



UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS  
General Certificate of Education  
Advanced Subsidiary Level and Advanced Level

**MATHEMATICS**

**9709/12**

Paper 1 Pure Mathematics 1 (P1)

**October/November 2011**

**1 hour 45 minutes**

Additional Materials: Answer Booklet/Paper  
Graph Paper  
List of Formulae (MF9)



**READ THESE INSTRUCTIONS FIRST**

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 75.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **4** printed pages.



- 1 (i) Find the first 3 terms in the expansion of  $(2 - y)^5$  in ascending powers of  $y$ . [2]
- (ii) Use the result in part (i) to find the coefficient of  $x^2$  in the expansion of  $(2 - (2x - x^2))^5$ . [3]

- 2 The functions  $f$  and  $g$  are defined for  $x \in \mathbb{R}$  by

$$f : x \mapsto 3x + a,$$

$$g : x \mapsto b - 2x,$$

where  $a$  and  $b$  are constants. Given that  $ff(2) = 10$  and  $g^{-1}(2) = 3$ , find

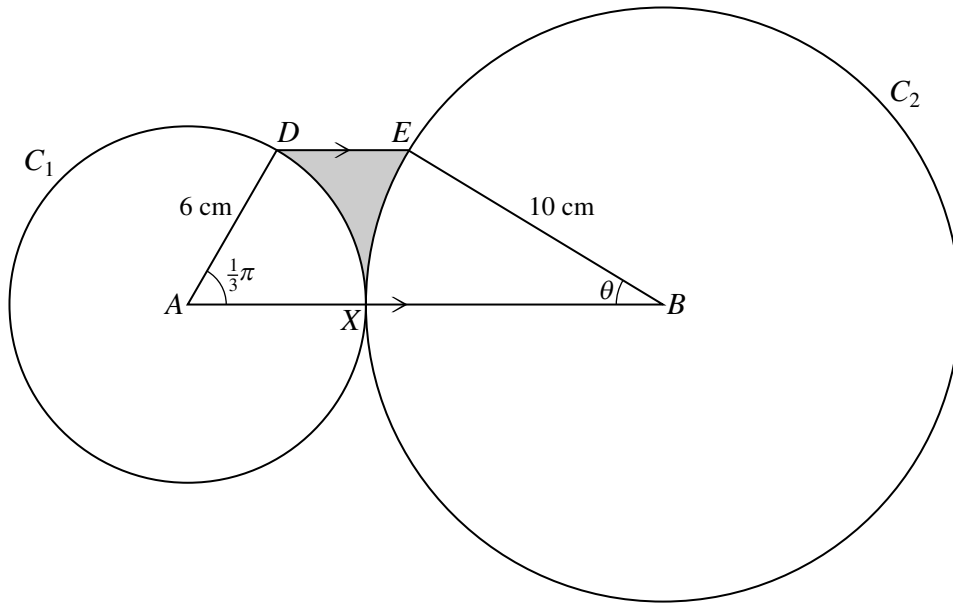
- (i) the values of  $a$  and  $b$ , [4]
- (ii) an expression for  $fg(x)$ . [2]
- 3 Relative to an origin  $O$ , the position vectors of points  $A$  and  $B$  are given by

$$\overrightarrow{OA} = 5\mathbf{i} + \mathbf{j} + 2\mathbf{k} \quad \text{and} \quad \overrightarrow{OB} = 2\mathbf{i} + 7\mathbf{j} + p\mathbf{k},$$

where  $p$  is a constant.

- (i) Find the value of  $p$  for which angle  $AOB$  is  $90^\circ$ . [3]
- (ii) In the case where  $p = 4$ , find the vector which has magnitude 28 and is in the same direction as  $\overrightarrow{AB}$ . [4]
- 4 The equation of a curve is  $y^2 + 2x = 13$  and the equation of a line is  $2y + x = k$ , where  $k$  is a constant.
- (i) In the case where  $k = 8$ , find the coordinates of the points of intersection of the line and the curve. [4]
- (ii) Find the value of  $k$  for which the line is a tangent to the curve. [3]
- 5 (i) Sketch, on the same diagram, the graphs of  $y = \sin x$  and  $y = \cos 2x$  for  $0^\circ \leq x \leq 180^\circ$ . [3]
- (ii) Verify that  $x = 30^\circ$  is a root of the equation  $\sin x = \cos 2x$ , and state the other root of this equation for which  $0^\circ \leq x \leq 180^\circ$ . [2]
- (iii) Hence state the set of values of  $x$ , for  $0^\circ \leq x \leq 180^\circ$ , for which  $\sin x < \cos 2x$ . [2]

