MATHEMATICS

Key messages

- Candidates need to be careful to eliminate arithmetic and numerical errors in their answers.
- Candidates need to take care to avoid algebraic errors.

General comments

This paper as in previous years allowed those candidates who had covered the full breadth of the syllabus and who had practised by working through past papers, to demonstrate their ability and knowledge. At the same time it exposed weaknesses in the mathematical foundations of a proportion of candidates. For example, errors were made in algebraic manipulations which in some cases showed a lack of understanding.

Comments on specific questions

Question 1

A significant number of candidates made a mistake on the very first line by taking d to be 4 rather than –4. Errors in algebra occurred in simplifying the equation $n/2[122 + (n-1)(-4)]$, and then solving it. Dividing throughout by n was seen quite often, which in general is not good practice since it loses the solution $n = 0$. In this particular case, $n = 0$ is not a valid answer so candidates were not penalised.

Answer: 31.

Question 2

This question was well done. Some candidates integrated correctly, but forgot to include the constant of integration and the consequent substitution of (2, 4).

Answer: $y = \dfrac{4}{x^2} - x + 5$.

Question 3

This question was found difficult by many candidates. The fact that the question was given in context meant that candidates first had to define the variables (A, r, t) for themselves before writing down the appropriate chain rule in order to find dr/dt. It was this initial 'setting-up' process which prevented many candidates from making good progress with this question.

Answer: 300π.

Question 4

Most candidates made a good attempt at this question. A common error was to 'lose' the minus sign.

Answers: **(i)** $64x^6 - 192x^7 + 240x^8$; **(ii)** 288.

Question 5

Most candidates were able to find dy/dx accurately. The word 'verify' is used in the question to indicate to candidates the easier method of substituting $x = 2$ to reach dy/dx = 0 and hence to conclude that the curve has a stationary point at $x = 2$. Many candidates tried the more complex method of starting with dy/dx = 0 and tried to prove that $x = 2$ was a solution. The word 'determine' in the question signals that some justification is required for the candidate's answer. A simple statement of 'minimum' is not sufficient. In this question, probably the easiest way is to show that the second derivative is positive, but other valid methods were accepted.

Answer: Minimum.

Question 6

This question was often not completed entirely successfully, although most candidates found some of the lengths correctly and thereby achieved some marks. It was often the case that candidates found the lengths as decimal numbers without seeming to understand the significance of the requirement that the answer is given 'as an exact value'. In this case this statement meant that the answer had to be given in terms of π and √3.

Answers: **(i)** $r - r\cos\theta$; **(ii)** $7\pi/3 + 2\sqrt{3} - 2$.

Question 7

Part **(i)** was usually done correctly, although a number of candidates omitted a second solution or got the second solution wrong. Relatively few candidates were successful with part **(ii)**. The first step was to realise that the value of n was 3 and then that it was necessary to find the largest solution for 3θ in the range 0° to 1080°. Division by 3 then gave the required answer.

Answers: **(i)** 30°, 150°; **(ii)** 290°.

Question 8

In part **(i)** the majority of candidates chose the more difficult route and tried to form a quadratic equation in x. On this particular occasion forming an equation in y is far simpler and then the required value of x corresponds to the larger value, 3, of y. In part **(ii)**, there are various methods of finding the required area, but probably the most straightforward is to find the direct integral of the curve between $\frac{1}{2}$ and 5 and subtract from this value the area of the triangle. Unfortunately numerous errors, both in integration and basic arithmetic, were present in the work of many candidates in this part.

Answer: **(ii)** 9/4.

Question 9

In part **(i)** most candidates found vector **CD** correctly but a number of candidates did not appear to know how to find the corresponding unit vector. In part **(ii)** a very common mistake indeed was to think **OE** = 1/2**CD**. Many candidates demonstrated that they knew how to find the angle between two vectors, for which they obtained some method marks, but one of the vectors used was often incorrect.

Answers: **(i)** 1/7(2**i** – 3**j** +6**k**); **(ii)** 8.6°.

Question 10

Parts **(i)** and **(ii)** were done reasonably well, although a significant proportion of candidates thought the range was g(x) ≥ −25. In part **(iii)**, much of the work for the inverse function was done correctly up to the point of taking the square root. At this point few candidates remembered to insert ±, and so took the positive square root by default when, in fact, it was necessary to take the negative square root.

Answers: **(i)** $4(x - 3)^2 - 25$, (3, –25); **(ii)** g(x) ≥ –9; **(iii)** $3 - 1/2\sqrt{x + 25}$, $x \geq -9$.

Question 11

Part **(i)** was done reasonably well with the majority of candidates being able to differentiate the function correctly and then to find correctly the gradients and the equations of the tangent and normal. Part **(ii)** was also done well. However, in part **(iii)**, the equation of *BC* was often not found correctly (or sometimes not attempted) and hence only a minority of candidates were able to reach the correct coordinates of the point *E*.

Answers: **(i)** $y = x/2 + 3/2$, $y = -2x + 4$; **(ii)** $5/2$; **(iii)** $(6/11, 12/11)$, *E* is not the mid-point.

MATHEMATICS

Paper 9709/12
Paper 12

Key messages

- Candidates need to take care to avoid arithmetic and algebraic errors.
- It is important for candidates to not spend too much time on any one question or part of question.
- Candidates need to be careful to eliminate numerical errors in their answers.
- Candidates need to be careful to read the question in detail and answer as indicated.

General comments

Although there were many very good scripts, many candidates found the standard of this paper to be too difficult. Both **Questions 10** and **11** presented candidates with some degree of difficulty. Many good scripts were affected by poor algebra and such basic errors as "$a + b = c$", therefore "$a = c \div b$" were common. Generally, scripts were well presented and easy to mark with necessary working shown in full. Although some candidates continue to split the page into two halves and work down each side, which makes marking difficult.

Comments on specific questions

Question 1

The use of the binomial distribution and of binomial coefficients was very good and there were many correct answers. Many candidates gave the whole expansion and it was a common error to deduce that the term in

x^5 resulted from the product of $(x^2)^5$ and $\left(\dfrac{-a}{x}\right)^2$. Other common errors were either to ignore the "$-$" sign in

$\left(\dfrac{-a}{x}\right)^3$ or to expand $\left(\dfrac{-a}{x}\right)^3$ as $\dfrac{-a}{x^3}$.

Answer: $a = 2$.

Question 2

(i) A significant number of students misread the original expression, $\sqrt{\dfrac{x+3}{2}}+1$ as either $\sqrt{\dfrac{x+3}{2}}$ or

as $\sqrt{\dfrac{x+3}{2}+1}$. Candidates were however confident in the way they attempted to make "x" the subject of the equation and, apart from a few basic algebraic errors, solutions were generally correct. The main error, especially from weaker candidates, was to replace $y = \sqrt{\dfrac{x+3}{2}}+1$ by

$y^2 = \dfrac{x+3}{2}+1$. Some candidates still did not realise that an expression for f$^{-1}(x)$ must be in terms of x and not y and many did not read the instruction to express the answer in the form $ax^2 + bx + c$.

(ii) This was poorly answered with only a small proportion realising that the domain of f^{-1} was the same as the range of f.

Answers: **(i)** $2x^2 - 4x - 1$; **(ii)** $x \geq 1$.

Question 3

(i) Those candidates who realised that the area of the playground could be obtained by subtracting the sum of the areas of three triangles from the area of park were usually correct. At least a third of all attempts however attempted to use Pythagoras to find the lengths of *AX*, *AY* and *XY* and made no further progress.

(ii) This part was well answered with candidates realising the need to set the differential to zero. A common error was to set the second differential to zero and a large number of candidates left the answer as "*x* = 15" thereby failing to find the value of *A*.

Answers: **(i)** Proof; **(ii)** 975 m².

Question 4

The majority of candidates attempted to use an algebraic method and were more successful than those using calculus.

(i) Most candidates eliminated *y* from the two equations to form a quadratic equation. The majority then attempted to set "$b^2 - 4ac$" to 0 and to solve for *k*, Simple algebraic errors affected many solutions and only about a third of all solutions led to $k^3 = -1$. Even then, $k = 1$ or $k = \pm1$ were common solutions. Candidates attempting calculus often made the mistake of equating "dy/dx" for one equation to "*y*" of the other.

(ii) When part **(i)** was correct, candidates usually coped correctly with part **(ii)** by returning to the solution of a quadratic equation.

Answers: **(i)** $k = -1$; **(ii)** P(−2, 1).

Question 5

This was well answered and the majority of candidates scored full marks. A few weaker candidates however assumed, with no reason, either that triangle *ABC* was isosceles or that *AX* was a quarter of *AC*. Such assumptions can gain no credit.

(i) Candidates were confident in finding the gradient of *AB,* the perpendicular gradient and hence the equation of *BC*.

(ii) Finding the equation of *AX* and hence the coordinates of *C* by the solving of simultaneous equations presented few problems.

Answers: **(i)** $2y + x = 27$; **(ii)** C(13, 7).

Question 6

(i) This was well answered with the vast majority of candidates using the trigonometric identities linking sin*x*, cos*x* and tan*x* to form a correct quadratic equation in sin*x*.

(ii) At least a quarter of all candidates did not realise the link between parts **(i)** and **(ii)** and made no subsequent progress. Of the rest, at least a half either did not realise the need to find values for *2y* (30º and 150º) before dividing by 2, or obtained the answer 15º and then subtracted this from 180º.

Answers: **(i)** $2\sin^2 x + 3\sin x - 2 = 0$; **(ii)** 15º, 75º.

Question 7

(i)   Virtually all candidates were confident in their ability to calculate an angle using a scalar product. Simple numerical errors in substituting $k = 2$, or in working out a modulus occurred frequently, but the most common error was in evaluating the scalar product as 8 by assuming that "$0 \times -2 = -2$".

(ii)   This was badly answered and there were few correct answers. Most candidates were able to obtain the vector **AB** in terms of k, about half then attempted to find the modulus in terms of k, but only a small handful realised that this modulus was equal to 1.

Answers: **(i)** 24.1º; **(ii)** $k = 1$ or $\dfrac{2}{3}$.

Question 8

(a) (i) This proved to be a straightforward question and correct answers were common. Candidates were able to write down correct equations for the 2nd and 4th terms and to eliminate r, the common ratio, to find the first term.

(ii)   Use of the correct formula for the sum to infinity was accurate.

(b)   Only a small proportion of candidates were able to obtain full marks. Many were unable to proceed further than realising that the common difference of the arithmetic progression was 2. Two main misunderstandings affected the remainder of the solutions. Many did not realise that the sum of n terms was 360º. Of the remaining solutions, it was common to equate 360 with the nth term, rather than with the sum of n terms.

Answers: **(a) (i)** 32; **(ii)** 128; **(b)** $n = 18$.

Question 9

(i)   This part of the question was well answered and most candidates recognised that y was a composite function and multiplied by the differential of the bracket. Most then obtained a correct expression for the equation of the tangent, though the error of finding the equation of the normal or of leaving the gradient as a function of x were common.

(ii)   This part was poorly answered with many candidates not realising the need to set x to zero in the equation of the tangent and then deciding whether the resulting value of y was larger or smaller than $1\dfrac{1}{2}$, the value of y halfway between 0 and 3.

(iii)   The majority of candidates used the correct formula for the volume of rotation and the integration was well done, with most candidates again realising the need to divide by "2", the differential of the bracket. The most common error was to incorrectly assume that the lower limit of 0 could be ignored.

Answers: **(i)** $9y + 2x = 15$; **(ii)** C is nearer to B; **(iii)** 9π.

Question 10

(i) The majority of candidates realised the need to integrate, to include a constant of integration and to use the coordinates of *P* to find the value of the constant. Common errors were to ignore the constant of integration completely or to assume that the equation of the curve is the same as the equation of the tangent.

(ii) This part of the question presented all candidates with difficulty and only a handful of correct solutions were seen. The majority of candidates did not realise that the question was requesting the minimum value of d*y*/d*x*, not of *y*. It was necessary therefore to show that $\dfrac{d^2 y}{dx^2}$ was zero when *x* = 2 and then to show that $\dfrac{d^3 y}{dx^3}$ was positive when *x* = 2.

Answers: **(i)** $y = \dfrac{x^2}{2} = \dfrac{4}{2} + 1$; **(ii)** Proof, Minimum value of the gradient = 3.

Question 11

All candidates found part **(i)** difficult, and most made no attempt at a solution. However, virtually all candidates realised that parts **(ii)** and **(iii)** were still accessible to them.

(i) This proved to be too difficult for all but the most able candidates. Obtaining the correct value of *x* meant using trigonometry in triangle *OCR* (sin 0.6 = $\dfrac{CR}{OC}$) and realising that this led to sin 0.6 = $\dfrac{x}{(20 - x)}$.

(ii) This was well answered with the majority of candidates realising the need to subtract the area of the circle from the area of the sector.

(iii) Most candidates realised the need to find the arc length *PSR*, but at least a half of the attempts did not realise that the angle subtended by this arc at the centre *C* was (π – 1.2) radians. Finding the lengths of *OP* and *OR* also caused problems and many candidates incorrectly assumed these lengths to be 10cm, half of the radius *OQ*. Only a small proportion used trigonometry in triangle *OCR* to find these lengths.

Answers: **(i)** Proof; **(ii)** 76.3 cm²; **(iii)** 35.1 cm.

MATHEMATICS

> **Paper 9709/13**
> **Paper 13**

Key messages

- Candidates need to be careful to read the question in detail and answer as indicated.
- Candidates need make sure they do not make approximations during their working, as this can impact on the accuracy of the final result.

General comments

Most candidates were able to make a very good attempt at the majority of the paper and there were many more good marks than poor. In general the first five questions were very well done, but **Questions 6**, **7 (ii)**, **9 (i)** and **11 (iv)** proved more challenging for many candidates. The overall level of presentation was very good with only a few candidates continuing to make 2 columns of work on the same page.

In order to improve their performance candidates should remember the paramount importance of reading each question carefully before it is started. In some cases parts of some questions had not been fully attempted. Candidates also need to be very careful to give the answers in the correct form for the question, for example in degrees or in radians. Full working should always be shown, especially for given answers. Some candidates worked out answers and then copied them up. This is a very time-consuming process.

Comments on specific questions

Question 1

Candidates often achieved full marks on this question. Some candidates created unnecessary work for themselves by writing out the full expansion, sometimes failing then to identify the appropriate term. Omissions of the minus sign or not evaluating $\left(\frac{1}{2}\right)^3$ were other errors. A small number of candidates tried to take the 2 out of the bracket and expand $\left(1+\frac{x}{2}\right)^7$ but generally they were defeated by the algebra.

Answer: -70.

Question 2

The majority of candidates performed very well on this question. They recognised that the most straightforward method of solution was to find the derivative and this was usually done correctly. Some candidates omitted to say that the negative gradient implied a decreasing function and so lost the final mark. A minority simply evaluated the function for various values and tried to imply that it was therefore increasing, but this received no credit.

