MARK SCHEME for the October/November 2015 series

9709 MATHEMATICS

9709/12

Paper 1, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2015 series for most Cambridge IGCSE[®], Cambridge International A and AS Level components and some Cambridge O Level components.

® IGCSE is the registered trademark of Cambridge International Examinations.



| Page 2 | Mark Scheme | | Paper |
|--------|--|------|-------|
| | Cambridge International AS/A Level – October/November 2015 | 9709 | 12 |

Mark Scheme Notes

Marks are of the following three types:

- M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B Mark for a correct result or statement independent of method marks.
- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- Note: B2 or A2 means that the candidate can earn 2 or 0. B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

| Page 3 | Mark Scheme | | Paper |
|--------|--|------|-------|
| | Cambridge International AS/A Level – October/November 2015 | 9709 | 12 |

The following abbreviations may be used in a mark scheme or used on the scripts:

| AEF | Any Equivalent Form (of answer is equally acceptable) |
|-----|---|
| AG | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid) |
| BOD | Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear) |
| CAO | Correct Answer Only (emphasising that no "follow through" from a previous error is allowed) |
| CWO | Correct Working Only – often written by a 'fortuitous' answer |
| ISW | Ignore Subsequent Working |
| MR | Misread |
| PA | Premature Approximation (resulting in basically correct work that is insufficiently accurate) |
| SOS | See Other Solution (the candidate makes a better attempt at the same question) |
| SR | Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |

Penalties

- MR-1 A penalty of MR-1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through ↓^{*}" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.
- PA-1 This is deducted from A or B marks in the case of premature approximation. The PA-1 penalty is usually discussed at the meeting.

| Ρ | age 4 | | Syllabus Paper | | | |
|---|--------------------|---|----------------|--|--|--|
| | | Cambridge International AS/A Level – Octo | ber/Novem | ber 2015 9709 12 | | |
| | | | | | | |
| 1 | | $f: x \mapsto 3x + 2, g: x \mapsto 4x - 12$ | B1 | | | |
| | | $f^{-1}(x) = \frac{x-2}{3}$ | B1 | | | |
| | | gf(x) = 4(3x+2) - 12 | M1 | Equates, collects terms, +soln | | |
| | | Equate $\rightarrow x = \frac{2}{7}$ | A1 | | | |
| | | Equale $\rightarrow x - \frac{1}{7}$ | [4] | | | |
| 2 | | $(x+2k)^{7}$ | | | | |
| - | | Term in $x^5 = 21 \times 4k^2 = 84k^2$ | B1 | | | |
| | | Term in $x^4 = 35 \times 8k^3 = 280k^3$ | B 1 | | | |
| | | Equate and solve $\rightarrow k = 0.3$ or $\frac{3}{10}$ | M1 A1 | Correct method to obtain k. | | |
| | | 1 10 | [4] | | | |
| 3 | (i) | $\tan 60 = \frac{x}{h} \rightarrow x = h \tan 60$ | B1 | Any correct unsimplified length | | |
| | | | MT1 | Correct method for area | | |
| | | $A = h \times x$ $V = 40\sqrt{(3h^2)}$ | M1 | | | |
| | | $V = 40\sqrt{(3h^2)}$ | A1 [3] | ag | | |
| | | dV as \sqrt{a} | | | | |
| | (ii) | $\frac{\mathrm{d}V}{\mathrm{d}h} = 80\sqrt{(3h)}$ | B1 | | | |
| | | If $h = 5$, $\frac{dh}{dt} = \frac{1}{2\sqrt{3}}$ or 0.289 | M1A1 | B1 M1 (must be \div , not \times). | | |
| | | $\ln n = 5, \frac{1}{dt} = \frac{1}{2\sqrt{3}} \text{or } 0.239$ | [3] | | | |
| | | $\begin{pmatrix} 1 & 1 \end{pmatrix}^2 \begin{pmatrix} 1 & c \end{pmatrix}^2$ | | | | |
| 4 | | $\left(\frac{1}{\sin x} - \frac{1}{\tan x}\right)^2 = \left(\frac{1}{s} - \frac{c}{s}\right)^2$ | M1 | Use of $\tan = \frac{\sin}{\cos}$ | | |
| | | $\frac{(1-c)^2}{s^2} = \frac{(1-c)^2}{1-c^2}$ | M1 | Use of $s^2 = 1 - c^2$ | | |
| | | | | | | |
| | | $=\frac{(1-c)(1-c)}{(1-c)(1+c)} \text{ or } \frac{(1-c)^2}{(1-c)(1+c)}$ | A1 | | | |
| | | | | | | |
| | | $\equiv \frac{1 - \cos x}{1 + \cos x}$ | A1 [4] | ag | | |
| | | $1 + \cos x$ | | | | |
| | <i>(</i>) | $\begin{pmatrix} 1 & 1 \end{pmatrix}^2 2$ | | | | |
| | (ii) | $\left(\frac{1}{\sin x} - \frac{1}{\tan x}\right)^2 = \frac{2}{5}$ | | | | |
| | | $1 - \cos x + 2 = -3$ | | | | |
| | | $\frac{1 - \cos x}{1 + \cos x} = \frac{2}{5} \to \cos x \frac{3}{7}$ | M1 | Making cosx the subject | | |
| | | $\rightarrow x = 1.13 \text{ or } 5.16$ | A1 A1√ | $2\pi - 1^{st}$ answer. | | |
| | | | [3] | | | |
| L | | | | 1 | | |

