Cambridge International Advanced Subsidiary Level

MARK SCHEME for the October/November 2015 series

9709 MATHEMATICS

9709/23

Paper 2, maximum raw mark 50

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

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Mark Scheme Notes

Marks are of the following three types:

- M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B Mark for a correct result or statement independent of method marks.
- When a part of a question has two or more "method" steps, the M marks are generally
 independent unless the scheme specifically says otherwise; and similarly when there are
 several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a
 particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme.
 When two or more steps are run together by the candidate, the earlier marks are implied and
 full credit is given.
- The symbol I implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0. B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

- MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √^k" marks. MR is not applied when the candidate misreads his own figures this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.
- PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Cambridge International AS Level - October/November 20159709231Integrate to obtain $k \ln(2x + 5)$ MIObtain correct $\frac{1}{2} \ln(2x + 5)$ A1Apply limits and use logarithm law for $\ln a - \ln b$ MIUse logarithm power lawMIObtain 125A12(i)EitherState or imply non-modulus equation $(2x + 3)^2 = (x + 8)^2$ or corresponding pair of linear equations Solve 3-term quadratic equation or 2 linear equations Obtain $x = -\frac{11}{4}$ and $x = 5$ OrObtain $x = -\frac{11}{4}$ and $x = 5$ OrObtain $x = -\frac{11}{4}$ similarlyB1Obtain $x = -\frac{11}{4}$ similarlyG1Use logarithms to solve equation of form $2^y = k$ where $k > 0$ Obtain $\frac{dx}{dt} = e^t + (t + 1)e^t$ or equivalentGbtain $\frac{dy}{dt} = t(t + 4)^{-\frac{1}{4}}$ B1Substitute $t = 0$ and divide to obtain gradient of tangentObtain $\frac{3x}{4} + 45 = 0$ or equivalent of required formA1Obtain all quotient $3x^2 + 11x$ Obtain complete quotient $3x^2 + 11x$ Obtain all quotient $3x^2 + 11x + 20$ with no errors seenCalculate discriminant of quadratic factor or equivalentObtain -119 or equivalent of negative or equivalentM1Obtain -119 or equivalent of expression of form $k_1e^{3x} + k_2e^x$ M1Obtain -119 or equivalent of expression of form $k_1e^{3x} + k_2e^x$ M1Obtain -119 or equivalent of expression of form $k_1e^{3x} + k_2e^x$ M1Obtain -119 or equivalent of expression of form $k_1e^{3x} + k_2e^x$ M1 <th>Ρ</th> <th>age 4</th> <th>۱<u> </u></th> <th colspan="3">Mark Scheme Syllabus</th> <th>er</th>	Ρ	age 4	۱ <u> </u>	Mark Scheme Syllabus			er
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Obtain $e^{3a} + 5e^{a} = 106$ or similarly simplified equivalentA1Rearrange and introduce logarithmsM1	5	(i)	•				
Rearrange and introduce logarithms M1				· -		M1	
Confirm given answer $a = \frac{1}{3} \ln(106 - 5e^{-1})$ AI							۲ <i>۳</i> ٦
			Confirm	given answer $a = \frac{1}{3} \ln(100 - 5e^{-1})$		AI	[5]

Page 5		Mark Scheme		Paper	
		Cambridge International AS Level – October/November 2015	9709	23	
		Use the iterative formula correctly at least once Obtain final answer 1.477 Show sufficient iterations to justify accuracy to 3 d.p. or show sign change in i (1.4765, 1.4775)	interval	M1 A1 A1	[3]
6		State or imply $R = 3$ Use appropriate formula to find α Obtain 41.81°		B1 M1 A1	[3]
	(ii)	 (a) Attempt to find one correct value of θ + α Obtain one correct value (30.7 or 245.6) of θ Carry out correct method to find second answer Obtain second correct answer and no others in range 		M1 A1 M1 A1	[4]
		(b) State greatest value is 13, following their value of <i>R</i>State least value is 7, following their value of <i>R</i>		B1 B1	[2]
7	(i)	Use quotient rule or equivalent to find first derivative Obtain $\frac{2\cos 2x(\cos x + 1) + \sin 2x \sin x}{(\cos x + 1)^2}$ or equivalent		M1 A1	
		Use at least one of $\cos 2x = 2\cos^2 x - 1$ and $2x = 2\sin x \cos x$ Express first derivative in terms of $\cos x$ only		B1 M1	
		Obtain $\frac{2\cos^3 x + 4\cos^2 x - 2}{(\cos x + 1)^2}$ or equivalent		A1	
		Factorise numerator or divide numerator by $(\cos x + 1)$ or equivalent		M1	
		Confirm given answer $\frac{2(\cos^2 x + \cos x - 1)}{\cos x + 1}$ correctly		A1	[7]
		Use quadratic formula or equivalent to find value of cosx Obtain x-coordinate 0.905		M1 A1	

Obtain x-coordinate -0.905 and no others in range

A1 [3]