Question 3

Nearly all candidates scored well on this question, with the majority scoring full marks. Candidates who did not use $\sin^2 x = 1 - \cos^2 x$ could not make any meaningful progress, but these were rare. Generally the algebra was completed satisfactorily leading to $\cos x = -\frac{1}{2}$. Many candidates used $\cos^{-1} -\frac{1}{2}$ to give 120° as their answer, but some were confused by what to do with an obtuse angle. These then often simply gave 120° as their only answer.

Answers: 120°, 240°.

Question 4

Candidates produced many wholly correct and efficient solutions. It was intended that they would find the area of the triangle and subtract from this the area of the sector (after finding angle *BAC*). Some candidates produced longer than necessary solutions by involving calculations for the angle at B and/or the length of *AB*, both of which were often subsequently used to achieve correct answers. The use of $\frac{1}{2}ab \sin C$ was seen occasionally and usually correctly done, but some candidates spoilt a valid (if longer) approach by using the given sides with the angle *A* and not calculating and using *AB*. Among weaker scripts, the common issues were: use of degrees in $\frac{1}{2}r^2\theta$; $\tan^{-1}\sqrt{3}$ incorrectly determined to be $\frac{\pi}{6}$ or 30°; and working in decimals and not in terms of π and $\sqrt{3}$.

Answer: $2\sqrt{3} - \frac{2\pi}{3}$.

Question 5

This question proved to be accessible to the vast majority of candidates. Many quickly divided $2\frac{1}{4}$ by $5\frac{1}{3}$ to find r^3 and then cube rooted to correctly find *r*. In part **(i)** a few did confuse $5\frac{1}{3}$ with $\frac{15}{3}$ and some found *r* to be > 1 but seemed unconcerned and went on to use their values in a correctly stated formula for S_∞. A few weaker candidates quoted and attempted to use the formulae for APs.

Answers: **(i)** 0.75 or $\frac{3}{4}$; **(ii)** $21\frac{1}{3}$.

Question 6

Nearly all of the candidates carried out the operations in the correct order in both parts **(i)** and **(ii)**.

In part **(i)** a number of them followed the correct statement that $\cos(\frac{1}{2}x + \frac{\pi}{6}) = 1$ with the incorrect one that $\cos(\frac{1}{2}x) + \cos(\frac{\pi}{6}) = 1$. A number of other candidates gave an extra incorrect answer of $\frac{\pi}{3}$ as well as the correct $-\frac{\pi}{3}$.

In part **(ii)** a number of otherwise correct solutions were spoiled by the common mistake of omitting to give the final answer in both its negative and positive form. Most candidates got correctly as far as cosx = . . ., but some prematurely approximated 0.9528 to 0.95. Another common error was giving the answer in degrees rather than radians.

Answers: **(i)** $-\frac{\pi}{3}$; **(ii)** ± 0.31.

Question 7

Although there were 1 or 2 sign errors the vast majority of candidates achieved all three marks for the first part of this question. However, few by contrast achieved all five marks for the second part. Many candidates simply did not offer a domain for $f^{-1}(x)$. While this could be due to a lack of ability, it is also possible that the candidates did not understand that this was required. The question stated that their answer be expressed in a 'similar way' and in order to achieve full marks the domains for the 2 parts of the inverse function were needed. Of those who did attempt to give values for the domain these were often copies of those given in the question (either p or 3). Other candidates seemed to try and equate the 2 parts of the function as they had in part **(i)** to achieve some type of combined inverse. This received no credit. Those candidates that lost marks in stating the functions usually did so due to incorrect signs such as $\pm$ before the square root.

Answers: **(i)** $p=3$, $q=2$; **(ii)** $f^{-1}(x) = \begin{cases} 5-x & x < 2 \\ \sqrt{11-x}, & 2 \le x \le 11 \end{cases}$

Question 8

Part **(i)** was generally answered very well with the vast majority of candidates obtaining at least some credit. A common error was to omit the factor of 3 derived from $\dfrac{d}{dx}(3x+4)$, arriving at the incorrect answer $3(3x+4)^{\frac{1}{2}} - 6$. A number of weaker candidates obtained $3(3(3x+4)^{\frac{1}{2}} - 6)$.

Part **(ii)** was a challenge. Some candidates did not correctly justify the nature of the stationary point and others simply quoted $\dfrac{dy}{dx} = 0$ or showed insufficient working to justify it. It is essential that candidates appreciate the key importance of showing full and detailed working when verifying or working towards a given answer. It was hoped that $2 + 6 - 8 = 0$ would be seen followed by the statement that $\dfrac{dy}{dx} = 0$ and hence $x = -1$ was a stationary point.

The majority of candidates obtained full marks in part **iii** through integration although some weaker candidates did incorrectly try to use the equation of a straight line. A number forgot about the $+c$ on the end of the integral and hence lost the final 2 marks.

Answers: **(i)** $9(3x+4)^{\frac{1}{2}} - 6$; **(ii)** Minimum; **(iii)** $y = \dfrac{4}{15}(3x+4)^{\frac{5}{2}} - 3x^2 - 8x - \dfrac{4}{15}$.

Question 9

Many good candidates were familiar with the required techniques in this question and a considerable number scored full marks. The key was to clearly identify the information provided in each part of the question. In part **(i)** *OA* is parallel to *OB* and an expression of the form *OA* = *kOB* is required. Those who realised this usually found the correct answer that $p = 2$ very quickly. A considerable number of candidates over complicated this part and spent a great deal of time fruitlessly with some very difficult equations. A significant number were unable to form the unit vector and answers of the form $c(2\mathbf{i}+\mathbf{j}+\mathbf{k})$ with $c = 1, 6, \dfrac{1}{6}$ and $\sqrt{6}$ were seen. Part **(ii)** seemed more familiar to many candidates and completely correct solutions were common. Some did however discard the $p = 0$ answer or incorrectly solved the quadratic to obtain only one value of p. A number of weaker candidates attempted *OA.OB* = 0 and this gained no credit. In part **(iii)** it was essential that candidates appreciated the cyclic nature of OABC and those who did soon realised that *OC* = *AB* to easily obtain the correct answer. A number thought that *OC* = *kAB* and spent time and effort usually unsuccessfully trying to find k.

Answers: **(i)** $2, \sqrt{6}\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$; **(ii)** 0 or 5; **(iii)** $\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$.

Question 10

Overall this question was the best done of the final 4 longer questions with many candidates scoring full marks. Part **(i)** was usually done well with only a few candidates showing either insufficient working to award the mark or making sign errors. Some candidates did use an excessive number of lines in order to arrive at the solution and receive the 1 mark available.

In part **(ii)** most candidates used the intended method of setting $b^2 - 4ac = 0$. A few then did $-(6 + k)^2$ rather than $(-(6 + k))^2$. If this error then led to non-integer values for k they rarely continued to part **(iii)**. Some candidates used $b^2 - 4ac > 0$ initially, but then often realised their error and found the 2 values of k. An alternative method was to differentiate the line and the curve and to set the gradients to be equal. Better candidates often scored full marks, but some weaker ones became confused by the meaning of the x values that they obtained.

In part **(iii)** some candidates confused k as x. Others did not use the values of k they had just found in the equation shown in part **(i)**, but reworked by equating again the curve and the line. This led to unnecessary work and sometimes to errors. Some candidates used the alternative method of equating the gradients of the line and curve as in part **(ii)**. This often worked very well, but again led to much repeated work with candidates re-finding the x and y values that they had already calculated in part **(ii)**. A number of candidates stopped when they had found the co-ordinates of A and B and did not find the equation of the line connecting them. Often this was candidates who had done much repeated work which had made the question very long for them.

Answers: **(i)** 2, 10; **(ii)** $(4, 2), (2, -2), y - 2 = 2(x - 4)$.

Question 11

Many candidates seemed to launch themselves into this question without pausing for thought. Many of them found the value for b in part **(i)** or in part **(iv)** as well (or sometimes instead of) in part **(ii)**. The wording in part **(i)**: 'State the value of a' should have suggested that this was relatively straightforward. Indeed it was anticipated that candidates would see that $y = 0$ and deduce that $x = 2$ without any working. Some candidates attempted to expand y only to then try to re-factorise it. The alternative method of differentiating and setting to 0 was often compromised by incorrect differentiation.

In parts **ii** and **iii** the most common error was attempting to differentiate or integrate the product $x(x - 2)^2$ as if each part was independent of the other. The incorrect answer, $2(x-2)$, for the differential was a common one, especially among weaker candidates. This received no credit. Although in part **(ii)** a few candidates did successfully use the product rule generally the most productive method was to expand y. This was also then often successfully used in part **(iii)** and integrated. The majority of candidates used the correct limits in part **(iii)** but a number did integrate between b and a rather than 0 and a.

In part **(iv)** many candidates did not realise that the minimum value of the gradient function was required. Even able candidates sometimes merely repeated the work from part **(ii)** and then substituted a or b into $\dfrac{d^2y}{dx^2}$ thereby receiving no further credit. Candidates who were successful used a variety of methods. These included setting $\dfrac{d^2y}{dx^2}$ equal to zero, or completing the square for $\dfrac{dy}{dx}$, or setting $\dfrac{dy}{dx}$ equal to m, then equating to 0 and using $b^2 - 4ac = 0$.

Answers: **(i)** 2; **(ii)** $\dfrac{2}{3}$; **(iii)** $\dfrac{4}{3}$; **(iv)** $\dfrac{-4}{3}$.

MATHEMATICS

Key messages

- Candidates need to be careful to read the question in detail and answer as indicated.
- Candidates need make sure they do not make approximations during their working, as this can impact on the accuracy of the final result.
- Candidates need to take care to avoid arithmetic and algebraic errors.

General comments

The marks obtained by candidates for this paper covered the entire range available. There were some high scoring scripts which showed a good coverage and understanding of the syllabus together with knowledge of how to apply the techniques learned appropriately. There were also some low scoring scripts which were evidence of candidates not being sufficiently prepared for the examination. Many candidates were able to attempt most if not all the questions on the paper. Candidates should be reminded to read the questions carefully and make sure that they are giving their final solutions in the required form and that they are also working to the required level of accuracy.

Comments on specific questions

Question 1

Most candidates chose to work with $(x-2)^2 \geq (x+5)^2$, usually obtaining the correct critical value. There were occasional errors with the inequality sign and arithmetic slips, but most candidates scored highly with this question. Those that chose the alternate approach of considering the possible linear equations were usually less successful especially when it came to deciding the direction of the inequality. Candidates could be encouraged to 'test' appropriate values in order to make a correct conclusion.

Answer: $x \leq -\dfrac{3}{2}$.

Question 2

A completely correct statement of $x\log 5 = (2x-1)\log 3$ was obtained by most candidates showing a good understanding of the basic laws of logarithms. Slips in algebraic manipulation rather than application and use of logarithms seemed to be the major cause of candidates not obtaining the final correct result.

Answer: 1.87.

Question 3

Use of the appropriate double angle formula was attempted by most, with those that chose to start with another form of the double angle formula usually manipulating their equation in order to obtain a quadratic equation in terms of $\cos\theta$. Algebraic and arithmetic slips were the main cause of incorrect quadratic equations. Candidates could be encouraged to ensure that they are considering the correct angles within the range, there were several instances of candidates stating incorrectly solutions of 60° and 120°.

Answer: 60°.

Question 4

(i) Many candidates were able to reach the given answer. Those who did not could be encouraged to check their working for errors.

(ii) Most candidates were able to equate the given gradient function to 3 and solve the resulting equation. Some candidates did not take account of the given information that $t < 0$ and chose the positive value of t rather than the negative value. Candidates need to answer as required as the exact coordinates of the point were needed and many chose to use their calculators and give a 3 significant figure answer.

Answer: **(ii)** (ln 3, -2).

Question 5

(i) A correct method for finding the area required was used by most candidates with occasional slips with the application of the limits correctly.

(ii) Although this part of the question was worth only 1 mark, candidates could be encouraged to give a short explanation with their response, such as 'Area of rectangle = ' rather than just use $\theta \cos \theta = 1 + \sin \theta$.

(iii) Most candidates were able to apply the iterative formula at least once, but many had their calculator in degree mode rather than radian mode. The information given at the start of the question stated that $0 \le \theta \le \dfrac{\pi}{2}$, implying that θ is in radians throughout the question. For those candidates using radians, many stopped short of the required number of iterations needed to be able to give the result correct to 2 decimal places.

Answers: **(i)** $1 - \sin \theta$; **(iii)** 0.56.

Question 6

(a) The trapezium rule was applied correctly by many candidates, with errors occurring mainly due to the miscalculation of the y ordinates. Again candidates could be encouraged to read the question carefully as some attempted straightforward integration.

(b) This part of the question proved to be one of the most difficult on the paper. Many attempted a 'reverse chain rule' or opted to integrate each term separately. There were very few scripts with a correct initial method applied (multiplication of the 2 brackets followed by a division of each term by e^{2x}) and often those candidates that did use a correct method were unable to integrate the exponential terms correctly.

Answers: **(a)** 0.11; **(b)** $x + 4e^{-x} - 2e^{-2x} + c$.

Question 7

(i) The great majority of candidates were able to score highly, showing a good understanding of remainder theorem and most were able to obtain the correct value of a and of b.

(ii) This part was more problematic. Most candidates attempted algebraic long division with varying degrees of success, most being able to obtain a quotient in the form of $2x - k$. The actual process of obtaining the remainder from the long division process proved to be more difficult with both algebraic and arithmetic errors being common.

Answers: **(i)** $a = -4, b = 6$, **(ii)** Quotient $2x - 4$, Remainder -2.

Question 8

(i) It was evident that some candidates had learned the given result by rote and did not know how to obtain it from $\dfrac{1}{\cos\theta}$, but many made correct attempts at differentiation either of a quotient or using the chain rule to obtain $\dfrac{\sin\theta}{\cos^2\theta}$ and hence the given result.

(ii) Different approaches were adopted, some choosing to use $\dfrac{\sin\theta}{\cos^2\theta}$ to obtain the second derivative - this approach meant a little more work in order to obtain the required result. Others chose to differentiate $\tan\theta\sec\theta$ usually with success, to obtain $\sec^3\theta + \tan^2\theta\sec\theta$. Many chose to leave their answer in this form believing it to be in the required form. Use of the appropriate identity was needed to substitute in for $\tan^2\theta$.

(iii) Candidates needed to re-write $1 + \tan^2\theta$ and also use the work from part **(i)** to re-write $3\sec\theta\tan\theta$ in order to integrate. The question specified an exact value, so candidates should realise that their answer must not contain rounded values obtained from a calculator.

Answers: **(ii)** $a = 2, b = -1$; **(iii)** $4 - 3\sqrt{2}$.

MATHEMATICS

> **Paper 9709/22**
> **Paper 22**

Key messages

- Candidates need to be careful to read the question in detail and answer as indicated.
- Candidates need to be careful to eliminate arithmetic and numerical errors in their answers.
- Candidates need to take care to avoid sign errors.

