UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2010 question paper for the guidance of teachers

9709 MATHEMATICS

9709/31 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the October/November 2010 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only - often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

<u>Penalties</u>

MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures - this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1	EITHER:	State or imply non-modular inequality $(2(x-3))^2 > (3x+1)^2$, or corresponding quadratic equation, or pair of linear equations $2(x-3) = \pm(3x+1)$	B1	
		Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations	M1	
		Obtain critical values $x = -7$ and $x = 1$	A1	
		State answer $-7 < x < 1$	A1	
	OR:	Obtain critical value $x = -7$ or $x = 1$ from a graphical method, or by inspection, or by solving a linear equation or inequality	B1	
		Obtain critical values $x = -7$ and $x = 1$	B2	
		State answer $-7 < x < 1$	B1	[4]
		[Do not condone: ‹ for <.]		

2	Use law for the logarithm of a power, a quotient, or a product correctly at least once	M1	
	Use ln e = 1 or e = exp(1)	M1	
	Obtain a correct equation free of logarithms, e.g. $1 + x^2 = ex^2$	A1	
	Solve and obtain answer $x = 0.763$ only	A1	[4]
	[For the solution $x = 0.763$ with no relevant working give B1, and a further B1 if 0.763 is shown to be the only root.]		
	[Treat the use of logarithms to base 10 with answer 0.333 only, as a misread.]		
	[SR: Allow iteration, giving B1 for an appropriate formula, e.g. $x_{n+1} = \exp((\ln(1 + x_n^2) - 1)/2)$, M1 for using it correctly once, A1 for 0.763, and A1 for showing the equation has no other root but 0.763.]		

3	Attempt use of $\cos(A + B)$ formula to obtain an equation in $\cos\theta$ and $\sin\theta$	M1	
	Use trig formula to obtain an equation in $\tan\theta$ (or $\cos\theta$, $\sin\theta$ or $\cot\theta$)	M1	
	Obtain $\tan\theta = 1/(4 + \sqrt{3})$ or equivalent (or find $\cos\theta$, $\sin\theta$ or $\cot\theta$)	A1	
	Obtain answer $\theta = 9.9°$	A1	
	Obtain $\theta = 189.9°$, and no others in the given interval	A1	[5]
	[Ignore answers outside the given interval. Treat answers in radians as a misread (0.173, 3.31).]		
	[The other solution methods are *via* $\cos\theta = \pm(4 + \sqrt{3})/\sqrt{\left(1 + \left(4 + \sqrt{3}\right)^2\right)}$ or $\sin\theta = \pm 1/\sqrt{\left(1 + \left(4 + \sqrt{3}\right)^2\right)}$.]		

4	**(i)**	Make recognisable sketch of a relevant graph over the given range	B1	
		Sketch the other relevant graph on the same diagram and justify the given statement	B1	[2]
	(ii)	Consider sign of $4x^2 - 1 - \cot x$ at $x = 0.6$ and $x = 1$, or equivalent	M1	
		Complete the argument correctly with correct calculated values	A1	[2]
	(iii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 0.73	A1	
		Show sufficient iterations to at least 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (0.725, 0.735)	A1	[3]

5 **(i)** State or imply dx = 2 cos θ dθ, or $\dfrac{dx}{d\theta} = 2\cos\theta$, or equivalent — B1

Substitute for x and dx throughout the integral — M1

Obtain the given answer correctly, having changed limits and shown sufficient working — A1 [3]

(ii) Replace integrand by 2 – 2 cos 2θ, or equivalent — B1

Obtain integral 2θ – sin 2θ, or equivalent — B1√

Substitute limits correctly in an integral of the form $a\theta \pm b$ sin 2θ, where $ab \neq 0$ — M1

Obtain answer $\dfrac{1}{3}\pi - \dfrac{\sqrt{3}}{2}$ or exact equivalent — A1 [4]

[The f.t. is on integrands of the form $a + c$ cos 2θ, where $ac \neq 0$.]