6



The diagram shows a circle  $C_1$  touching a circle  $C_2$  at a point  $X$ . Circle  $C_1$  has centre  $A$  and radius  $6$  cm, and circle  $C_2$  has centre  $B$  and radius  $10$  cm. Points  $D$  and  $E$  lie on  $C_1$  and  $C_2$  respectively and  $DE$  is parallel to  $AB$ . Angle  $DAX = \frac{1}{3}\pi$  radians and angle  $EBX = \theta$  radians.

- (i) By considering the perpendicular distances of  $D$  and  $E$  from  $AB$ , show that the exact value of  $\theta$  is  $\sin^{-1}\left(\frac{3\sqrt{3}}{10}\right)$ . [3]
- (ii) Find the perimeter of the shaded region, correct to 4 significant figures. [5]

7 A curve is such that  $\frac{dy}{dx} = 5 - \frac{8}{x^2}$ . The line  $3y + x = 17$  is the normal to the curve at the point  $P$  on the curve. Given that the  $x$ -coordinate of  $P$  is positive, find

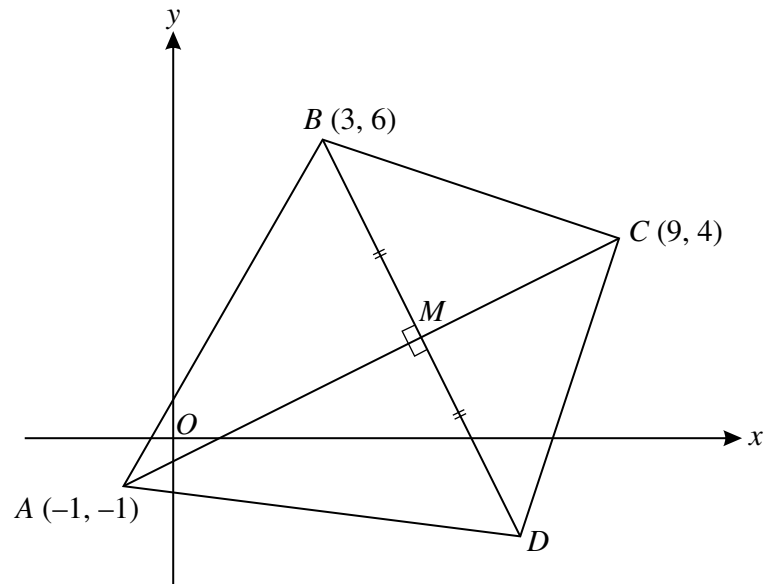
- (i) the coordinates of  $P$ , [4]
- (ii) the equation of the curve. [4]

8 The equation of a curve is  $y = \sqrt{(8x - x^2)}$ . Find

- (i) an expression for  $\frac{dy}{dx}$ , and the coordinates of the stationary point on the curve, [4]
- (ii) the volume obtained when the region bounded by the curve and the  $x$ -axis is rotated through  $360^\circ$  about the  $x$ -axis. [4]

[Questions 9 and 10 are printed on the next page.]

9



The diagram shows a quadrilateral  $ABCD$  in which the point  $A$  is  $(-1, -1)$ , the point  $B$  is  $(3, 6)$  and the point  $C$  is  $(9, 4)$ . The diagonals  $AC$  and  $BD$  intersect at  $M$ . Angle  $BMA = 90^\circ$  and  $BM = MD$ . Calculate

- (i) the coordinates of  $M$  and  $D$ , [7]  
 (ii) the ratio  $AM : MC$ . [2]

10 (a) An arithmetic progression contains 25 terms and the first term is  $-15$ . The sum of all the terms in the progression is 525. Calculate

- (i) the common difference of the progression, [2]  
 (ii) the last term in the progression, [2]  
 (iii) the sum of all the positive terms in the progression. [2]

(b) A college agrees a sponsorship deal in which grants will be received each year for sports equipment. This grant will be \$4000 in 2012 and will increase by 5% each year. Calculate

- (i) the value of the grant in 2022, [2]  
 (ii) the total amount the college will receive in the years 2012 to 2022 inclusive. [2]

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