General comments

The paper produced a greatly varying standard of work from the candidates. There were many candidates who appeared insufficiently prepared for the examination from the standard of some of the responses produced. However, there were also a large number of very well prepared candidates who were very familiar with the syllabus content and who were able to produce responses that reflected this. Quite a few candidates were able to gain full marks. Work produced by the better prepared candidates was usually well presented and easy to follow. Candidates should be made aware of the importance of the accuracy of their work and to also take care when dealing with trigonometric ratios and calculus. Candidates should also be encouraged to check that their calculator is in the correct mode when dealing with trigonometric questions.

Comments on specific questions

Question 1

Most candidates adopted the approach of squaring both $2x + 1$ and $2x - 5$, which usually produced the correct critical value of -1. Whilst the great majority of these candidates were able to continue and obtain the correct inequality, a significant number were under the impression that a change of inequality sign was necessary when dividing by 24.

The approach of considering $2x + 1 = \pm(2x - 5)$ was less common and occasionally caused candidates problems when one of the cases did not yield a solution for x.

Graphical solutions were rare, but did usually yield the correct critical value.

Answer: $x < 1$.

Question 2

Most candidates attempted to differentiate a quotient, however there were quite a few who were unable to quote and use the correct quotient formula. Some attempted to differentiate an appropriate product, which was usually successful. Most errors involved missing factors of 2 from the differentiation of the separate terms. Few candidates were able to solve the resulting expression equated to zero. Whilst most realised that they needed to solve the trigonometric equation $\cos 2x - \sin 2x = 0$, or equivalent, the actual process of reducing to the form $\tan 2x = 1$ was not often seen. Answers in degrees and radians in decimal form were frequent, costing candidates a final accuracy mark. Candidates should be reminded to check that they are giving their answers in the required form, to avoid such losses.

Answer: $\dfrac{\pi}{8}$.

Question 3

(i) Reasonable attempts at this question were made by most candidates using either algebraic long division (the most common), an identity with constants to be found by equating coefficients or by observation. Marks were obtained by most although it was clear from some responses that some candidates were not familiar with the word 'quotient'. Most errors were usually the result of arithmetic slips in the long division process.

(ii) Most candidates were able to gain at least 2 marks by using either their quotient from part (i) or the given divisor. Combining the two results from factorisation and equating to zero correctly, was necessary to obtain full marks.

Answers: **(i)** $x^2 - x - 2$; **(ii)** $x = \pm 1, 2$.

Question 4

Many candidates lost marks in this question due to the fact that they had their calculator in degree mode rather than radian mode.

(i) Most errors occurred either in the calculation of the ordinates or the application of the correct interval width and number of ordinates in the trapezium rule.

(ii) The great majority of candidates were familiar with the application of an iterative formula, but use of the incorrect calculator mode yielded a very common response of 1.41. Candidates usually gave sufficient iterations to justify their answer.

Answers: **(i)** 1.84; **(ii)** 1.06.

Question 5

The most common approach by candidates was to re-write the given equation in the form $\ln y = \ln A - x \ln b$. A correct gradient was found by most, but often equated to $\ln b$ rather than $-\ln b$. Often an incorrect substitution of $\ln 2.9$ or $\ln 1.4$ meant that an incorrect method of reaching $\ln A$ was used. Alternative approaches included substitution into $\ln y = \ln A - x \ln b$, which was equally acceptable provided the correct values were substituted in, although many candidates again lost marks due to the incorrect use of $\ln 2.9$ and $\ln 1.4$. Equally acceptable was the method of substitution of $e^{2.9}$ and similar into the original equation. Candidates should be careful that they are using the correct values when substituting into equations of this type.

Answers: $A = 33.12$, $b = 1.82$.

Question 6

(a) Most candidates realised that the answer contained $e^{-\frac{1}{2}x}$, with most errors occurring in the calculation of the numerical factor involved.

(b) In spite of the answer of $\ln 16$ being given, some candidates did not recognise the logarithmic form of the integral and were not able then to make any meaningful progress. Of the candidates that did realise the connection with logarithms, some were unable to obtain the correct numerical factor that was also needed, but this did not prevent them from gaining method marks for the correct application of limits and laws of logarithms provided their working was clear.

Many candidates contrived to gain the given result and as a result lost method marks which they could otherwise have gained had they worked with their own results correctly. This was especially true for those candidates who obtained $6 \ln(3x - 1)$ or $3 \ln(3x - 1)$ rather than the correct $2 \ln(3x - 1)$.

Answer: **(a)** $-8e^{-\frac{1}{2}x} (+c)$.

Question 7

(i)   Most candidates were familiar with the implicit differentiation needed for this question. Some, however, made errors when it came to the implicit differentiation of a product, including sign errors. Some candidates contrived to obtain the given answer from their incorrect work by often changing signs to conform to the expected response.

(ii)   Most candidates realised that $3x - 2y = 0$ and often went on to obtain a quadratic equation in either x or y. However, many candidates thought that the tangent being parallel to the x-axis meant that $y = 0$ and so were unable to gain any further credit for correct work.

Answers: **(ii)** $(2, 3)$, $(-2, -3)$.

Question 8

(a)   Application of the correct formula usually meant that candidates were able to gain some credit. There were many sign errors in the formula and often candidates used a substitution of $\tan t$ rather than t. Most candidates were able to gain at least 2 marks for $\dfrac{t + \tan B}{1 - t \tan B} = 4$, but many were often unable to manipulate this equation to obtain $\tan B$ explicitly.

(b)   In this part there were many sign errors in the attempt at the trigonometric formula required. However, many candidates were able to reduce their result to a quadratic equation in $\tan x$ and go on to solve this correctly. There were quite a number of candidates who did not make use of any formula at all choosing instead to state incorrectly that $\tan(45° - x) = \tan 45° - \tan x$. These candidates were unable to gain any marks.

Answers: **(a)** $\tan B = \dfrac{4 - t}{1 + 4t}$;   **(b)** $18.4°, 116.6°, 198.4°, 296.6°$.

MATHEMATICS

Key messages

- Candidates need to be careful to read the question in detail and answer as indicated.
- Candidates need make sure they do not make approximations during their working, as this can impact on the accuracy of the final result.
- Candidates need to take care to avoid arithmetic and algebraic errors.

General comments

The marks obtained by candidates for this paper covered the entire range available. There were some high scoring scripts which showed a good coverage and understanding of the syllabus together with knowledge of how to apply the techniques learned appropriately. There were also some low scoring scripts which were evidence of candidates not being sufficiently prepared for the examination. Many candidates were able to attempt most if not all the questions on the paper. Candidates should be reminded to read the questions carefully and make sure that they are giving their final solutions in the required form and that they are also working to the required level of accuracy.

Comments on specific questions

Question 1

Most candidates chose to work with $(x-2)^2 \geq (x+5)^2$, usually obtaining the correct critical value. There were occasional errors with the inequality sign and arithmetic slips, but most candidates scored highly with this question. Those that chose the alternate approach of considering the possible linear equations were usually less successful especially when it came to deciding the direction of the inequality. Candidates could be encouraged to 'test' appropriate values in order to make a correct conclusion.

Answer: $x \leq -\dfrac{3}{2}$.

Question 2

A completely correct statement of $x\log 5 = (2x-1)\log 3$ was obtained by most candidates showing a good understanding of the basic laws of logarithms. Slips in algebraic manipulation rather than application and use of logarithms seemed to be the major cause of candidates not obtaining the final correct result.

Answer: 1.87.

Question 3

Use of the appropriate double angle formula was attempted by most, with those that chose to start with another form of the double angle formula usually manipulating their equation in order to obtain a quadratic equation in terms of $\cos\theta$. Algebraic and arithmetic slips were the main cause of incorrect quadratic equations. Candidates could be encouraged to ensure that they are considering the correct angles within the range, there were several instances of candidates stating incorrectly solutions of 60° and 120°.

Answer: 60°.

Question 4

(i) Many candidates were able to reach the given answer. Those who did not could be encouraged to check their working for errors.

(ii) Most candidates were able to equate the given gradient function to 3 and solve the resulting equation. Some candidates did not take account of the given information that $t < 0$ and chose the positive value of t rather than the negative value. Candidates need to answer as required as the exact coordinates of the point were needed and many chose to use their calculators and give a 3 significant figure answer.

Answer: **(ii)** (ln 3, -2).

Question 5

(i) A correct method for finding the area required was used by most candidates with occasional slips with the application of the limits correctly.

(ii) Although this part of the question was worth only 1 mark, candidates could be encouraged to give a short explanation with their response, such as 'Area of rectangle = ' rather than just use $\theta \cos \theta = 1 + \sin \theta$.

(iii) Most candidates were able to apply the iterative formula at least once, but many had their calculator in degree mode rather than radian mode. The information given at the start of the question stated that $0 \le \theta \le \dfrac{\pi}{2}$, implying that θ is in radians throughout the question. For those candidates using radians, many stopped short of the required number of iterations needed to be able to give the result correct to 2 decimal places.

Answers: **(i)** $1 - \sin \theta$; **(iii)** 0.56.

Question 6

(a) The trapezium rule was applied correctly by many candidates, with errors occurring mainly due to the miscalculation of the y ordinates. Again candidates could be encouraged to read the question carefully as some attempted straightforward integration.

(b) This part of the question proved to be one of the most difficult on the paper. Many attempted a 'reverse chain rule' or opted to integrate each term separately. There were very few scripts with a correct initial method applied (multiplication of the 2 brackets followed by a division of each term by e^{2x}) and often those candidates that did use a correct method were unable to integrate the exponential terms correctly.

*Answer*s: **(a)** 0.11; **(b)** $x + 4e^{-x} - 2e^{-2x} + c$.

Question 7

(i) The great majority of candidates were able to score highly, showing a good understanding of remainder theorem and most were able to obtain the correct value of a and of b.

(ii) This part was more problematic. Most candidates attempted algebraic long division with varying degrees of success, most being able to obtain a quotient in the form of $2x - k$. The actual process of obtaining the remainder from the long division process proved to be more difficult with both algebraic and arithmetic errors being common.

Answers: **(i)** $a = -4, b = 6$, **(ii)** Quotient $2x - 4$, Remainder -2.

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Question 8

(i)   It was evident that some candidates had learned the given result by rote and did not know how to obtain it from $\dfrac{1}{\cos\theta}$, but many made correct attempts at differentiation either of a quotient or using the chain rule to obtain $\dfrac{\sin\theta}{\cos^2\theta}$ and hence the given result.

(ii) Different approaches were adopted, some choosing to use $\dfrac{\sin\theta}{\cos^2\theta}$ to obtain the second derivative - this approach meant a little more work in order to obtain the required result. Others chose to differentiate $\tan\theta\sec\theta$ usually with success, to obtain $\sec^3\theta + \tan^2\theta\sec\theta$. Many chose to leave their answer in this form believing it to be in the required form. Use of the appropriate identity was needed to substitute in for $\tan^2\theta$.

(iii) Candidates needed to re-write $1 + \tan^2\theta$ and also use the work from part **(i)** to re-write $3\sec\theta\tan\theta$ in order to integrate. The question specified an exact value, so candidates should realise that their answer must not contain rounded values obtained from a calculator.

Answers: **(ii)** $a = 2, b = -1$;   **(iii)** $4 - 3\sqrt{2}$.

MATHEMATICS

| Paper 9709/31 |
| Paper 31 |

Key messages

- Attention needs to be paid to making sure workings are carried out at a sufficient level of accuracy to ensure the accuracy of the final answer.
- Candidates need to take care to avoid arithmetic and algebraic errors.
- Candidates need to be careful to read the question in detail and answer as indicated.

General comments

The standard of work on this paper varied considerably and resulted in a wide spread of marks from zero to full marks. No question or part of a question seemed to be of undue difficulty and most questions discriminated well. The questions or parts of questions that were generally done well were **Question 1** (inequality), **Question 4** (binomial expansion), and **Question 7 (i)** (implicit differentiation). Those that were done least well were **Question 5** (trigonometric identities), **Question 6** (differential equation), **Question 9 (ii)** (complex numbers), and **Question 10** (vector geometry). In general the presentation of work was good.

The detailed comments that follow draw attention to common errors and might lead to a cumulative impression of indifferent work on a difficult paper. In fact there were a fair number of scripts showing a complete understanding of all the topics being tested.

Where numerical and other answers are given after the comments on specific questions it should be understood that alternative forms are often acceptable and that the form given is not necessarily the only 'correct answer'.

Comments on specific questions

Question 1

This was generally well answered by a variety of methods. Most candidates squared the given inequality and then solved a non-modular quadratic equation or inequality. The main error here was the failure to square the factor of 3. In such cases, where the 3 was not squared, accurate algebra leads to the irrational critical values $(-10 \pm \sqrt{108})/2$. The appearance of irrational values in problems of this form indicates that an error has been made.

Answer: $\dfrac{2}{5} < x < 4$.

Question 2

This was well answered by those with a sound understanding of indices and logarithms, but there were also many poor attempts. Common errors included taking $\ln(5^x - 5)$ to be $x \ln 5 - \ln 5$ and, after letting $y = 5^x$, taking 5^{x-1} to be y^{-1}.

Answer: 1.14.

Question 3

Almost all began correctly and made some progress. However, candidates need to check they have not made a sign error when expanding expressions of the form cos(*A* − *B*). The subsequent rearrangement to obtain tan *θ* was relatively well done, but the final answer was often lost because the angle was not given to the required degree of accuracy or was incorrect because of premature approximations earlier on.

Answer: 105.9°.

Question 4

This was well answered by most candidates. When expanding $(1 + ax)^{-2}$ some failed to square or cube *a* when forming the coefficients of x^2 and x^3. In fact the distinction between 'term' and coefficient of a term in an expansion did not seem to be universally known and equations such as $2ax = 4ax^3$, or $2ax = 4a^3x^3$ were quite often seen. In part **(i)** the question states that the coefficients of *x* and x^3 are equal. The coefficients were frequently misread as those of x^2 and x^3 or those of x and x^2. Candidates need to be reminded of the need to read and respond to questions carefully.

Answers: **(i)** $\dfrac{1}{\sqrt{2}}$; **(ii)** $1 - \sqrt{2}x + \dfrac{3}{2}x^2$.

Question 5

This question was structured so as to help candidates to complete the final integration without difficulty and a minority of candidates did work through the four parts succinctly and successfully. However, the majority of candidates were challenged by this question. In part **(i)** the application of the quotient rule sometimes contained the error $\dfrac{d}{dx}(1) = 1$. In part **(ii)** many candidates lacked a sound strategy for tackling the identity, often giving up when they had converted the left hand side to $\dfrac{\cos x}{1 - \sin x}$. Lack of an overall strategy was a problem in part **(iii)** and in part **(iv)** most candidates did not use the result of part **(i)** to write down the integral of sec *x* tan *x*.

Question 6

This question was poorly answered by most candidates. Having separated variables correctly, integrated one side with respect to *x* and obtained the term ln *x*, the majority of candidates failed to produce any work of value. The integral $\dfrac{1}{1 - y^2}$ of was usually taken incorrectly to be $\dfrac{\ln(1 - y^2)}{-2y}$ or simply ln(1 − y^2). Apart from a very small group who applied the formula for integrals of this type, successful candidates realised that they had to (a) factorise 1 − y^2 and (b) express $\dfrac{1}{1 - y^2}$ in partial fractions, before integrating with respect to *y*.