| Ρ | age 5 | Mark Scheme | | | Syllabus | Paper | |
|---|-------|---|-----------------------------|-------------------------|--|-------|--|
| | | Cambridge International AS/A Level – October/November 2015 | | 9709 | 12 | | |
| 5 | (i) | Length of $OB = \frac{6}{\cos 0.6} = 7.270$ | M1 [1] | | ag Any valid method | | |
| | (ii) | $AB = 6\tan 0.6 \text{ or } 4.1$ Arc length = 7.27 × (½ π – 0.6) = (7.06) Perimeter = 6 + 7.27 + 7.06 + 6tan 0.6 = 24.4 | B1 M1 A1 [3] | Use of $s=$ | Sight of in (ii) Use of $s = r\theta$ with sector angle | | |
| | (iii) | Area of $AOB = \frac{1}{2} \times 6 \times 7.27 \times \sin 0.6$ Area of $OBC = \frac{1}{2} \times 7.27^2 \times (\frac{1}{2}\pi - 0.6)$ \rightarrow area = 12.31 + 25.65 = 38.0 | M1 M1 A1 [3] | Use of $\frac{1}{2}r^2$ | Use of any correct area method Use of $\frac{1}{2}r^2\theta$. | | |
| 6 | (i) | A(-3, 7), B(5, 1) and C(-1, k) AB = 10 $6^2 + (k - 1)^2 = 10^2$ k = -7 and 9 | B1 M1 A1 [3] | - | Use of Pythagoras | | |
| | (ii) | $m \text{ of } AB = -\frac{3}{4} m \text{ perp} = \frac{4}{3}$ M = (1, 4) Eqn $y - 4 = \frac{4}{3}(x - 1)$ Set y to 0, $\rightarrow x = -2$ | B1 M1 B1 M1 A1 [5] | Complete | se of $m_1m_2 =$ method leadi | | |
| 7 | (i) | $\overrightarrow{OA} = \begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix}, \overrightarrow{OB} = \begin{pmatrix} 2 \\ 5 \\ -2 \end{pmatrix}, \overrightarrow{OC} = \begin{pmatrix} 3 \\ p \\ q \end{pmatrix}.$ $\overrightarrow{AB} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \overrightarrow{AC} \begin{pmatrix} 3 \\ p-2 \\ q+3 \end{pmatrix} \overrightarrow{BC} \begin{pmatrix} 1 \\ p-5 \\ q+2 \end{pmatrix}$ $\rightarrow p = 6\frac{1}{2} \text{ and } q = -\frac{1}{2}$ | B1B1 B1 B1 [4] | | 3 relevant vec | tors | |
| | (ii) | 6+3p-6+q+3=0 $\rightarrow q = -3p-3$ | M1 A1 [2] | | $y_2 + y_1y_2 + z_1z_2$ | = 0 | |
| | (iii) | $AB^{2} = 4 + 9 + 1 AC^{2} = 9 + 1 + (q + 3)^{2}$ $\rightarrow (q + 3)^{2} = 4$ $\rightarrow q = -1 \text{ or } -5$ | M1 A1 A1 [3] | For attemp | ot at either | | |

