CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the October/November 2013 series

9709 MATHEMATICS

9709/33 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2013 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1 Apply at least one logarithm property correctly *M1

Obtain $\frac{(x+4)^2}{x} = x + a$ or equivalent **without logarithm** involved A1

Rearrange to express x in terms of a M1 d*M

Obtain $\frac{16}{a-8}$ or equivalent A1 [4]

2 Carry out complete substitution including the use of $\frac{\mathrm{d}u}{\mathrm{d}x} = 3$ M1

Obtain $\int\left(\frac{1}{3} - \frac{1}{3u}\right)\mathrm{d}u$ A1

Integrate to obtain form $k_1u + k_2\ln u$ or $k_1u + k_2\ln 3u$ where $k_1k_2 \neq 0$ M1

Obtain $\frac{1}{3}(3x+1) - \frac{1}{3}\ln(3x+1)$ or equivalent, condoning absence of modulus signs and + c A1 [4]

3 **(i)** Substitute –2 and equate to zero or divide by $x + 2$ and equate remainder to zero or use –2 in synthetic division M1

Obtain $a = -1$ A1 [2]

(ii) Attempt to find quadratic factor by division reaching $x^2 + kx$, or inspection as far as $(x+2)(x^2 + Bx + c)$ and equations for one or both of B and C, or $(x+2)(Ax^2 + Bx + 7)$ and equations for one or both of A and B. M1

Obtain $x^2 - 3x + 7$ A1

Use discriminant to obtain –19, or equivalent, and **confirm one root** cwo A1 [3]

4 Differentiate y^3 to obtain $3y^2\frac{\mathrm{d}y}{\mathrm{d}x}$ B1

Use correct product rule at least once *M1

Obtain $6\mathrm{e}^{2x}y + 3\mathrm{e}^{2x}\frac{\mathrm{d}y}{\mathrm{d}x} + \mathrm{e}^{x}y^3 + 3\mathrm{e}^{x}y^2\frac{\mathrm{d}y}{\mathrm{d}x}$ as derivative of LHS A1

Equate derivative of LHS to zero, substitute $x = 0$ and $y = 2$ and find value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ M1(d*M)

Obtain $-\frac{4}{3}$ or equivalent as **final answer** A1 [5]

5 **(i)** Use integration by parts to obtain $ax\mathrm{e}^{-\frac{1}{2}x} + \int b\mathrm{e}^{-\frac{1}{2}x}\mathrm{d}x$ M1*

Obtain $-8x\mathrm{e}^{-\frac{1}{2}x} + \int 8\mathrm{e}^{-\frac{1}{2}x}\mathrm{d}x$ or unsimplified equivalent A1

Obtain $-8x\mathrm{e}^{-\frac{1}{2}x} - 16\mathrm{e}^{-\frac{1}{2}x}$ A1

Use limits correctly and equate to 9 M1(d*M)

Obtain given answer $p = 2\ln\left(\frac{8p+16}{7}\right)$ correctly A1 [5]

(ii) Use correct iteration formula correctly at least once M1
Obtain final answer 3.77 A1
Show sufficient iterations to 5sf or better to justify accuracy 3.77 or show sign change in interval (3.765, 3.775) A1 [3]
[$3.5 \rightarrow 3.6766 \rightarrow 3.7398 \rightarrow 3.7619 \rightarrow 3.7696 \rightarrow 3.7723$]

6 (i) Find scalar product of the normals to the planes M1
Using the correct process for the moduli, divide the scalar product by the product of the moduli and find $\cos^{-1}$ of the result. M1
Obtain 67.8° (or 1.18 radians) A1 [3]

(ii) EITHER Carry out complete method for finding point on line M1
Obtain one such point, e.g. $(2,-3,0)$ or $\left(\frac{17}{7},0,\frac{6}{7}\right)$ or $(0,-17,-4)$ or … A1…

Either State $3a-b+2c=0$ and $a+b-4c=0$ or equivalent B1
Attempt to solve for one ratio, e.g. $a:b$ M1
Obtain $a:b:c=1:7:2$ or equivalent A1
State a correct final answer, e.g. r = [2, –3, 0] + λ[1, 7, 2] A1√

Or 1 Obtain a second point on the line A1
Subtract position vectors to obtain direction vector M1
Obtain [1, 7, 2] or equivalent A1
State a correct final answer, e.g. r = [2, –3, 0] + λ[1, 7, 2] A1√

Or 2 Use correct method to calculate vector product of two normals M1
Obtain two correct components A1
Obtain [2, 14, 4] or equivalent A1
State a correct final answer, e.g. r = [2, –3, 0] + λ[1, 7, 2] A1√
[√ is dependent on both M marks in all three cases]

OR 3 Express one variable in terms of a second variable M1
Obtain a correct simplified expression, e.g. $x=\frac{1}{2}(4+z)$ A1
Express the first variable in terms of third variable M1
Obtain a correct simplified expression, e.g. $x=\frac{1}{7}(17+y)$ A1
Form a vector equation for the line M1
State a correct final answer, e.g. r = [0, –17, –4] + λ[1, 7, 2] A1

OR 4 Express one variable in terms of a second variable M1
Obtain a correct simplified expression, e.g. $z=2x-4$ A1
Express third variable in terms of the second variable M1
Obtain a correct simplified expression, e.g. $y=7x-17$ A1
Form a vector equation for the line M1
State a correct final answer, e.g. r = [0, –17, –4] + λ[1, 7, 2] A1 [6]