These candidates usually obtained a correct solution in some form, but the removal of logarithms involved in making *y* the subject proved difficult for some.

Answer: $y = \dfrac{x^2 - 4}{x^2 + 4}$.

Question 7

Part **(i)** was well answered by many candidates. Part **(ii)** on the other hand was found to be difficult. The question states that the tangent is parallel to the axis of *y*. The majority of candidates could not interpret the statement correctly in algebraic terms and equated the gradient variously to 0, or to 1, or to *x*, or to *y*, and scored nothing. The few successful attempts usually began by equating the denominator of the expression for $\dfrac{dy}{dx}$ to zero, or occasionally, forming $\dfrac{dx}{dy}$ and equating that to zero. The subsequent calculations of the coordinates were nearly always correct.

Answer: **(ii)** (5.47, 0.693).

Question 8

(i)   Most candidates appeared to combine use of the chain rule with either the product or quotient rule and the initial attempt at the first derivative was often correct. Errors in differentiating $e^{-\frac{1}{2}x^2}$ were common. In the work seeking the required x-value, mistakes in algebraic manipulation were frequent, e.g. cancelling $(1+2x^2)^{\frac{1}{2}}$ and $(1+2x^2)^{-\frac{1}{2}}$.

(ii)   This task discriminated well. Many began by stating a suitable starting point such as $\alpha = \sqrt{(\ln(4+8\alpha^2))}$ (1) or $0.5 = e^{-\frac{1}{2}\alpha^2}(1+2\alpha^2)^{\frac{1}{2}}$ (2). Some candidates progressed to an intermediate stage such as $e^{\alpha^2} = 4+8\alpha^2$, but few completed the transition from (1) to (2) or *vice versa*.

(iii)   Most candidates answered this part very well.

Answers: **(i)** $\dfrac{1}{\sqrt{2}}$; **(ii)** $\alpha = \sqrt{(\ln(4+8\alpha^2))}$; **(iii)** α = 1.86.

Question 9

(i)   Many scored well on this part, with x^4 and x^2 being expanded correctly in most attempts. It was surprising how many left x^2 as $1 + 2\sqrt{2}\ i - 2$ when further squaring it to obtain x^4. Those who simplified it to $-1 + 2\sqrt{2}\ i$ before continuing had less to do and were less prone to error. A few candidates deduced that it would be sufficient to show that $x^2 - 2x + 3$ was a factor of p(x) and achieved a neat solution which later helped them gain credit in part **(ii)**.

(ii)   Only the strongest candidates completed this successfully. They usually used the known roots $1 \pm \sqrt{2}\ i$ to deduce or write down the factor $x^2 - 2x + 3$. Division of p(x) by this factor or the method of unknown coefficients then gave the other factor $x^2 + 2x + 3$ and hence the other two roots. A neat alternative way of finding the factor $x^2 + 2x + 2$ was to divide p(x) successively by $x - 1 - \sqrt{2}\ i$ and $x - 1 + \sqrt{2}\ i$ using Synthetic Division or Long Division.

Answers: **(i)** $1 - \sqrt{2}\ i$; **(ii)** $-1 + i, -1 - i$.

Question 10

In general this question was poorly answered. Both the parts were soluble by routine methods, yet candidates did not apply them to the right vectors.

In part **(i)** the unknown plane is parallel to $\overrightarrow{OC}$, hence its normal is perpendicular to $\overrightarrow{OC}$. The plane contains $\overrightarrow{AB}$ hence its normal is perpendicular to $\overrightarrow{AB}$. Thus, there is a choice of methods based on (a) a pair of zero scalar products of the unknown normal with $\overrightarrow{AB}$ and with $\overrightarrow{OC}$, (b) the vector product of $\overrightarrow{AB}$ and $\overrightarrow{OC}$, or (c) a two parameter equation involving the vectors $\overrightarrow{AB}$, $\overrightarrow{OC}$ and the position vector of a point on $\overrightarrow{AB}$, e.g. $\overrightarrow{OA}$. Yet many candidates did not grasp the geometry of the situation and worked with irrelevant pairs of direction vectors such as $\overrightarrow{AB}$ and $\overrightarrow{AC}$, or $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

A similar situation arose in part **(ii)**, though on the whole candidates did better in this section. There was a considerable variety of approaches. The most popular method was to take a general point P on $\overrightarrow{AB}$ with parameter λ say, equate the scalar product $\overrightarrow{CP}.\overrightarrow{AB}$ to zero, solve for λ and calculate $\overrightarrow{CP}$. Here again candidates applied the method to wrong pairs of vectors, e.g. $\overrightarrow{CP}$ and $\overrightarrow{OC}$, or $\overrightarrow{OP}$ and $\overrightarrow{AB}$.

Answers: **(i)** $3x + z = 13$; **(ii)** $3\sqrt{2}$.

MATHEMATICS

| Paper 9709/32 |
| Paper 32 |

Key messages

- Attention needs to be paid to making sure workings are carried out at a sufficient level of accuracy to ensure the accuracy of the final answer.
- Candidates need to take care to avoid arithmetic and algebraic errors.
- Candidates need to be careful to read the question in detail and answer as indicated.

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Answer: $\dfrac{2}{5} < x < 4$.

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Answers: **(i)** $1 - \sqrt{2}$ i; **(ii)** $-1 + $ i, $-1 - $ i.

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Answers: **(i)** $3x + z = 13$; **(ii)** $3\sqrt{2}$.

MATHEMATICS

Paper 9709/33
Paper 33

Key messages

- Candidates need to be careful to read the question in detail and answer as indicated.
- Candidates need to take care to avoid numerical and algebraic errors.
- Marks depend on giving answers which are correct to 3 significant figures. In order to gain as many marks as possible all working should be either exact or correct to at least 4 significant figures.

General comments

The majority of candidates offered responses to all ten questions on this paper. Many candidates were clearly tackling their strongest topics first, but it was more common to see the questions worked in the order set. A full range of achievement was seen, with several candidates achieving full marks. The responses to **Question 8**, on vectors, were particularly good, but the attempts at solving a differential equation using separation of variables (**Question 4**) and integration by substitution (**Question 7**) were less successful. The candidates were very proficient in dealing with familiar tasks, but questions with a less familiar format (**Question 6**, the end of **Question 7** and the end of **Question 10**) proved to be quite challenging.

All candidates should be reminded of the need to take great care in reading the questions, both in terms of ensuring that they find what is required, and to guard against misreads. In **Question 1** it was common to see answers giving e in terms of *x*. In **Question 9** there were several misreads, some of which changed the nature of the question.

Comments on specific questions

Question 1

Many fully correct responses were seen. The most popular approach was to start by combining $\ln(x+5)$ and $\ln x$ and then go on to use $\ln \mathrm{e} = 1$. Some candidates who started well did not reach an expression for *x* in terms of e, usually because they stopped as soon as they had an expression for e in terms of *x*.

A significant minority of candidates demonstrated a weak understanding of the laws of logarithms.

Answer: $\dfrac{5}{\mathrm{e}-1}$.

Question 2

(i) The majority of candidates found the values of *R* and *α* correctly. The most common errors were to state $\tan \alpha = \dfrac{-7}{24}$, $\tan \alpha = \dfrac{24}{7}$, $\sin \alpha = 7$ or $\cos \alpha = 24$, without showing any other working. Some candidates lost a mark because they did not give the value of *α* to the required degree of accuracy. The value of *R* needed to be stated as part of the solution here, but sometimes it did not appear until part **(ii)**.

(ii) Most candidates recognised the need to find the value of $\sin^{-1}\left(\dfrac{17}{25}\right)$. Many correct answers were seen, but full marks were only available for a fully correct solution.

Answers: **(i)** 25 sin (*θ* – 16.26°); **(ii)** 59.1°.

Question 3

(i) Most candidates demonstrated a good understanding of how to work with equations in parametric form. Many had correct methods for finding $\dfrac{dy}{dt}$ and $\dfrac{dx}{dt}$, but there were many numerical and algebraic slips. It was common to see the incorrect answer $\dfrac{dy}{dt} = \dfrac{2}{2t+3}$. The application of the chain rule was usually correct, but many solutions already had errors by this point.

A small number of candidates started by eliminating the parameter to form a Cartesian equation. Some of these did not then do as the question asked and express $\dfrac{dy}{dx}$ in terms of t.

(ii) There were many correct answers, and many candidates used the correct value for t in their expression for $\dfrac{dy}{dx}$. Several candidates assumed that when $x = 1$, t will also equal 1. Another common problem was candidates who went from $2t = 3$ to $t = \dfrac{2}{3}$.

Answers: **(i)** $\dfrac{1}{3}(2t+3)$; **(ii)** 2.

Question 4

There were many concise and correct solutions to this differential equation, but also a number of candidates struggling with the algebra or simply not aware that they needed to start by separating the variables. Candidates who separated the variables correctly often went on to give fully correct solutions. The most common problem with the integration was that candidates did not recognise $\displaystyle\int \dfrac{x}{x^2+4}\,dx$ as a logarithmic function. Those candidates who integrated correctly usually went on to find the constant of integration correctly, but the final step of transforming an expression for $\ln y$ into an expression for y proved more difficult, usually because the constant was not dealt with correctly.

Answer: $y = \dfrac{1}{2}\left(x^2+4\right)^3$.

Question 5

(i) Most candidates recognised the need to use the product rule, and it was common to see a correct expression for $f'(x)$, but a number of candidates appeared to have misunderstood the notation and attempted to find $f^{-1}(x)$. Errors in differentiation usually involved the use of $-\dfrac{1}{2}$ in place of -2.

There were a number of errors in the substitution, commonly involving the use of $\dfrac{1}{2}$ in place of $-\dfrac{1}{2}$ or giving the answer in decimal form rather than the exact form requested.

(ii) Most candidates demonstrated an understanding of the method for integration by parts, but there were several errors with the factors and signs. The limits on the integral were usually used correctly, but here again there were problems in simplifying the terms due to the large number of negatives involved. Some candidates mistakenly took the absolute value of their answer at the end, presumably thinking that they were finding an area. Candidates need to be reminded that the use of decimal approximations is not acceptable when they are asked to find an exact value for an integral.

Answers: **(i)** 6e; **(ii)** $-\dfrac{3}{4}$.

Question 6

(i)   Given that they had been told that α was a negative integer, it was surprising not to find more candidates considering f(-1) and f(-2). Those candidates who attempted to use the factor theorem were often successful, but many candidates could not find a way to start this question. There were many attempts to use calculus or algebraic manipulation which led nowhere. A few candidates using algebraic manipulation did arrive correctly at the conclusion that $(x + 2)$ is a factor of the quartic, but there were many incorrect approaches involving false assumptions.

(ii)   Some candidates with a factor $(x + 2)$ in **(i)** went on to find the cubic factor correctly, but they did not always go on to rearrange this and obtain the given answer. Candidates working back from the given answer frequently reached the factor $x^3 + 2x - 8$, but very few went on to show that this is a factor of the quartic.

(iii)   This part of the question was answered well by many candidates. A few had difficulties because they changed from $\sqrt[3]{(8 - 2x)}$ to $\sqrt[3]{(8 - x)}$ after the first iteration. Some candidates lost marks through not working to the required degree of accuracy, commonly working to 4 significant figures rather than to 4 decimal places.

Answers: **(i)** –2; **(iii)** 1.67.

Question 7

(i)   Most candidates found this question very challenging.   Having been given the substitution $u = \sin 2x$, many candidates did score the mark for finding $\dfrac{\mathrm{d}u}{\mathrm{d}x}$ correctly, although only a minority of candidates made any further progress. The double angle caused several problems with the substitution. Some candidates were able to use $\cos^2 2\theta = 1 - \sin^2 2\theta$ correctly, but some lost their way attempting to use the double angle formulae and there were many incorrect statements of trigonometrical formulae. Some candidates who dealt correctly with the trigonometry then lost the factor of $\dfrac{1}{2}$. The error $u^3 \times u^2 = u^6$ was quite common. Of the candidates who completed the integration correctly, many did not identify the correct limits for their integral, so the correct value for *A* was only found by a minority of candidates.

(ii)   Many candidates thought that this part of the question required further integration, despite the allocation of only two marks. Success in part **(ii)** was often unrelated to success in part **(i)**, but in general only the stronger candidates recognised that they were being asked to consider the symmetry of the function.

Answers: **(i)** $\dfrac{1}{24}$; **(ii)** 10.

Question 8

Many candidates scored full marks on this question. The most common cause of errors was candidates misreading the given equations or disregarding minus signs.

(i)   The method of expressing the coordinates of a general point on each line in parametric form was well understood. Many candidates obtained correct values for *s* and *t* at the point of intersection, although arithmetic errors were common. Having been asked to find both the value of *p* and the coordinates of the point of intersection, some candidates found only one of these.

(ii)   Although candidates used a variety of methods to find the normal vector, the most common approach was to use the vector product of the direction vectors of the two lines. This was often completed correctly, although there were slips, and some candidates used position vectors rather than direction vectors. Having obtained the normal vector, most candidates went on to obtain the equation of the plane correctly.

Answers: **(i)** 9, (7, −1, 2); **(ii)** $11x - 10y - 7z = 73$.

Question 9

(i)   Those candidates who started with a correct form for the partial fractions often completed the process correctly. Some candidates started with an additional constant term, but could still score full marks provided they concluded that this was zero. The element which caused most difficulty was the denominator $\left(1 + x^2\right)$ of one of the partial fractions. Some treated it as $\left(1 - x^2\right)$, and some treated it as $(1 + x)^2$. If the denominator of this fraction was correct, part of the numerator was sometimes missing.

(ii)   Many candidates dealt with the binomial expansion very proficiently. Some candidates struggled with the expansion of $(3 - x)^{-1}$, frequently writing it as $3\left(1 - \dfrac{x}{3}\right)^{-1}$ or $3(1 - x)^{-1}$, but most difficulties were caused by the expansion of $\dfrac{-2x + 1}{1 + x^2}$ because the numerator was frequently miscopied as $-(2x + 1)$.

Answers: **(i)** $\dfrac{6}{3 - x} + \dfrac{-2x + 1}{1 + x^2}$; **(ii)** $3 - \dfrac{4}{3}x - \dfrac{7}{9}x^2 + \dfrac{56}{27}x^3$.

Question 10

Most candidates were able to complete at least one part of this question correctly, but fully correct solutions were unusual.

(a) Many candidates reached the equation $iw^2 = -8i$, and the majority of these found one solution for w correctly, but large numbers of candidates overlooked the second square root. Of the few candidates who adopted the alternative approach of starting from $\sqrt{i}w = 2 - 2i$ only a small number had a method for dealing with $\sqrt{i}$.

(b)(i) The majority of candidates recognised that the inequality represented the inside of a circle of radius 2, and most of these placed the centre in the first quadrant. The third quadrant or on one of the axes were the most common incorrect positions for the centre. Several candidates drew a correct circle but then gave no indication of whether the required region was inside or outside of the circle.