| 8f: $x \rightarrow x^2 + ax + b$,B1 for $(x + 3)^2$. B1 for -17 (i) $x^2 + 6x - 8 = (x + 3)^2 - 17$ B1 B1B1 for $(x + 3)^2$. B1 for -17 or $2x + 6 = 0 \rightarrow x = -3 \rightarrow y = -17$ B1 B1B1 for $x = -3$, B1 $y = -17$ (ii) $(x - k)(x + 2k) = 0$ B1 \sqrt{k} Following through visible mether(ii) $(x - k)(x + 2k) = 0$ M1Realises the link between roots the equation $x = 5$ $y = -2k^2 = -50$ A1Comparing coefficients of x | Page 6 | | | Syllabus Paper | | |
|--|--------|---|-----------|--|--|--|
| (i) $ \begin{array}{c c} x^{2} + 6x - 8 = (x + 3)^{2} - 17 \\ \text{or } 2x + 6 = 0 \rightarrow x = -3 \rightarrow y = -17 \\ \rightarrow \text{ Range } f(x) \ge -17 \end{array} $ B1 B1 B1 $\begin{array}{c c} B1 B1 \\ B1 for (x + 3)^{2} & B1 for -17 \\ or B1 for x = -3, B1 y = -17 \\ B1 \sqrt{n} \end{array} $ Following through visible methers [3] (ii) $ \begin{array}{c c} (x - k)(x + 2k) = 0 \\ \equiv x^{2} + 5x + b = 0 \\ \rightarrow k = 5 \\ \rightarrow b = -2k^{2} = -50 \end{array} $ M1 Realises the link between roots the equation comparing coefficients of x | | Cambridge International AS/A Level – Octo | ber/Novem | ber 2015 9709 12 | | |
| (i) $x^{2} + 6x - 8 = (x + 3)^{2} - 17$ or $2x + 6 = 0 \rightarrow x = -3 \rightarrow y = -17$ \rightarrow Range $f(x) \ge -17$ (ii) $(x - k)(x + 2k) = 0$ $\equiv x^{2} + 5x + b = 0$ $\rightarrow k = 5$ $\rightarrow b = -2k^{2} = -50$ (ii) $x^{2} + 6x - 8 = (x + 3)^{2} - 17$ B1 B1 B1 B1 B | ſ | | | | | |
| $\begin{array}{c c c c c c c c c c c c c c c c c c c $ | 8 | $f: x \to x^2 + ax + b ,$ | | | | |
| $\begin{array}{c c c c c c c c c c c c c c c c c c c $ | | | | $D1 f_{rm}(-1.2)^2 = D1 f_{rm} = 17$ | | |
| or $2x + 6 = 0 \rightarrow x = -3 \rightarrow y = -17$ $B1 \sqrt{k}$ Following through visible mether(ii) $(x - k)(x + 2k) = 0$ $[3]$ Following through visible mether $(ii) = x^2 + 5x + b = 0$ $M1$ Realises the link between roots the equation $comparing coefficients of x$ $(ii) = -2k^2 = -50$ $A1$ Comparing coefficients of x | (1) | 2 · (2 · (2 · 2) 2 · 17 | D4 D4 | . , | | |
| \rightarrow Range $f(x) \ge -17$ $B1\sqrt[h]{}$ Following through visible mether(ii) $(x-k)(x+2k) = 0$ M1Realises the link between roots $\equiv x^2 + 5x + b = 0$ A1comparing coefficients of x $\rightarrow k = 5$ A1comparing coefficients of x | | | BI BI | or B1 for $x = -3$, B1 $y = -17$ | | |
| (ii) $(x-k)(x+2k) = 0$ $\equiv x^2 + 5x + b = 0$ $\rightarrow k = 5$ $\rightarrow b = -2k^2 = -50$ [3] M1 Realises the link between roots the equation comparing coefficients of x | | 2 | R1√ | Following through visible method | | |
| (ii) $(x-k)(x+2k) = 0$ $\equiv x^2 + 5x + b = 0$ $\rightarrow k = 5$ $\rightarrow b = -2k^2 = -50$ M1 Realises the link between roots the equation A1 A1 Comparing coefficients of x | | | | Following through visible method. | | |
| $ = x^{2} + 5x + b = 0 $ $ \rightarrow k = 5 $ $ \rightarrow b = -2k^{2} = -50 $ A1 A1 A1 A1 A1 A1 A1 | | | | | | |
| $ \begin{array}{c} \rightarrow k = 5 \\ \rightarrow b = -2k^2 = -50 \end{array} $ A1 comparing coefficients of x A1 | (ii) | | M1 | | | |
| $\rightarrow b = -2k^2 = -50$ | | | A 1 | - | | |
| | | | | comparing coefficients of x | | |
| | | | | | | |
| | | 2 | | | | |
| | (iii) | | | Replaces "x" by " $x + a$ " in 2 terms | | |
| Uses $b^2 - 4ac \rightarrow 9a^2 - 4(2a^2 + b - a)$ DM1 Any use of discriminant | | | | Any use of discriminant | | |
| $\rightarrow a^2 < 4(b-a) \tag{A1}$ [3] | | $\rightarrow a^2 \leq 4(b-a)$ | | | | |
| | | | [2] | | | |
| 9 $f''(x) = \frac{12}{x^3}$ | 9 | $f''(x) = \frac{12}{x}$ | | | | |
| x^3 | - | x^3 | | | | |
| 6 | | 6 | | | | |
| (i) $f'(x) = -\frac{6}{x^2}$ (+ c) B1 Correct integration | (i) | $f'(x) = -\frac{0}{x^2} (+c)$ | B1 | Correct integration | | |
| | | | | | | |
| = 0 when $x = 2 \rightarrow c = \frac{3}{2}$ M1 A1 Uses $x = 2$, f'($x = 0$) | | $= 0$ when $x = 2 \rightarrow c = \frac{3}{2}$ | M1 A1 | Uses $x = 2$, f'($x = 0$) | | |
| $f(x) = \frac{6}{r} + \frac{3x}{2}$ (+A) B1 $\sqrt[h]{B1}$ For each integral | | $f(x) = \frac{6}{3} + \frac{3x}{3}$ | D1.∱D1.∱ | For each integral | | |
| | | $X = \Sigma$ | | For each integral | | |
| $= 10 \text{ when } x = 2 \rightarrow A = 4 $ A1 | | $= 10 \text{ when } x = 2 \rightarrow A = 4$ | | | | |
| [6] | | | [0] | | | |
| (i) $6 + 3 = +2$ | (::) | 6 + 3 + 2 | M1 | Sate their 2 terms $f'(x)$ to 0 | | |
| (ii) $-\frac{6}{x^2} + \frac{3}{2} = 0 \rightarrow x = \pm 2$ M1 Sets their 2 term f'(x) to 0. | (11) | $-\frac{1}{x^2} + \frac{1}{2} = 0 \implies x = \pm 2$ | IVII | Sets their 2 term $f(x)$ to 0. | | |
| Other point is $(-2, -2)$ A1 | | Other point is $(-2, -2)$ | | | | |
| [2] | | | [2] | | | |
| (iii) At $x = 2$, f''(x) = 1.5 Min B1 | (iii) | At $x = 2$, f''(x) = 1.5 Min | B1 | | | |
| At $x = -2$, f''(x) = -1.5 Max B1 | | | B1 | | | |
| [2] | | | [2] | | | |