6 **(i)** State modulus is 2 — B1

State argument is $\frac{1}{6}\pi$, or 30°, or 0.524 radians — B1 [2]

(ii) **(a)** State answer $3\sqrt{3} + \mathrm{i}$ — B1

(b) *EITHER*: Multiply numerator and denominator by $\sqrt{3} - \mathrm{i}$, or equivalent — M1

Simplify denominator to 4 or numerator to $2\sqrt{3} + 2\mathrm{i}$ — A1

Obtain final answer $\frac{1}{2}\sqrt{3} + \frac{1}{2}\mathrm{i}$, or equivalent — A1

OR 1: Obtain two equations in x and y and solve for x or for y — M1

Obtain $x = \frac{1}{2}\sqrt{3}$ or $y = \frac{1}{2}$ — A1

Obtain final answer $\frac{1}{2}\sqrt{3} + \frac{1}{2}\mathrm{i}$, or equivalent — A1

OR 2: Using the correct processes express iz*/z in polar form — M1

Obtain $x = \frac{1}{2}\sqrt{3}$ or $y = \frac{1}{2}$ — A1

Obtain final answer $\frac{1}{2}\sqrt{3} + \frac{1}{2}\mathrm{i}$, or equivalent — A1 [4]

(iii) Plot A and B in relatively correct positions — B1

EITHER: Use fact that angle AOB = arg(iz*) – arg z — M1

Obtain the given answer — A1

OR 1: Obtain tan $A\hat{O}B$ from gradients of OA and OB and the correct tan(A – B) formula — M1

Obtain the given answer — A1

OR 2: Obtain cos $A\hat{O}B$ by using correct cosine formula or scalar product — M1

Obtain the given answer — A1 [3]

7	**(i)**	State correct equation in any form, e.g. $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k} + \lambda(2\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$		B1	[1]
	(ii)	*EITHER*:	Equate a relevant scalar product to zero and form an equation in λ	M1	
		OR 1:	Equate derivative of OP^2 (or OP) to zero and form an equation in λ	M1	
		OR 2:	Use Pythagoras in *OAP* or *OBP* and form an equation in λ	M1	
		State a correct equation in any form		A1	
		Solve and obtain $\lambda = -\frac{1}{6}$ or equivalent		A1	
		Obtain final answer $\overrightarrow{OP} = \frac{2}{3}\mathbf{i} + \frac{5}{3}\mathbf{j} + \frac{7}{3}\mathbf{k}$, or equivalent		A1	[4]
	(iii)	*EITHER*:	State or imply $\overrightarrow{OP}$ is a normal to the required plane	M1	
			State normal vector $2\mathbf{i} + 5\mathbf{j} + 7\mathbf{k}$, or equivalent	A1√	
			Substitute coordinates of a relevant point in $2x + 5y + 7z = d$ and evaluate d	M1	
			Obtain answer $2x + 5y + 7z = 26$, or equivalent	A1	
		OR 1:	Find a vector normal to plane *AOB* and calculate its vector product with a direction vector for the line *AB*	M1*	
			Obtain answer $2\mathbf{i} + 5\mathbf{j} + 7\mathbf{k}$, or equivalent	A1	
			Substitute coordinates of a relevant point in $2x + 5y + 7z = d$ and evaluate d	M1(dep*)	
			Obtain answer $2x + 5y + 7z = 26$, or equivalent	A1	
		OR 2:	Set up and solve simultaneous equations in a, b, c derived from zero scalar products of $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ with **(i)** a direction vector for line *AB*, **(ii)** a normal to plane *OAB*	M1*	
			Obtain $a : b : c = 2 : 5 : 7$, or equivalent	A1	
			Substitute coordinates of a relevant point in $2x + 5y + 7z = d$ and evaluate d	M1(dep*)	
			Obtain answer $2x + 5y + 7z = 26$, or equivalent	A1	
		OR 3:	With $Q\ (x, y, z)$ on plane, use Pythagoras in *OPQ* to form an equation in x, y and z	M1*	
			Form a correct equation	A1√	
			Reduce to linear form	M1(dep*)	
			Obtain answer $2x + 5y + 7z = 26$, or equivalent	A1	
		OR 4:	Find a vector normal to plane *AOB* and form a 2-parameter equation with relevant vectors, e.g., $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k} + \lambda(2\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}) + \mu(8\mathbf{i} - 6\mathbf{j} + 2\mathbf{k})$	M1*	
			State three correct equations in x, y, z, λ and μ	A1	
			Eliminate λ and μ	M1(dep*)	
			Obtain answer $2x + 5y + 7z = 26$, or equivalent	A1	[4]