(ii) Only a minority of candidates demonstrated a clear understanding of the geometry involved in this part of the question. Few diagrams gave any indication of where the candidates were looking to find the values for p and q, although several did show tangents passing through the origin. Some candidates were able to find p, but not q. When trying to find $\arg z$ there were many attempts which incorrectly assumed contact between circle and tangents at $(4 + 2i)$ and $(2 + 2i)$.

Answers: **(a)** $\pm i\sqrt{8}$; **(b) (ii)** 3.66, 7.66, 0.424, 1.15.

MATHEMATICS

Paper 9709/41
Paper 41

Key messages

- Answers should be tidy and legibly presented in the examination.
- Candidates need to take care to avoid arithmetic and sign errors.
- It is important for candidates to read the question carefully, and answer as required.

General comments

Candidates should take care with the presentation of their work. This helps the candidate with the process of developing the work in a question from start to finish and in the paper as a whole, from question to question.

Comments on specific questions

Question 1

This was one of the best attempted questions. Candidates who realised that the time of motion for the object and the time of motion for the target are the same, usually answered the question correctly.

Answer: 35 m.

Question 2

This question was a good source of marks for the better candidates.

Answers: **(i)** 1.68 N; **(ii)** 1.2 N.

Question 3

Candidates who attempted this question recognised the need to resolve forces perpendicular to the plane and parallel to the plane in parts **(i)** and **(ii)** respectively. However, there were many wrong answers in parts **(i)** and **(ii)**, leading to an incorrect answer in **(iii)**.

Answers: **(i)** 4.63 N; **(ii)** 0.824 N; **(iii)** 0.178.

Question 4

Candidates who made progress in the first part of the question did so by resolving forces in the 'x' and the 'y' directions, but frequently incorrect values were found. There was some confusion in attempting to proceed to find R and θ. A few candidates gave the answer for θ as 56.1°, understandably so because the symbol θ is often used as the angle between a resultant and the positive x-axis.

Answer: $R = 5.37$, $\theta = 33.9°$.

Question 5

(i) This part was the best attempted among all the parts of all the questions.

(ii) Very few candidates made any progress in this part.

Answers: **(i)** 60 s, 210 m; **(ii)** $k = 0.0000648$, 14 ms^{-1}.

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Question 6

(i)   Most candidates recognised the need for the use of the principle of conservation of energy. However very few candidates obtained the correct answer for the decrease in speed.

(ii)   Very few candidates made any progress at all in this part.

Answers: **(i)** 1.16 ms^{-1}; **(ii)** 1.41 ms^{-1}.

Question 7

(i)/(ii)   These parts were sources of marks for the more able candidates.

(iii)   Very few candidates made progress in this part.

Answers: **(i)** 0.4 ms^{-2}; **(ii)** 742.5 m.

MATHEMATICS

| Paper 9709/42 |
| Paper 42 |

Key messages

- Answers should be tidy and legibly presented in the examination.
- A considerable proportion of the total marks in this paper depend on giving answers which are correct to 3 significant figures. In order to gain as many marks as possible all working should be either exact or correct to at least 4 significant figures.

General comments

The standard of presentation was generally good, although there were exceptions. Candidates should not draw ruled lines downward in the middle of the page on which they are working. Work on different questions must not be shown alongside each other on different sides of the same page.

The demand for answers to be correct to three significant figures is usually met in the sense that the answer calculated is given to three significant figures or more. However, the most common reason for failing to obtain an answer correct to 3 significant figures is premature approximation. Working with intermediate figures that are correct to three significant figures does not guarantee a final value correct to three significant figures. Candidates are advised to retain intermediate values on the calculator until the next stage toward the required answer is found.

In part **(i)** of **Question 6** very many candidates obtained an answer of 129 kJ or 130 kJ, which does not round to the correct answer to three significant figures of 128 kJ. The incorrect answer arises from premature approximation and led to the loss of the relevant accuracy mark, unless a mark had already been lost by the candidate as a result of an earlier premature approximation.

Comments on specific questions

Question 1

Almost all the candidates answered this question correctly. The usual error made by the very small number of candidates who failed to score full marks was to use 'Work Done. = Fd sinα' instead of 'Work Done = Fd cosα'.

Answer: 1090 J.

Question 2

This question was well attempted.

(i) Almost all candidates answered this part correctly and many recognised the need for the use of the answer in the applications of Newton's second law in the next part.

(ii) This part was well attempted, although many candidates made the mistake, on applying Newton's second law to particle B, of having for the 'mass × acceleration' term as $2m$ instead of $2(1 - m)$.

The mistake of reading the mass of B as $(m - 1)$ instead of $(1 - m)$ arose occasionally.

Answers: **(i)** 2 ms^{-2}; **(ii)** $m = 0.6$, tension = 4.8 N.

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Question 3

This question was well attempted, but less well than **Questions 1** and **2**. Almost all candidates obtained the equation $5u + 12.5a = 55$. However very many such candidates also obtained $5u + 12.5a = 65$, without seeming to appreciate the contradiction.

The number of candidates who scored two marks for the first of the above equations, but failed to score further marks, is approximately the same as the number of candidates who scored all six marks for the question.

Some candidates asserted that the speed is constant and equal to 11 ms^{-1} (55/5) from *A* to *B* and 13 ms^{-1} (65/5) from *B* to *C*. This often led to $a = 0.4$ from incorrect methods.

Answer: $a = 0.4$, $u = 10$.

Question 4

There seemed to be many candidates who did not have a clear understanding of the concepts of magnitude, component and resultant, all of which are very important in the study of Mechanics.

(i)   The candidates who understood the requirements of the question and, in the process of answering the question, found each of the angles that the forces of magnitudes 68 N and 100 N make with the x-axis, were more numerous than those who used Pythagoras' Theorem.

Very many of such candidates obtained the required components as -32, 75 and -28.1, the error in the third component arising from premature approximation of the relevant angle.

(ii)   Candidates who understood the concepts usually obtained correct values for the *x*-component and the *y*-component of the resultant of the three forces.

Candidates who reached this stage usually applied $R^2 = X^2 + Y^2$ and $\theta = \tan^{-1}(Y/X)$ satisfactorily.

Answers: **(i)** -32, 75, -28; **(ii)** Magnitude 39 N, direction 22.6° anticlockwise from the positive *x*-axis.

Question 5

A critical element of this question is to realise the importance of the answer in part **(i)**, when answering part **(ii)**.

(i)   This part was very well attempted and most candidates obtained $\theta = 30$, using a correct method.

(ii)   Most candidates recognised the need to find the acceleration of the particle while moving from *B* to *C*. A few candidates just used $a = 5$, continuing with the same value as that for the movement from *A* to *B*.

A very significant number of candidates obtained $a = -(5÷6)$, failing to subtract 0.8s from 4.8s in finding the time taken by the particle in moving from *B* to *C*.

Almost all candidates used Newton's second law and $F = \mu R$ appropriately, albeit with an error in the value of the acceleration in many cases.

Answers: **(i)** 30°; **(ii)** 0.693.

Question 6

As expected for later questions, this question proved to be significantly more difficult than any of the earlier questions. Most candidates used or attempted to use, at some stage, the linear relationship:-
'Kinetic energy gain = Work done by the car's engine – Potential energy gain – Work done against resistance'.

(i) With the subject of the above relationship as written being 'kinetic energy gain', many candidates assumed this form would be fruitful in finding the required answer to this part. Unfortunately candidates taking this approach made no progress until using a different approach.

Most candidates did realise that the use of Newton's second law and $P = Fv$ are necessary, and many were successful in obtaining the values of v at the bottom and the top of the hill. Unfortunately a few candidates did not use Newton's second law and used the given values of the accelerations as the values of the velocities.

A very considerable proportion of candidates, who used Newton's second law and $P = Fv$, failed to include the component of the weight of the car in applying Newton's second law. A smaller proportion of candidates omitted not only the component of the weight of the car, but also the resistance to the car's motion.

Among those who did include the component and the resistance, many used $12500 \sin 3.4°$ for the component instead of $12500 × (30 ÷ 500)$. Such candidates obtained the eventual answer of 130 000 J to 3 significant figures, instead of the correct answer 128 000 J to 3 significant figures.

(ii) Most candidates used the linear relationship referred to above, with 'Work done by the car's engine' the subject instead of 'Kinetic energy gain'. The mark for 'PE gain = 1250g × 30 and WD against resistance = 1000 × 500' was scored by almost all candidates who attempted this part of the question. A value of KE gain was brought forward from part **(i)** and a very high proportion of candidates scored all three marks in this part, many benefitting from the 'follow through' attached to the final accuracy mark.

Answers: **(i)** 128 kJ; **(ii)** 1000 kJ.

Question 7

Some candidates who answered **Questions 1** to **6** very well did not answer **Question 7** well. In contrast, some candidates who failed to score well in **Questions 1** to **6** could score well in **Question 7**.

Many candidates failed to answer part **(i)** satisfactorily, so it is pleasing that candidates were able to use the given value of k in the subsequent parts.

A value of t is significant in each of the four parts of the question. The correct values are 40, 60, 60 and 80 respectively, but very few candidates used the correct value in all of the four parts.

It is disappointing to report that some candidates used constant acceleration formulae in this question.

(i) It seemed that for many candidates it is not obvious that $dv/dt = 0$ when P is at its maximum velocity. This gave rise to difficulty when solving for k with $k(60 \times t_{at\,max}^2 - t_{at\,max}^3) = 6.4$.

(ii) Many candidates obtained the answer 128 m by using the same value of t as used in part **(i)**. Others used a constant acceleration formula, but the majority of candidates answered correctly.

(iii) Most candidates differentiated correctly to find $a(t)$, then used the same value of t that was used in **(ii)**.

(iv) This proved to be the most difficult part. Very few appreciated the need to integrate $v(t)$ and thus did not find $t = 80$ for the time when reaching O after P had travelled from O to A and back to O.

Some candidates used $t = 120$, presumably believing that the length of time outward to A is the same as the time back to O.

Answers: **(ii)** 216 m; **(iii)** 0.72 ms^{-2}; **(iv)** 25.6 ms^{-1}.

MATHEMATICS

Key messages

- Candidates should ensure that answers are given correct to 3 significant figures as required and they should be aware that early approximation can affect the accuracy of final answers (e.g. **Question 3** part **(ii)** and **Question 6**).
- A complete force diagram can assist in forming accurate equations of motion (e.g. **Question 7** part **(i)**).
- Candidates need to understand that for a particle in equilibrium on an inclined plane (as in **Question 6**) there are <u>two</u> possible directions for the frictional force.

General comments

The paper allowed candidates to show their knowledge with some parts of each question being accessible to most candidates whilst a few parts challenged even the strongest candidates. **Questions 2**, **3** and **5** part **(i)** were the most well answered questions. **Question 1, Question 5** part **(iii)**, **Question 6** and **Question 7** part **(iii)** were found to be more difficult.

Comments on specific questions

Question 1

This was one of the less well answered questions. Candidates generally recognised that they needed to consider energy but the semi-circular situation caused some confusion.

(i) Many candidates found the required difference between potential energies at *A* and *C*. Some, however, found the potential energy at only one of *A* or *C* and some added instead of calculating the difference. Others unsuccessfully attempted to use the arc length to find the work done.

(ii) Candidates who used KE gain = PE loss – WD against resistance often obtained the correct answer, although errors occurred from e.g. division by 0.6 instead of multiplication when finding the work done. Some of the work / energy equations seen contained two instead of three terms and some contained sign errors leading to answers such as 7.07 ms^{-1} (from $0.8(10)(2.5)= \frac{1}{2} (0.8)v^2$) and 2.90 ms^{-1} (from $0.6 \times 5.6 = \frac{1}{2} (0.8)v^2$).

Answers: **(i)** 5.6 J; **(ii)** 6.45 ms^{-1}.

Question 2

The majority of candidates recognised that differentiation was needed in part **(i)** to find acceleration and that integration was needed in part **(ii)** to find the displacement. Fully correct solutions were often seen.

(i) Although differentiation was usually completed correctly, 'initial' acceleration was not always understood. Some candidates made no further progress; others attempted to use $a = 0$ rather than $t = 0$. The question contained the answer $t = 25$. The best solutions showed an equation formed in t and solved to obtain $t = 25$ rather than starting with $t = 25$ and verifying that the acceleration then was $2.5 \times$ initial acceleration.

(ii) Solutions were frequently correct. Errors seen included the mis-copy of $v = 0.2t + 0.006t^2$ as $v = 0.2t + 0.06t^2$, leading to $s = 375$ m, and the use of an incorrect limit e.g. $t = 33\frac{1}{3}$ (from $v = 0$) rather than substitution of $t = 25$.

Answer: **(ii)** 93.8 m.

Question 3

This question was often well answered, with appropriate use of the constant acceleration formulae for motion under gravity.

(i) A variety of constant acceleration formulae were used, usually to obtain the distance OA (3.2 m) and then the speed for which the particle was either 1.6 m from O moving upwards or from A moving downwards. Some candidates believed erroneously that half of the distance would take half of the time despite acceleration. A final answer of 5.66 ms^{-1} was expected (three significant figures) rather than $4\sqrt{2}$ ms^{-1}.

(ii) The majority used $v = u - gt$ but some found the time taken from A to the midpoint rather than from O to the midpoint. Those who used $s = ut + \frac{1}{2}at^2$ to find the time taken did not always choose between the two solutions obtained. The time was sometimes stated as 0.23 s rather than a three significant figure answer as required.

Answers: **(i)** 5.66 ms^{-1}; **(ii)** 0.234 s.

Question 4

The most usual correct method seen was to resolve forces horizontally and vertically and then to solve the pair of simultaneous equations. Occasional errors were made when calculating angles in the two key triangles, in mixing sine and cosine or in assuming that the tensions in the two strings were the same. More commonly, errors occurred during the solution of the two simultaneous equations. It was also common to see two separate 'systems' each supporting 21 N (e.g. $T_1 = 21/\cos 67.4$ leading to $T_1 = 54.6$ and $T_2 = 21/\cos 36.9^\circ$ leading to $T_2 = 26.3$).

Answer: $S_1 = 13$ N, $S_2 = 20$ N.

Question 5

(i) The increase in kinetic energy was usually calculated accurately. Occasionally candidates calculated $\frac{1}{2}m(v-u)^2$ instead of $\frac{1}{2}m(v^2-u^2)$.

(ii) Since the question said 'hence find .....', candidates were expected to use their answer to part **(i)**. Many, however, used Newton's Second Law $F = ma$ with $a = g\sin\theta$ as an alternative method, often gaining a correct answer but not full marks. Some candidates who did equate the increase in kinetic energy to the decrease in potential energy found the change in height (2 m) rather than the distance AB.

(iii) The 'horizontal' force was very often misinterpreted as a force directed up the plane. Consequently the incorrect answer 20.8 N was frequently seen.

Answers: **(i)** 240 J; **(ii)** 11.5 m; **(iii)** 21.2 N.

Question 6

In this question the diagram showed a force of P N directed up the plane. The candidate was then left to consider the possible effects of friction. Most were able to gain some marks, usually by assuming that the direction of the frictional force was down the plane. Full marks required a clear understanding that the direction of the frictional force could be either up or down the plane and also that for equilibrium, given a frictional force (F) and normal reaction (R), $F \leq \mu R$.