| Page 7 | Mark Scheme | | | | Paper | |
|--------|--|-----------------|-------------------------|---|-------|--|
| | Cambridge International AS/A Level – Octo | ber/Noven | ber 2015 9709 12 | | | |
| 10 | $y = \sqrt{(9 - 2x^2)} P(2, 1)$ | | | | | |
| (i) | $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2\sqrt{(9-2x^2)}} \times -4x$ | B1 B1 | Without " Allow even | $\times -4x$ " n if B0 above | 2. | |
| | At P, $x = 2$, $m = -4$ Normal grad = $\frac{1}{4}$ Eqn AP $y - 1 = \frac{1}{4}(x - 2)$ | M1 M1 | | For $m_1m_2 = -1$ calculus needed Normal, not tangent Full justification. | | |
| | $\rightarrow A (-2, 0) \text{ or } B (0, \frac{1}{2})$ Midpoint <i>AP</i> also $(0, \frac{1}{2})$ | A1 A1 [6] | Full justifi | | | |
| (ii) | $\int x^2 dy = \int \left(\frac{9}{2} - \frac{y^2}{2}\right) dy$ $= \frac{9y}{2} - \frac{y^3}{6}$ | M1 | Attempt to | Attempt to integrate x^2 | | |
| | $=\frac{9y}{2}-\frac{y^3}{6}$ | A1 | Correct integration | | | |
| | Upper limit = 3 Uses limits 1 to 3 \rightarrow volume = 4 ² / ₃ π | B1 DM1 A1 | | valuates upper limit ses both limits correctly | | |
| | | [5] | | | | |