \mapsto 5 - 3\sin 2x$		
		B1 DB1 B1 [3]	1 complete oscillation 1st quadrant, not touching x-axis. Needs to be "down" first and curves. nb If no labels, assume 0 to π.
	(iii) No inverse – not 1 : 1.	B1 [1]	co. Independent of graph.

5	**(i)** $(\sin x + \cos x)(1 - \sin x \cos x)$ $= \sin x + \cos x - \sin^2 x \cos x - \cos^2 x \sin x$ $\sin^2 x = 1 - \cos^2 x$ and $\cos^2 x = 1 - \sin^2 x$ $\rightarrow \sin^3 x + \cos^3 x$	M1 M1 A1 [3]	Needs 4 terms from the product. Needs to be used once. All ok.
	(ii) $(\sin x + \cos x)(1 - \sin x \cos x) = 9 \sin^3 x$ Uses part **(i)** $\rightarrow 8 \sin^3 x = \cos^3 x$ $\rightarrow \tan^3 x = \frac{1}{8} \rightarrow \tan x = ½$ $\rightarrow x = 26.6^\circ$ and 206.6°	 M1 A1 B1√ [3]	 Uses $\tan x = \sin x \div \cos x \rightarrow \tan^3 x = k$. Co. √ for 180º + first answer and providing there are no other answers in range.
6	**(i)** $\overrightarrow{OQ} = 3\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}$ $\overrightarrow{PQ} = -3\mathbf{i} + \mathbf{j} + 6\mathbf{k}$	B1 B2, 1 [3]	co Loses one for each error.
	(ii) $(3\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}).(-3\mathbf{i} + \mathbf{j} + 6\mathbf{k})$ $= -9 + 3 + 36 = 30$ $30 = \sqrt{54}\sqrt{46}\cos\theta$ $\theta = 53.0^\circ$ Cosine rule M1 modulus M1 attempt at 3 sides M1 A1 answer.	 M1 M1 M1A1 [4]	 Use of $x_1x_2 + y_1y_2 + z_1z_2$ co. Correct method for modulus (once) and all correctly linked. co. nb $\overrightarrow{QO}.\overrightarrow{QP}$ can gain 4/4. but $\overrightarrow{OQ}.\overrightarrow{PO}$ can only gain 3/4. Use of other vectors (e.g. $\overrightarrow{OP}.\overrightarrow{OQ}$) M3 ok.
7	**(i)** $2r + r\theta = 50$ $\theta = \frac{1}{r}(50 - 2r)$ $A = \frac{1}{2} r^2\theta$ $\rightarrow A = 25r - r^2$	M1 A1 M1 A1 [4]	Must use $s = r\theta$ and link with perimeter co Used with θ as f(r) co (answer given)
	(ii) $\frac{dA}{dr} = 25 - 2r$ $= 0$ when $r = 12.5$ $A = 156¼$ 2nd differential negative → Maximum	B1 M1 A1 B1 [4]	co sets differential to 0 + solution co Could be quoted directly from quadratic.

8	$x \mapsto \dfrac{3}{2x+5}$		
	(i) f′(x) = –3(2x + 5)$^{-2}$ × 2 f′(x) is negative → decreasing	B1 B1 B1√ [3]	B1 for –3(2x + 5)$^{-2}$. B1 for ×2 √ providing bracket is squared. (using value or values only B0)
	(ii) $y = \dfrac{3}{2x+5} \rightarrow 2x+5 = \dfrac{3}{y}$	M1	Attempt at making x the subject.
	→ f^{-1} (x) = $\dfrac{1}{2}\left(\dfrac{3}{x} - 5\right)$ or $\dfrac{3-5x}{2x}$	A1 [2]	co including f(x) not f(y)
	(iii) $\int \pi \dfrac{9}{(2x+5)^2} \mathrm{d}x$ = (–9π(2x + 5)$^{-1}$ ÷ 2) Limits 0 to 2 → π (−½ − −0.9) → = 0.4π (or 1.26)	B1 B1 M1 A1 [4]	For –9(2x + 5)$^{-1}$ For ÷ 2 in ∫ of y^2 Use of correct limits with ∫ of y^2. co
9	**(i)** y-coordinate same as the y-coordinate of the mid-point of AC.	B1 [1]	co
	(ii) m of $AD = \dfrac{8}{h}$ or $\dfrac{h-12}{8}$	M1 A1	any use of y-step ÷ x-step for M mark
	m of $CD = \dfrac{8}{12-h}$ or $\dfrac{-h}{8}$ nb AC = 20, M(6, 6) MD = 10 → D(16, 6) and B(−4, 6)	A1 [3]	co
	(iii) Product of gradients = −1 → h^2 – 12h – 64 = 0 → h = 16 or − 4 so x_D = 16 and x_B = –4 or Pyth $h^2 + 8^2 + 8^2 + (12 - h)^2 = 400$	M1 M1 DM1A1 [4]	Used correctly with the two gradients Forming a quadratic equation Solution of equation. co
	(iv) Area = √320 × √80 → 160 (or Area = 2 × area of a triangle with base = BD, → 2 × ½ × 20 × 8 = 160) (or matrix method)	M1 M1 A1 [3]	M1 for method for one of the lengths M1 for base × height. co

10	**(i)**	**(a)** $2y = x + 5$, $y = x^2 - 4x + 7$ Sim equations → $2x^2 - 9x + 9 = 0$ → $x = 3$ or $x = 1½$.	M1 DM1 A1 [3]	Complete elimination of x or y Correct method for quadratic. co.
		(b) $\frac{dy}{dx} = 2x - 4$ → $y - 4 = 2(x - 3)$ nb use of $y + 4$ or x, y interchanged M1 A0	B1 M1 A1 [3]	co Correct form of eqn with m numeric. co
		(c) $m = 2$ → angle of 63.4° $m = ½$ → angle of 26.6° → angle between = 37° $(\mathbf{i}+2\mathbf{j}).(2\mathbf{i}+\mathbf{j}) \rightarrow 4=\sqrt{5}\sqrt{5}\cos\theta$ M1M1A1 or use of $\tan(A-B)$ M2A1 or Cosine rule with 3 sides found.	M1 M1A1 [3]	Finds angle with x-axis once. Subtracts two angles. co.
	(ii)	$y = x^2 - 4x + 7$ $2y = x + k$ Sim eqns → $2x^2 - 9x + 14 - k = 0$ Uses $b^2 - 4ac$, $81 - 8(14 - k)$ Key value is $k = 3.875$ or 31/8. $k < 3.875$	M1 A1 M1 A1 [4]	Eliminates y or x completely. Co (= 0) Uses $b^2 - 4ac = 0$, or < 0 or > 0 Co condone $\leqslant$.