It was usual to see answers of $P = 4.49$, $P = 4.50$ or $P = 4.5$ from candidates who directed the frictional force down the plane. 4.50 and 4.5 were obtained either from rounding too soon (1.96 + 2.54) or from answers corrected to two instead of three significant figures. Of those who also considered the frictional force directed up the plane to obtain $P = 0.578$, only a few understood that there was a range of values rather than just one or two possible values for P.

Answer: $0.578 \leq P \leq 4.49$.

Question 7

(i) Two equations for the acceleration (a) and the tension (T) were generally formed and then solved simultaneously. The most common error was to see the connected particles treated as if they both hung vertically so that the equation of motion for particle B was given as $T - 0.48g = 0.48a$ or $0.48g - T = 0.48a$ instead of $T = 0.48a$.

(ii) Having found the acceleration of A, candidates were usually able to use one or more appropriate constant acceleration formulae to calculate the time taken for A to reach the floor. Since $a = -2$ was often found in part **(i)**, $s = \frac{1}{2}at^2$ led to the solution of $0.98 = -t^2$. This was commonly dealt with either by using $a = 2$ instead of $a = -2$ or by following $0.98 = -t^2$ with $t = \sqrt{0.98}$. Candidates could benefit from considering whether or not a negative value suggests an error in their earlier work.

(iii) Candidates often attempted this part of the question but the majority assumed acceleration for both stages of motion, rather than appreciating that when the string becomes slack, the speed of B becomes constant. Some split the motion into two parts and used their answer from part **(ii)** whilst others worked erroneously with constant acceleration for the 1.4 metres.

Answers: **(i)** $a = 4$ ms^{-2}, $t = 1.92$ N;   **(ii)** 0.7 s;   **(iii)** 0.85 s.

MATHEMATICS

Key messages

- Candidates need to make full use of the supplied list of formulae.
- Candidates need to take care to avoid arithmetic and sign errors.
- Answers should be tidy and legibly presented in the examination.

General comments

The questions that candidates found easier were 1, 2**(i)** and 3**(i)**. The questions that candidates found harder were 2**(ii)**, 5**(ii)**, 6**(ii)** and 7

A formula booklet is provided. Many incorrect formulae were seen; it is recommended that candidates refer to this booklet.

Comments on specific questions

Question 1

The better candidates used the correct formula to find the distance of the centre of mass *OG* from *O* and then used $v = r\omega$ with $r = OG$. Weaker candidates simply used the radius of the semi-circle and $v = r\omega$; no mention of the centre of mass was seen.

Answer: $v = 0.382$ ms^{-1}.

Question 2

(i) Most candidates realised that they needed to take moments about *B*. Better candidates scored all 3 marks. Weaker candidates had difficulty in finding the perpendicular distance from *B* to the force.

(ii) This part of the question could be solved by resolving horizontally and vertically to find the friction force, *F*, and the normal reaction, *R*. It was then necessary to use $F = \mu R$ to find *μ*, the coefficient of friction. Many candidates had incorrect angles when trying to resolve.

Answers: **(i)** 2.12 N; **(ii)** 0.313.

Question 3

(i) Most candidates scored 2 marks for this part of the question.

(ii) Usually an integral in *v* and *t* was set up and an attempt to solve it was seen. On a number of occasions when integrating $1/(10 - 4v)$, the result was incorrect. Too often $-\ln(10 - 4v)$ or $-4\ln(10 - 4v)$ was seen. On occasions, no constant of integration was seen. Good candidates scored well on this part of the question.

Answer: **(ii)** $v = 2.27$.

Question 4

The stronger candidates scored well on this question. Weaker candidates found the question rather difficult and scored few marks. Quite often when resolving vertically, only 2 forces instead of 3 appeared in the equation. Again only 2 forces instead of 3 appeared in the equation when using Newton's second law horizontally.

Answer: 6(.00) rads^{-1}.

Question 5

(i)   Most candidates found $v = 8$, but some were not able to explain that this was in the downward direction.

(ii)   Too many candidates used the equation $8 = 30\sin 60 - gt$ instead of $-8 = 30\sin 60 - gt$. The first equation gave the time when the particle was moving upwards with a velocity of 8 ms^{-1}. Having found the vertical distance, many candidates thought that this was the distance *OP*. *OP* could be found by using $OP = \sqrt{(x^2 + y^2)}$.

Answers: **(i)** $v = -8$; **(ii)** *OP* = 59.4 m.

Question 6

(i)   This part of the question was well done. A number of candidates used the incorrect formula for finding the centre of mass of the semi-circular lamina. Moments were usually taken about *O* resulting in an equation giving $OG = 0$, where *G* is the centre of mass of the whole lamina.

(ii)   This part of the question was not as well done as the first part. Many candidates did not realise that the centre of mass of the semi-circular lamina was at *O* and so their solution did not produce a correct answer.

Answer: **(ii)** 0.117 m.

Question 7

This question proved to be the hardest question on the paper, with very few marks being scored.

(i)   The good candidates did manage to score all 4 marks by using an energy equation with 4 terms. Many candidates often had the wrong number of terms.

(ii)   To do this part of the question it was necessary to realise that the greatest speed occurred at the equilibrium position. This position could be found by using the equation $T_A = T_B + 6$. If e was the extension of *PA* then $1 - e$ was the extension of *PB* and so $45e/1.5 = 45(1 - e)/1.5 + 6$. This gave $e = 0.6$ and so $AP = 1.5 + 0.6 = 2.1$ m. An energy equation was now needed at the equilibrium position in order to give the greatest speed.

(iii)   Very rarely was this part of the question even attempted and when it was, usually only the acceleration at the top was found. It was also necessary to find the acceleration at the bottom and then decide on the magnitude of the greatest acceleration.

Answers: **(i)** 1.22 m; **(ii)** 2.1 m, 6 ms^{-1}; **(iii)** 60 ms^{-2}.

MATHEMATICS

Key messages

- Candidates need to make full use of the supplied list of formulae.
- Candidates need to take care to avoid arithmetic and sign errors.
- Answers should be tidy and legibly presented in the examination.

General comments

The questions that candidates found easier were 1, 2**(i)** and 3**(i)**. The questions that candidates found harder were 2**(ii)**, 5**(ii)**, 6**(ii)** and 7

A formula booklet is provided. Many incorrect formulae were seen; it is recommended that candidates refer to this booklet.

Comments on specific questions

Question 1

The better candidates used the correct formula to find the distance of the centre of mass *OG* from *O* and then used $v = r\omega$ with $r = OG$. Weaker candidates simply used the radius of the semi-circle and $v = r\omega$; no mention of the centre of mass was seen.

Answer: v = 0.382 ms^{-1}.

Question 2

(i)   Most candidates realised that they needed to take moments about *B*. Better candidates scored all 3 marks. Weaker candidates had difficulty in finding the perpendicular distance from *B* to the force.

(ii)   This part of the question could be solved by resolving horizontally and vertically to find the friction force, *F*, and the normal reaction, *R*. It was then necessary to use $F = \mu R$ to find *μ*, the coefficient of friction. Many candidates had incorrect angles when trying to resolve.

Answers: **(i)** 2.12 N; **(ii)** 0.313.

Question 3

(i)   Most candidates scored 2 marks for this part of the question.

(ii)   Usually an integral in *v* and *t* was set up and an attempt to solve it was seen. On a number of occasions when integrating $1/(10 - 4v)$, the result was incorrect. Too often $-\ln(10 – 4v)$ or $-4\ln(10 – 4v)$ was seen. On occasions, no constant of integration was seen. Good candidates scored well on this part of the question.

Answer: **(ii)** v = 2.27.

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Question 4

The stronger candidates scored well on this question. Weaker candidates found the question rather difficult and scored few marks. Quite often when resolving vertically, only 2 forces instead of 3 appeared in the equation. Again only 2 forces instead of 3 appeared in the equation when using Newton's second law horizontally.

Answer: 6(.00) rads^{-1}.

Question 5

(i)   Most candidates found $v = 8$, but some were not able to explain that this was in the downward direction.

(ii)   Too many candidates used the equation $8 = 30\sin60 - gt$ instead of $-8 = 30\sin60 - gt$. The first equation gave the time when the particle was moving upwards with a velocity of 8 ms^{-1}. Having found the vertical distance, many candidates thought that this was the distance *OP*. *OP* could be found by using $OP = \sqrt{(x^2 + y^2)}$.

Answers: **(i)** $v = -8$; **(ii)** *OP* = 59.4 m.

Question 6

(i)   This part of the question was well done. A number of candidates used the incorrect formula for finding the centre of mass of the semi-circular lamina. Moments were usually taken about *O* resulting in an equation giving $OG = 0$, where *G* is the centre of mass of the whole lamina.

(ii)   This part of the question was not as well done as the first part. Many candidates did not realise that the centre of mass of the semi-circular lamina was at *O* and so their solution did not produce a correct answer.

Answer: **(ii)** 0.117 m.

Question 7

This question proved to be the hardest question on the paper, with very few marks being scored.

(i)   The good candidates did manage to score all 4 marks by using an energy equation with 4 terms. Many candidates often had the wrong number of terms.

(ii)   To do this part of the question it was necessary to realise that the greatest speed occurred at the equilibrium position. This position could be found by using the equation $T_A = T_B + 6$. If e was the extension of *PA* then $1 - e$ was the extension of *PB* and so $45e/1.5 = 45(1 - e)/1.5 + 6$. This gave $e = 0.6$ and so $AP = 1.5 + 0.6 = 2.1$ m. An energy equation was now needed at the equilibrium position in order to give the greatest speed.

(iii)   Very rarely was this part of the question even attempted and when it was, usually only the acceleration at the top was found. It was also necessary to find the acceleration at the bottom and then decide on the magnitude of the greatest acceleration.

Answers: **(i)** 1.22 m; **(ii)** 2.1 m, 6 ms^{-1}; **(iii)** 60 ms^{-2}.

MATHEMATICS

Key messages

- Candidates need to make full use of the supplied list of formulae.
- Candidates need to take care to avoid arithmetic and sign errors.
- Answers should be tidy and legibly presented in the examination.

General comments

The questions that the candidates found easier were 1**(i)**, 3**(i)**, 5 and 6**(i)**. The questions that the candidates found harder were 2 and 4**(iii)**

A formula booklet is provided. Many incorrect formulae were seen; it is recommended that candidates refer to this booklet.

Comments on specific questions

Question 1

(i) Most candidates tried to take moments about a point. At times the incorrect formulae were used to find the centres of mass of the arc and lamina. The majority of candidates managed to score full marks.

(ii) Some candidates tried to take moments about *O*, not realising that there would be a force acting at *A*. Another mistake was often made when calculating the perpendicular distance from the force *F* to the point *A*.

Answer: **(ii)** *F* = 5.36.

Question 2

This question proved to be rather difficult for many of the candidates.

(i) Candidates tried to set up an energy equation. Too often an energy term was omitted and sometimes an additional term was seen. A number of candidates simply used $v^2 = u^2 + 2as$ which appears to give the correct answer. This approach failed to score any marks.

(ii) Only a minority of the candidates used $T_B = T_A + 6$ to give a correct result. If the above was not used then no further progress was made.

Answers: **(i)** *v* = 6.32 ms^{-1}; **(ii)** Greatest KE = 36.3 J.

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Question 3

(i)   This part of the question was generally well done.

(ii) $0 = (25 \sin70)t – gt^2/2$ was often used to find the required time instead of $1.2 = (25 \sin70)t – gt^2/2$.

(iii)   If the formula $R = v^2 \sin2\alpha/g$ was used then the correct answer was usually seen. If $(15 \sin\alpha)t – 5t^2 = 0$ and $20 = (15 \cos\alpha)t$ were used then mistakes often occurred.

Answers: **(i)**   5.22 m;   **(ii)**   4.65 s;   **(iii)**   31.4°.

Question 4

(i)   Most candidates found r to be 0.45.

(ii)   Many candidates attempted to take moments about A and found an equation in h. Their equation either had a mistake in it or the candidate could not manipulate it to give the required answer.

(iii) $\tan\alpha = 0.5 = 0.45/OG$ was rarely seen and so very little progress was made. The good candidates, having got $0.5 = 0.45/OG$, often went on to find the value of h.

Answers: **(i)** $r = 0.45$;   **(iii)** $h = 0.545$.

Question 5

This question was quite well done.

(i)(a) Candidates often resolved vertically to find the required tension. Sometimes a sign error occurred in the equation.

(i)(b) Newton's second law was confidently used to give a correct equation which resulted in the accurate value of v.

(ii)   Two equations in T_P and T_Q were usually attempted. Some errors did appear. Many candidates managed to score full marks. A number of candidates used $T_P = 7$ which gave a wrong answer.

Answers: **(i)**   3 N;   **(ii)**   5.10 ms^{-1}; **(iii)**   1.39 N.

Question 6

(i)   This part of the question was quite well done, with candidates setting up two equations using Newton's second law for each particle. These candidates usually went on to score all 4 marks. The candidates who used Newton's second law round the corner were only able to score 2 of the 4 marks.

(ii)   Many candidates were able to set up an integral and were then able to go on to calculate the value of t. In trying to calculate the distance too often errors occurred when integrating the exponential function. On many occasions the constant of integration was assumed to be 0.

Answers: **(i)** $dv/dt = -3v$;   **(ii)** $t = 0.231$, 0.833 m.

MATHEMATICS

Key messages

- Candidates need to be careful to read the question in detail and answer as indicated.
- Candidates need to show working to obtain the full marks available for each question.
- Candidates need to make full use of the supplied list of formulae.

General comments

This paper was well answered by many candidates who had prepared for the exam and covered the syllabus. There were still some candidates who were entered and had done little preparation, but the proportion of such entries was lower than in previous years.

Comments on specific questions

Question 1

Some candidates did not appreciate that 'without replacement' means that each probability changes after a pen is taken out, and so were only able to score a mark for realising that the number of green pens taken could only be 0, 1, 2 or 3. It is vital that candidates read the question carefully to see whether the objects taken are kept out or replaced. Both alternatives are equally possible.

Answers: P(0, 1, 2, 3) = 7/24, 21/40, 7/40, 1/120.

Question 2

This question was a challenge to many candidates. One question of this type occurs in most years. Candidates need to know that coded means and variance are used exclusively with coded data, and raw means and variance are used exclusively with raw data. The formulae for raw data are given in the tables. Many candidates used a mix of raw data and coded data which scored no marks. Alternative methods, of expanding the brackets, were successfully used by some candidates although many were unable to expand $\sum (x-36)^2$ correctly.

Answers: 804, 27011.76 (27000).

Question 3

Part **(i)** of this question was a standard normal distribution question which was well answered by candidates. Part **(ii)** involved using a binomial situation with a sample of 8 objects. The probability of 'success' was only given indirectly in that candidates had to appreciate that, with the given mean of 75 m the probability of having a length longer than 77 m is the same as the probability of having a length shorter than 73 m, which was given as 0.15.

Answers: **(i)** 1.93; **(ii)** 0.895.