8	**(i)**	State or imply the form $\dfrac{A}{1+x}+\dfrac{Bx+C}{1+2x^2}$	B1	
		Use any relevant method to evaluate a constant	M1	
		Obtain one of $A = -1$, $B = 2$, $C = 1$	A1	
		Obtain a second value	A1	
		Obtain the third value	A1	[5]
	(ii)	Use correct method to obtain the first two terms of the expansion of $(1+x)^{-1}$ or $(1+2x^2)^{-1}$	M1	
		Obtain correct expansion of each partial fraction as far as necessary	A1√ + A1√	
		Multiply out fully by $Bx + C$, where BC þ 0	M1	
		Obtain answer $3x - 3x^2 - 3x^3$	A1	[5]
		[Symbolic binomial coefficients, e.g., $\begin{pmatrix}-1\\1\end{pmatrix}$ are not sufficient for the first M1. The f.t. is on A, B, C.]		
		[If B or C omitted from the form of fractions, give B0M1A0A0A0 in **(i)**; M1A1√A1√ in **(ii)**, max 4/10.]		
		[If a constant D is added to the correct form, give M1A1A1A1 and B1 if and only if $D = 0$ is stated.]		
		[If an extra term $D/(1 + 2x^2)$ is added, give B1M1A1A1, and A1 if $C + D = 1$ is resolved to $1/(1 + 2x^2)$.]		
		[In the case of an attempt to expand $3x(1 + x)^{-1}(1 + 2x^2)^{-1}$, give M1A1A1 for the expansions up to the term in x^2, M1 for multiplying out fully, and A1 for the final answer.]		
		[For the identity $3x \equiv (1 + x + 2x^2 + 2x^3)(a + bx + cx^2 + dx^3)$ give M1A1; then M1A1 for using a relevant method to find two of $a = 0$, $b = 3$, $c = -3$ and $d = -3$; and then A1 for the final answer in series form.]		
9	**(i)**	Use correct product rule	M1	
		Obtain correct derivative in any form	A1	
		Equate derivative to zero and find non-zero x	M1	
		Obtain $x = \exp\left(-\frac{1}{3}\right)$, or equivalent	A1	
		Obtain $y = -1/(3e)$, or any ln-free equivalent	A1	[5]
	(ii)	Integrate and reach $kx^4 \ln x + l\int x^4 . \frac{1}{x}\,dx$	M1	
		Obtain $\frac{1}{4}x^4 \ln x - \frac{1}{4}\int x^3\,dx$	A1	
		Obtain integral $\frac{1}{4}x^4 \ln x - \frac{1}{16}x^4$, or equivalent	A1	
		Use limits $x = 1$ and $x = 2$ correctly, having integrated twice	M1	
		Obtain answer $4\ln 2 - \dfrac{15}{16}$, or exact equivalent	A1	[5]

10	**(i)**	State or imply $\frac{dx}{dt} = k(20 - x)$	B1	
		Show that $k = 0.05$	B1	[2]
	(ii)	Separate variables correctly and integrate both sides	B1	
		Obtain term $-\ln(20 - x)$, or equivalent	B1	
		Obtain term $\frac{1}{20}t$, or equivalent	B1	
		Evaluate a constant or use limits $t = 0$, $x = 0$ in a solution containing terms $a \ln(20 - x)$ and bt	M1*	
		Obtain correct answer in any form, e.g. $\ln 20 - \ln(20 - x) = \frac{1}{20}t$	A1	[5]
	(iii)	Substitute $t = 10$ and calculate x	M1(dep*)	
		Obtain answer $x = 7.9$	A1	[2]
	(iv)	State that x approaches 20	B1	[1]