7 **(i)** Use $\sec\theta = \dfrac{1}{\cos\theta}$ and $\text{cosec}\theta = \dfrac{1}{\sin\theta}$ B1

Use $\sin 2\theta = 2\sin\theta\cos\theta$ and to form a horizontal equation in $\sin\theta$ and $\cos\theta$ or fractions with common denominators M1

Obtain given equation $2\sin\theta + 4\cos\theta = 3$ correctly A1 [3]

(ii) State or imply $R = \sqrt{20}$ or 4.47 or equivalent B1

Use correct trigonometry to find α M1

Obtain 63.43 or 63.44 with no errors seen A1 [3]

(iii) Carry out a correct method to find one value in given range M1

Obtain 74.4° (or 338.7°) A1

Carry out a correct method to find second value in given range M1

Obtain 338.7° (or 74.4°) and no others between 0° and 360° A1 [4]

8 **(i)** Either State or imply form $\dfrac{A}{1+x} + \dfrac{B}{(1+x)^2} + \dfrac{C}{2-3x}$ B1

Use any relevant method to find at least one constant M1

Obtain $A = -1$ A1

Obtain $B = 3$ A1

Obtain $C = 4$ A1

Or State or imply form $\dfrac{A}{1+x} + \dfrac{Bx}{(1+x)^2} + \dfrac{C}{2-3x}$ B1

Use any relevant method to find at least one constant M1

Obtain $A = 2$ A1

Obtain $B = -3$ A1

Obtain $C = 4$ A1

Or State or imply form $\dfrac{Dx+E}{(1+x)^2} + \dfrac{F}{2-3x}$ B1

Use any relevant method to find at least one constant M1

Obtain $D = -1$ A1

Obtain $E = 2$ A1

Obtain $F = 4$ A1 [5]

(ii) Either Use correct method to find first two terms of expansion of $(1+x)^{-1}$ or $(1+x)^{-2}$ or $(2-3x)^{-1}$ or $\left(1-\frac{3}{2}x\right)^{-1}$ M1

Obtain correct unsimplified expansion of first partial fraction up to x^2 term A1√
Obtain correct unsimplified expansion of second partial fraction up to x^2 term A1√
Obtain correct unsimplified expansion of third partial fraction up to x^2 term A1√

Obtain final answer $4-2x+\frac{25}{2}x^2$ A1

Or 1 Use correct method to find first two terms of expansion of $(1+x)^{-2}$ or $(2-3x)^{-1}$ or $\left(1-\frac{3}{2}x\right)^{-1}$ M1

Obtain correct unsimplified expansion of first partial fraction up to x^2 term A1√
Obtain correct unsimplified expansion of second partial fraction up to x^2 term A1√
Expand and obtain sufficient terms to obtain three terms M1

Obtain final answer $4-2x+\frac{25}{2}x^2$ A1

Or 2 (expanding original expression)
Use correct method to find first two terms of expansion of $(1+x)^{-2}$ or $(2-3x)^{-1}$ or $\left(1-\frac{3}{2}x\right)^{-1}$ M1

Obtain correct expansion $1-2x+3x^2$ or unsimplified equivalent A1

Obtain correct expansion $\frac{1}{2}\left(1+\frac{3}{2}x+\frac{9}{4}x^2\right)$ or unsimplified equivalent A1

Expand and obtain sufficient terms to obtain three terms M1

Obtain final answer $4-2x+\frac{25}{2}x^2$ A1

Or 3 (McLaurin expansion)
Obtain first derivative $f'(x)=(1+x)^{-2}-6(1+x)^{-3}+12(2-3x)^{-2}$ M1
Obtain $f'(0)=1-6+3$ or equivalent A1
Obtain $f''(0)=-2+18+9$ or equivalent A1
Use correct form for McLaurin expansion M1

Obtain final answer $4-2x+\frac{25}{2}x^2$ A1 [5]

9 (a) Solve using formula, including simplification under square root sign M1*

Obtain $\frac{-2\pm 4\mathrm{i}}{2(2-\mathrm{i})}$ or similarly simplified equivalents A1

Multiply by $\frac{2+\mathrm{i}}{2+\mathrm{i}}$ or equivalent in at least one case M1(d*M)

Obtain final answer $-\frac{4}{5}+\frac{3}{5}\mathrm{i}$ A1

Obtain final answer –i A1 [5]

(b) Show w in first quadrant with modulus and argument relatively correct — B1
Show w^3 in second quadrant with modulus and argument relatively correct — B1
Show w^* in fourth quadrant with modulus and argument relatively correct — B1
Use correct method for area of triangle — M1
Obtain 10 by calculation — A1 [5]

10 Use $2\cos^2 x = 1 + \cos 2x$ or equivalent — B1
Separate variables and integrate at least one side — M1
Obtain $\ln\left(y^3 + 1\right) = \ldots$ or equivalent — A1
Obtain $\ldots = 2x + \sin 2x$ or equivalent — A1
Use $x = 0$, $y = 2$ to find constant of integration (or as limits) in an expression containing at least two terms of the form $a\ln\left(y^3 + 1\right)$, bx or $c\sin 2x$ — M1*
Obtain $\ln\left(y^3 + 1\right) = 2x + \sin 2x + \ln 9$ or equivalent e.g. implied by correct constant — A1
Identify at least one of $\frac{1}{2}\pi$ and $\frac{3}{2}\pi$ as x-coordinate at stationary point — B1
Use correct process to find y-coordinate for at least one x-coordinate — M1(d*M)
Obtain 5.9 — A1
Obtain 48.1 — A1 [10]