Question 4

The stem-and-leaf diagram in part **(i)** was well answered by candidates. Most candidates realised that the 'leaf' part could only consist of 1 digit and thus the 'stem' part had to be 14, 15, 16 and so on. Many candidates however did not specify dollars when giving their key.

The lower quartile was successfully found by most candidates. There were problems with understanding what was required in part **(iii)**, but most candidates earned at least one mark for multiplying three fractions together. Only the best candidates realised that there were 3C2 ways of obtaining the three fractions.

Answers: **(ii)** 15400; **(iii)** 4/33 (0.121).

Question 5

Many candidates scored well on part **(i)** of this question, recognising the binomial situation and finding the correct probabilities. There were a variety of methods for part **(ii)** and most candidates scored part marks, with only the best candidates scoring full marks.

Answers: **(i)** 0.264; **(ii)** $2830.

Question 6

The normal approximation to the binomial was recognised by most candidates. Most candidates scored well in this part of the question. The conditional probability in part **(ii)** was recognised by many candidates, who knew the formula and found an appropriate fraction. Not all the probabilities in the fraction were always correct, but candidates generally showed their working and were awarded part marks appropriately.

Answers: **(i)** 0.531; **(ii)** 0.136.

Question 7

Parts **(a)** and **(b)** of this question were well answered with the majority of candidates answering them correctly, or gaining good method marks. Part **(c)** was a good discriminating question and only the very best candidates scored full marks. However, most candidates managed to recognise the three cases of (1, 1, 7), (1, 3, 5) and (3, 3, 3). Some also recognised that these options can occur in any order and so should be multiplied by 3!/2!, 3! and 1 respectively. Fewer candidates recognised that the number of ways of choosing 1 then 3 then 5 say in that order was 9C1, 8C3, 5C5.

Answers: **(a)** 1260; **(b) (i)** 1680; **(ii)** 360; **(c)** 4920.

MATHEMATICS

Key messages

- It is important for candidates to show sufficient working to make their methods clear.
- It is important to read the question carefully before answering, to keep referring back to the question whilst answering and before moving on to the next question.
- When a question asks for a probability the answer should be a fraction or decimal with a value between 0 and 1.

General comments

This paper was relatively straightforward and all candidates who had covered the syllabus were able to attempt the questions and score reasonable marks on them. There were still many candidates entered who appeared to not have done any work or preparation, and scored 0 marks. Candidates generally worked with 4 or 5 figures and so were able to give answers correct to 3 significant figures without losing an accuracy mark for rounding too early.

Comments on specific questions

Question 1

Many candidates scored full marks for this question. For those who did not score full marks, many were able to gain marks for recognising a conditional probability situation and finding the probability of drinking coffee immediately as being part of a fraction.

Answers: **(i)** 0.1; **(ii)** 1/30.

Question 2

A number of candidates appeared not to read that the variable was given in thousands of dollars so that a mean of 6.4 actually meant 6400 dollars. When standardising, these candidates obtained a z-value of over 1500 which could not give a probability. It could have helped candidates to re-read the question at this stage to find out if they had missed anything. Some candidates then divided by 1000 to make the numbers smaller, but could not reach the correct answer. However, they could be awarded a method mark for subtracting two probabilities. In part **(ii)** a large proportion of candidates did not appreciate that making a loss meant that the profit had to be < 0. They did however appreciate that making a loss on exactly 1 of the next 4 consecutive days involved a binomial situation with P(X = 1) needing to be calculated and a method mark was awarded if candidates used the correct expression with their (wrong) probability found from part **(i).**

Answers: **(i)** 0.104; **(ii)** 0.309.

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Question 3

This was a straightforward question on routine work. However, a large number of candidates were unable to score full marks. Some candidates knew a histogram was frequency divided by something but did not appear to remember whether it was frequency divided by class width (the correct one), or divided by upper class limit, or divided by lower class limit, or divided by class mid-interval. The diagram of the histogram was generally well done with few candidates having gaps between the bars. However, labels were commonly wrong or missing, and not all candidates plotted the heights of the histogram correctly due to poor choices of scale. For part **(iii)** candidates had to estimate the mean time to travel, which involved summing the products of the mid-points of the time interval with the frequencies and dividing the total by 112. Only about half the candidates managed this correctly. Others used upper points, lower points, the time interval itself, the semi time interval, or divided by 6 instead of 112.

Answers: **(i)** 15-20 mins, 25-40 mins; **(iii)** 22.0 minutes.

Question 4

The three pieces of information given (Normal distribution, m = 4s, and p(X > 5) = 0.15) should have been combined to give $\Phi\left(\dfrac{5-4s}{s}\right) = 0.85$ which results in $\dfrac{5-4s}{s} = 1.036$. Many candidates managed to get some of this part **(i)** correct, but many only managed to use some of the information. For part **(ii)**, many candidates recognised that this was the normal approximation to the binomial, but failed to find the correct initial probability of P(X < 5), despite being told that P(X > 5) was 0.15. Many found a value for np and npq even though it was not correct, and gained method marks for the subsequent working, including one for a continuity correction of 159.5.

Answers: **(i)** 0.993, 3.97; **(ii)** 0.981.

Question 5

This question was commonly well answered. Candidates were confident with using permutations and combinations and had no trouble with parts **(a)** or **(b)(i)**. Part **(ii)** rarely gained full marks although many candidates did get credit for realising they had to find the number of ways together and subtracting from the number of ways found in part **(b)(i)**. The number of ways Frances and Mary (out of 8 people) were together in a row of 12 seats is 11P7 which should be doubled because they could swap round. This proved a good discriminating question. The last part **(b)(iii)** only rarely scored full marks because candidates thought that 5 empty spaces could be arranged in 5! different ways and thus wrote 8!×5! instead of 8!×5.

Answers: **(a)** 10920; **(b)(i)** 19958400; **(ii)** 16632000; **(iii)** 201600.

Question 6

Parts **(i)** and **(ii)** were understood by the large majority of candidates, who explained how the given answer was obtained, and filled in the table. Many explanations listed the options, but failed to mention probabilities and so scored only 2 out of 3 marks. A significant minority calculated the outcomes on two dice (2, 3, 4, 5, 6, 7, 8) and then showed the scores possible when combined with the third dice. However, without clear tabulation to show the number of outcomes or the probabilities of each score, their demonstration that P(X = 9) = 10/64 was not sufficient to gain full marks. In part **(ii)** candidates could have realised that the table was symmetrical.

Part **(iii)** required candidates to see whether 2 events were independent. Many confused independence with mutual exclusivity. For those who knew what independence meant, testing for independence involved finding the probability of each event separately, finding the probability of both events happening simultaneously, and checking whether the product of the 2 separate events was equal to the product of both events happening simultaneously. Only the best candidates gained full marks for this part.

Answers: **(i)** 10/64; **(ii)** P(5, 6, 8, 9, 10, 11, 12) = (6/64, 10/64, 12/64, 10/64, 6/64, 3/64, 1/64);
(iii) Not independent.

MATHEMATICS

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Paper 63

Key messages

- It is important for candidates to show sufficient working to make their methods clear.
- It is important to read the question carefully before answering, to keep referring back to the question whilst answering and before moving on to the next question.
- When a question asks for a probability the answer should be a fraction or decimal with a value between 0 and 1.

General comments

Questions 1, **2** and **4** were usually answered well. Many candidates found **Questions 3** and **6** very challenging. Normally the questions were attempted in the order on the paper, although sometimes a second or third attempt at **Question 2** was made after answering the other questions. The use of small sketches to show that part of the normal distribution under consideration in **Questions 1** and **5** was usually beneficial. Several candidates obtained an incorrect answer by not working to more than 3 significant figures. The presentation of answers varied considerably, in both neatness and length.

Comments on specific questions

Question 1

For the majority of candidates this was a good first question, with a high proportion obtaining the correct answer. A frequent mistake was to omit the "0" from the z-value, using 1.36 or 1.37 instead of 1.036 or 1.037 respectively. Z-values should be correct to at least 3 decimal places, rounding to 1.04 resulted in a wrong answer. A z-value of 1.032 featured in a few solutions. Several candidates did not pay attention to their choice of sign, usually resulting in a negative value for the standard deviation. Only a few candidates confused variance with standard deviation.

Answer: 3.57.

Question 2

This question was answered well by many candidates, with full marks gained quite often. The most frequent error was to omit the subtraction of $\{E(X)\}^2$ when writing down an equation for Var(X). Several candidates used the equations for $E(X)$ and Var(X) to find values for p and r, but then did not have a third equation and so either omitted to find a value for q or assumed that it was 0. Sometimes the value of 0.4 was overlooked when substituting into the equation involving $\sum$ probabilities $= 1$. Some candidates were alerted by a value for a probability which was not between 0 and 1 and made a second or third attempt at this question. Failure to work with values to at least 3 significant figures often resulted in the loss of either one or both of the marks for the correct answers. Those candidates working with fractions avoided this problem.

Answers: $p = \dfrac{1}{30}$, $q = \dfrac{1}{6}$, $r = \dfrac{2}{5}$.

Question 3

Many candidates found this question very challenging. A careful consideration of the required groups of countries should have been made, with answers involving 170 being particularly inappropriate for parts **(ii)** and **(iv)**. A few candidates assumed that each of the nine combinations had the same probability.

(i)   This part was usually answered correctly.

(ii)   Although 170 featured in some answers, most candidates realised that the required group was restricted to those 53 + 43 = 96 countries with a low or high GDP. Others realised that from these, the 3 + 35 = 38 countries with a low birth rate, but not a medium GDP were required. Only a minority of candidates had both of these numbers correct. 58, not 38 often featured in the numerator, and sometimes 58 appeared in the denominator.

(iii)   Whilst there were many very good responses regarding the conditions for two events to be exclusive, many answers gave the conditions for two events to be independent. Others included a mixture of several inappropriate conditions. The most frequently given correct condition was for the probability of the intersection to be zero. Less popular was for the probability of the union to be the sum of the individual probabilities. Occasionally the condition involving conditional probability was used.

(iv)   This part was often not attempted. Only a minority of the candidates produced answers which included 42 and 41 in the numerators with 74 and 54 in the denominators. Quite often, when the product of two fractions was given, this contained either the same numerators or the same denominators (or both), indicating that 'without replacement' had not been recognised.

Answers: **(i)** $\dfrac{74}{170}$; **(ii)** $\dfrac{38}{96}$; **(iii)** Exclusive, since P(high GDP and high birth rate) = 0; **(iv)** $\dfrac{1722}{3996}$.

Question 4

(i)   This part was well answered by the majority of candidates. The most frequent error was to give the mean correct to 3 significant figures and then use this approximate value for the mean when calculating the standard deviation. Fewer candidates either omitted the mean or did not square it when using the given formula for the standard deviation. There were a few arithmetic errors when calculating e.g. $\dfrac{31+50}{2}$ for the mid-point. A few used class widths instead of mid-points. Some candidates only calculated the mean.

(ii)   This part was usually answered well, with a very high proportion of responses being histograms rather than bar charts. The most frequent choice of scale was 1 unit per centimetre on the vertical axis with either 2.5 or 5 units per centimetre on the horizontal axis. Using 5 units per centimetre often involved extending the axis by 1 millimetre. A few avoided this by rotating their graph paper through 90°.

Candidates sometimes made drawing the diagram harder by choosing a difficult scale, e.g. 0.75 units per centimetre. Nearly all candidates calculated the frequency densities, only a few working with 59, 67, 19, 9 and 2.75. Some incorrectly used the class width minus 1 or the mid-point. The horizontal lines were generally correct. Most candidates had their vertical lines at 0.5, 5.5, 10.5, 20.5, 30.5 and 50.5. The first class starting at 0.5 did cause a few either scale or class width problems. Sometimes the vertical axis was labelled "frequency" instead of "frequency density" and the horizontal axis either unlabelled or "class boundary" instead of "percentage of meat".

Answers: **(i)** Mean 11.4 and Standard Deviation 9.78 or 9.79; **(ii)** Histogram.

Question 5

There were few fully correct answers to this question. It was not unusual for a candidate to answer one part correctly and either omit or answer the other parts poorly. Part **(iii)** was slightly more common in this respect. Small sketches often helped, especially in part **(iii)**. A small group of candidates used the variance instead of the standard deviation throughout the question. Another, smaller, group used 84 from part **(i)** instead of 87 in parts **(ii)** and/or **(iii)**. *Z*-values following linear interpolation are quite acceptable.

(i)   The majority of responses involved standardising with 84 rather than using 83.5 and 84.5. Several candidates took the upper limit to be 84.4 instead of 84.5. A few calculated $P(X < 84.5) - P(X > 83.5)$.

(ii)   The majority of candidates attempting this part calculated $P(X > 87)$ correctly. Most wrong values followed either an inappropriate continuity correction or giving $P(X < 87)$. Several candidates did not proceed to the binomial probability calculation. A proportion of answers only involved the probability of no observations being greater than 87 rather than none or one.

(iii)   Attempts at this part usually gained most of the marks available. Weaker candidates confused *z*-values and probabilities or used the probability corresponding to a *z*-value of 0.3 (or 0.7). This confusion often continued into the later stages of their answer with $\dfrac{k - 82}{\sqrt{126}}$ being set equal to a probability rather than a *z*-value. Some candidates considered $P(X > 87)$ and subtracted 0.3 from 0.3282. This is a valid alternative method. A few candidates omitted the "0" from otherwise correct *z*-value of 1.908 or 1.909.

Answers: **(i)** 0.0350; **(ii)** 0.471; **(iii)** 103.

Question 6

Candidates should be strongly encouraged to indicate their approach in questions involving permutations and combinations.

(a)   This part was answered correctly by many candidates who summed the number of ways including neither twin and the number of ways involving them both. A valid alternative was to calculate the number of ways which would involve one of the twins and subtract this from the total number of ways. Some candidates confused these two approaches whilst others ignored the difference between the girls and boys, choosing 2 or 4 from 11 or 13. A few answers involved either permutations rather than combinations or a product rather than the sum or difference.

(b) (i)   Candidates found this part very difficult. The majority of correct answers involved considering separately those numbers ending in 2, 6 or 8 and those ending in 4. Several candidates interchanged the number of possible ways for these, associating 6! with the number of ways for a 7-digit number ending with 2, etc. Approaches involving subtracting the number of ways resulting in an odd number from the total number of ways and that where the 4s are initially treated as different were quite common. A further four methods were identified.

(ii)   This part was usually answered well, with 2×120 the most common wrong answer.

(c)   This was the most difficult part question on the paper for most candidates. Attempts often involved trying to calculate the number of different arrangements which would have no two adjacent tiles the same colour. Sometimes these were divided by 8! Quite often the answer given was an integer, not a probability. Several candidates would have obtained the correct answer had they realised that the first tile could be chosen in 3 ways, giving an answer derived from $\dfrac{1}{3} \times \left(\dfrac{2}{3}\right)^7$ instead of $\left(\dfrac{2}{3}\right)^7$.

Answers: **(a)** 165; **(b)(i)** 1800; **(ii)** 120; **(c)** $\dfrac{128}{2187}$.

MATHEMATICS

Key messages

- Candidates need to be careful to read the question in detail and answer as indicated.
- Candidates need to be careful to eliminate arithmetic and numerical errors in their answers.

General comments

On this paper, candidates were on the whole able to demonstrate and apply their knowledge in the situations presented, though two topics in particular (covered in **Questions 3** and **4**) were not well received. There was a complete range of scripts from good responses to poor responses. In general, candidates scored well on **Questions 2** and **5** whilst **Questions 3** and **4,** as mentioned above, proved particularly demanding.

Accuracy was not as great an issue in this paper. On the whole, presentation was good and an adequate amount of working was shown by candidates.

Detailed comments on individual questions follow. Whilst the comments indicate particular errors and misconceptions, it should be noted that there were also some good and complete answers.

Comments on specific questions

Question 1

The method used by most candidates on this question was to find the equation of the straight line from 0 to 2 ($y = 1/2\ x$), then find the median by integration. This was successfully done by many candidates, though some candidates used incorrect limits. The limits should have been 0 to m, and not -1 and m. Other errors included the use of an incorrect equation of the line ($y = 2x$ was often seen).

Answer: m = $\sqrt{2}$.

Question 2

Many candidates made a good attempt at this question. The Null and Alternative Hypotheses were usually defined. It should be remembered that in defining H_0 and H_1 it should be clear that it is the 'population mean' (or μ) that is being tested. It is also important that a clear comparison between *z*-values, or areas, is shown in order to justify the conclusion drawn. Conclusions should, ideally, be non-definite. For example 'There is evidence that Hiergo has increased the heights' is preferable to 'Heirgo has increased the heights'.

Answer: There is evidence that Hiergo has increased the heights.

Question 3

Candidates found this question challenging. Calculating the value of the mean in both of parts **(i)** and **(ii)** was better attempted than the standard deviation. It was important that the question was carefully read and understood, as many candidates did not appreciate that the variance in part **(i)** was calculated by $3^2 \times 35^2$ whereas in part **(ii)** the variance was calculated by 6×105^2. A common error was to incorporate +500 in the calculation of the variance or standard deviation.

Answers: **(i)** 926, 105; **(ii)** 5556, 257.

Question 4

Candidates found this question challenging. It was important that in order to find the critical region, using B(20, 0.25), the cumulative probabilities were found. Here P(X≤1) and P(X≤2) were required in order to show that the critical region was X<2. Merely calculating P(0), P(1), and P(2) was not sufficient to justify this critical region.

Candidates did not always refer back to their critical region in part **(iii)** when considering the conclusions that Lola should draw.

Some credit was given to the candidates who used a Normal Approximation, though an approximation was not suggested in the question.

Answers: **(i)** Critical region 0 or 1 packets contain gift; **(ii)** 0.0243; **(iii)** No evidence to reject claim.

Question 5

Part **(i)** was well attempted, with many candidates showing, convincingly, that $k=1/\ln2$. However, some errors in the manipulation of the logarithmic expression were made by some candidates.

Part **(ii)** was often correctly started, but candidates needed to manipulate a logarithmic expression, which again demonstrated misunderstandings on the part of some candidates. Incorrect limits were also a cause of loss of marks in this part; if using limits 'a' and 5 then the integral should be equated to 0.25.

Answer: **(ii)** 4.36.

Question 6

Part **(i)** was well answered. Candidates realised that the telephone directory will exclude some people (people who choose not to have their names in the book, or people without phones etc.)

Finding the Confidence Interval was reasonably well attempted. The correct *z*-value was usually found. It is important that candidates know the correct formula to use, with proportions (38/200) used rather than the number of people (38). It is also important that the final answer is written as an interval, not as two separate values.

In part **(iii)**, many candidates found a correct equation for the width of the interval and solved this to find $z = 1.802$. The process to then find $x\%$ was not often understood, with candidates suggesting $x = 96.42$ rather than the correct value of 92.84 (93%).

Answers: **(i)** Excludes people without phones; **(ii)** 0.119 to 0.261; **(iii)** 93.

Question 7

This question was often a good source of marks for candidates of all abilities.

In part **(i)**, the correct expression for P(0, 1, 2, or 3) was usually used, but the correct parameter (4.8) was not always used.

Part **(ii)** proved to be slightly more challenging. Finding 'n' involved solving an equation involving powers of 'n'. Some candidates omitted brackets and wrote $1.6n^2$ rather than $(1.6n)^2$ and $1.6n^4$ rather than $(1.6n)^4$ causing subsequent working to be incorrect.

Part **(iii)** was particularly well attempted. Omission of a continuity correction (or use of the wrong one) was the major error. In general the correct method was understood and used.

Answers: **(i)** 0.294; **(ii)** 5; **(iii)** 0.0753.

MATHEMATICS

Paper 9709/72
Paper 72

Key messages

- Candidates need to be careful to read the question in detail and answer as indicated.
- Candidates need to be careful to eliminate arithmetic and numerical errors in their answers.

General comments

On this paper, candidates were on the whole able to demonstrate and apply their knowledge in the situations presented, though two topics in particular (covered in **Questions 3** and **4**) were not well received. There was a complete range of scripts from good responses to poor responses. In general, candidates scored well on **Questions 2** and **5** whilst **Questions 3** and **4,** as mentioned above, proved particularly demanding.

Accuracy was not as great an issue in this paper. On the whole, presentation was good and an adequate amount of working was shown by candidates.

Detailed comments on individual questions follow. Whilst the comments indicate particular errors and misconceptions, it should be noted that there were also some good and complete answers.

Comments on specific questions

Question 1

The method used by most candidates on this question was to find the equation of the straight line from 0 to 2 ($y = 1/2\ x$), then find the median by integration. This was successfully done by many candidates, though some candidates used incorrect limits. The limits should have been 0 to m, and not –1 and m. Other errors included the use of an incorrect equation of the line ($y = 2x$ was often seen).

Answer: m = √2.

Question 2

Many candidates made a good attempt at this question. The Null and Alternative Hypotheses were usually defined. It should be remembered that in defining H_0 and H_1 it should be clear that it is the 'population mean' (or μ) that is being tested. It is also important that a clear comparison between z-values, or areas, is shown in order to justify the conclusion drawn. Conclusions should, ideally, be non-definite. For example 'There is evidence that Hiergo has increased the heights' is preferable to 'Heirgo has increased the heights'.

Answer: There is evidence that Hiergo has increased the heights.

Question 3

Candidates found this question challenging. Calculating the value of the mean in both of parts **(i)** and **(ii)** was better attempted than the standard deviation. It was important that the question was carefully read and understood, as many candidates did not appreciate that the variance in part **(i)** was calculated by 3^2 x 35^2 whereas in part **(ii)** the variance was calculated by 6 x 105^2. A common error was to incorporate +500 in the calculation of the variance or standard deviation.

Answers: **(i)** 926, 105; **(ii)** 5556, 257.

Question 4

Candidates found this question challenging. It was important that in order to find the critical region, using B(20, 0.25), the cumulative probabilities were found. Here P(X≤1) and P(X≤2) were required in order to show that the critical region was X<2. Merely calculating P(0), P(1), and P(2) was not sufficient to justify this critical region.

Candidates did not always refer back to their critical region in part **(iii)** when considering the conclusions that Lola should draw.

Some credit was given to the candidates who used a Normal Approximation, though an approximation was not suggested in the question.

Answers: **(i)** Critical region 0 or 1 packets contain gift; **(ii)** 0.0243; **(iii)** No evidence to reject claim.

Question 5

Part **(i)** was well attempted, with many candidates showing, convincingly, that $k=1/\ln2$. However, some errors in the manipulation of the logarithmic expression were made by some candidates.

Part **(ii)** was often correctly started, but candidates needed to manipulate a logarithmic expression, which again demonstrated misunderstandings on the part of some candidates. Incorrect limits were also a cause of loss of marks in this part; if using limits 'a' and 5 then the integral should be equated to 0.25.

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Part **(iii)** was particularly well attempted. Omission of a continuity correction (or use of the wrong one) was the major error. In general the correct method was understood and used.

Answers: **(i)** 0.294; **(ii)** 5; **(iii)** 0.0753.

MATHEMATICS

> **Paper 9709/73**
> **Paper 73**

Key messages

- Candidates need to be careful to read the question in detail and answer as indicated.
- Candidates need to be careful to eliminate arithmetic and numerical errors in their answers.
- Candidates need to take care to avoid sign errors.

General comments

Many candidates showed a sound understanding of the syllabus and answered the questions in an efficient manner. In many cases work was well presented and clear to follow.

Many candidates scored well on **Questions 2(ii)**, **3(i)**, **6**, **7(i)** and **7(ii)**, whilst some candidates found **Questions 2(i)**, **2(iii)**, **4**, and **7(iii)** more demanding.

Question 1 required knowledge of the difference between the sum of 8 values of X and $8 \times X$.

When applying the significance test based on the discrete Poisson distribution in **Question 7(iii)** it was necessary to use the relevant tail probabilities to perform the test comparison.

Comments on specific questions

Question 1

Many candidates stated the correct normal distribution for the total length of the logs and found the mean and the variance of this distribution. Some candidates stated the standard deviation (0.339) instead of the variance. This was accepted provided that this value was clearly stated as the standard deviation.

Some mistakes occurred due to confusion between the sum of 8 lengths and $8 \times L$. Thus the variance was found from 8×0.12^2 not from $8^2 \times 0.12^2$. Some candidates thought that the distribution of sample means was required. This was incorrect.

Answers: Normal distribution, Mean 28, Variance 0.115.

Question 2

(i) Some candidates gave the correct expressions for the expectation and variance in terms of μ, σ and n. Some candidates made incorrect attempts at the variance, such as $n^2 \times \sigma^2$ or $n \times \sigma^2$ or multiplied by $(n – 1)/n$. Some candidates suggested formula sheet expressions involving Σx and Σx^2.

(ii) Many candidates used the correct variance ($6.1^2/12$) and found the correct probability. Some candidates incorrectly used 6.1 instead of 6.1^2 and some candidates incorrectly inserted a continuity correction factor.

(iii) As the heights of the adult males referred to in part **(ii)** were given to be normally distributed it was not necessary to use the Central Limit Theorem. The question required candidates to state "not necessary" and to give the reason. Some candidates needed to be more precise about which distribution was normal. It was not sufficient to state "it was normal". Some candidates referred to the sample size, but this was not the relevant factor here.

Answers: **(i)** μ, σ^2/n; **(ii)** 0.153; **(iii)** Not necessary population heights given as normally distributed.

Question 3

(i) Many candidates showed a clear understanding of the meaning of a Type 1 error in the context of this question and attempted to find the relevant probability of obtaining fewer than 2 sixes. This required the first two terms of the binomial distribution B(25, 1/6). Some candidates also included the probability of 2 sixes. Some candidates omitted the probability of 0 sixes. Some candidates found "1 – P(0 or 1)" which did not demonstrate the understanding of the Type 1 error. Some candidates converted to a normal approximation or a Poisson approximation. This was not appropriate.

Many candidates realised that the calculated probability converted to the significance level of the test. Some candidates incorrectly changed to 93.7% for the significance level. Some candidates incorrectly gave 10% or 8% (from 2/25).

(ii) Many candidates showed knowledge of the overall form for a confidence interval. Many candidates used the correct z-value of 1.96. Some candidates correctly worked with the probability of 9/100 from the given experimental results. Other candidates incorrectly used 1/6 or 6/9 or did not use a probability value (e.g. used 9 or 16 or 17).

Some candidates used the correct form for the variance (0.09 × 0.91/100). Other candidates omitted the 100, or multiplied by 100 or did not apply the square root correctly to their variance.

Answers: **(i)** 6.29%; **(ii)** (0.0339, 0.146) also acceptable here is (0.034, 0.146).

Question 4

Many candidates realised that it was necessary to create a new variable ($M_1 - 2M_2$) or equivalent.

Some candidates correctly found the mean (–180) and variance (7750) for this variable. Other candidates found incorrect values for the new variance by subtracting variances or by using 2 instead of 2^2.

Some candidates used the given variance as 1550^2 instead of 1550. Most candidates did standardise and did attempt to find a probability. Few candidates realised that the final probability was double this value.

Answer: 0.041 (2 significant figures sufficient here).

Question 5

(i) Most candidates correctly found the unbiased estimates of the population mean and variance.

(ii) Many candidates showed a good understanding of the techniques required for a significance test. Many candidates used the correct variance (2.9045/200) for the sample means distribution and showed a comparison of the test statistic and the critical value. Most candidates gave the conclusion in suitable context. A good number of candidates carried this out correctly. Other candidates made various errors, including omitting or wrongly stating the hypotheses, omitting the 200, comparing with the wrong critical value or omitting the necessary comparison, or making an incorrect conclusion.

Answers: **(i)** 2.3, 2.90; **(ii)** There is evidence that the mean weight loss is more than 2 kg.

Question 6

(i)   Many candidates correctly performed the integration of f(x), applied the limits 0 and a, and obtained the resulting total probability as 1. This result supported the idea that f(x) could be a probability density function. But most candidates omitted to point out that f(x) itself was also greater than or equal to 0 for the defined range – another requirement.

Some candidates did not apply the integration to the general case, but tried only a few selected values of a. This approach only verified the result for those a values and did not prove the result for all a values. A geometrical approach to find the total area as 1 was possible, but rarely seen.

(ii)   Many candidates performed the integration of x.f(x), used the given E(X) = 8, and found a = 12. A few candidates incorrectly used the 8 as a limit together with an area of 0.5 in an attempt at the process for the median.

(iii)   Many candidates found this probability correctly. Some candidates used the wrong limits, e.g. from 6 to 8, or from 7 to 12. Some candidates incorrectly tried to set up a normal distribution by finding the variance for the distribution and then found a probability from the normal distribution.

Answers: **(ii)** a = 12; **(iii)** $\frac{3}{4}$.

Question 7

(i)   A good number of candidates used the given binomial parameters to show why it was appropriate to approximate the binomial by a Poisson. The clearest answers showed the values or calculations for this. Thus n = 80 > 50 and np = 0.8 < 5.

A number of candidates stated general conditions for a Poisson distribution, referring to randomness, independence or constant rates. These were not relevant here.

(ii)   Many candidates changed to the 12 day period parameter 9.6 and found the probability of 3, 4 or 5 absent workers correctly. Some candidates did not change the parameter and other candidates changed incorrectly.

Some candidates tried to find P(> 2) and separately P(< 6) and then tried to combine these probabilities. These were not independent events, so this was invalid.

(iii)   Some candidates performed this significance test efficiently and accurately. These candidates used the Poisson distribution with parameter 8, found P(0,1,2) = 0.0138, compared with 0.02 and formed the correct conclusion.

Other candidates omitted the hypotheses or stated them incorrectly. Some candidates did not change the Poisson parameter. A particular error made by some candidates was to use only the probability of 2 workers being absent, instead of using the "tail probability" P(0,1,2).

The question stated that the Poisson distribution was to be used. Thus no credit could be gained for using a normal approximation, which was invalid anyway.

Answers: **(i)** n > 50, np < 5; **(ii)** 0.0800; **(iii)** There is evidence that the mean number of absent workers has decreased.