
MATHEMATICS

9709/01

Paper 1

For examination from 2017

MARK SCHEME

Maximum Mark: 75

Specimen

This document consists of **13** printed pages and **1** blank page.

Mark Scheme Notes

Marks are of the following three types:

- M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
 - A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
 - B Mark for a correct result or statement independent of method marks.
- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
 - The symbol \surd implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.
 - Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

- AEF Any Equivalent Form (of answer is equally acceptable)
- AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
- BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
- CAO Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
- CWO Correct Working Only – often written by a ‘fortuitous’ answer
- ISW Ignore Subsequent Working
- MR Misread
- PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)
- SOS See Other Solution (the candidate makes a better attempt at the same question)
- SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

- MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through \checkmark ” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
- PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Partial Marks	Guidance
1	$(a+x)^5 = a^5 + {}^5C_1 a^4 x + {}^5C_2 a^3 x^2 + \dots$ soi	1	M1	Ignore subsequent terms
	$\left(-\frac{2}{a} \times (\text{their } 5a^4) + (\text{their } 10a^3)\right)(x^2)$	1	M1	
	0	1	A1	AG
		3		
2	$f(x) = x^3 - 7x + c$	1	B1	
	$5 = 27 - 21 + c$	1	M1	Sub $x = 3$, $y = 5$. Dep. on c present
	$c = -1 \rightarrow f(x) = x^3 - 7x - 1$	1	A1	
		3		
3	$4x^2 + x^2 = 1/2$ soi	1	B1	
	Solve as quadratic in x^2	1	M1	E.g. $(4x^2 - 1)(2x^2 + 1)$ or $x^2 =$ formula
	$x^2 = 1/4$	1	A1	Ignore other solution
	$x = \pm 1/2$	1	A1	
		4		

Question	Answer	Marks	Partial Marks	Guidance
4(i)	$4 \cos^2 \theta + 15 \sin \theta = 0$	1	M1	Replace $\tan \theta$ by $\frac{\sin \theta}{\cos \theta}$ and multiply by $\sin \theta$ or equivalent
	$4(1 - s^2) + 15s = 0 \rightarrow 4 \sin^2 \theta - 15 \sin \theta - 4 = 0$	2	M1A1	Use $c^2 = 1 - s^2$ and rearrange to AG (www)
		3		
4(ii)	$\sin \theta = -1/4$	1	B1	Ignore other solution
	$\theta = 194.5$ or 345.5	2	B1B1 ✓	Ft from 1st solution, SC B1 both angles in rads (3.39 and 6.03)
		3		
5(i)	$\frac{dy}{dx} = -\frac{8}{x^2} + 2$ cao	2	B1B1	
	$\frac{d^2y}{dx^2} = \frac{16}{x^3}$ cao	1	B1	
		3		

Question	Answer	Marks	Partial Marks	Guidance
5(ii)	$-\frac{8}{x^2} + 2 = 0 \rightarrow 2x^2 - 8 = 0$	1	M1	Set = 0 and rearrange to quadratic form
	$x = \pm 2$	1	A1	
	$y = \pm 8$	1	A1	If A0A0 scored, SCA1 for just (2, 8)
	$\frac{d^2y}{dx^2} > 0$ when $x = 2$ hence MINIMUM	1	B1 ✓	$\left. \begin{array}{l} \text{Ft for "correct" conclusion if} \\ \frac{d^2y}{dx^2} \text{ incorrect or} \\ \text{any valid method inc. a good sketch} \end{array} \right\}$
	$\frac{d^2y}{dx^2} < 0$ when $x = -2$ hence MAXIMUM	1	B1 ✓	
		5		
6(i)	$x^2 - x + 3 = 3x + a \rightarrow x^2 - 4x + (3 - a) = 0$	1	B1	AG
6(ii)	$5 + (3 - a) = 0 \rightarrow a = 8$	1	B1	Sub $x = -1$ into (i)
	$x^2 - 4x - 5 = 0 \rightarrow x = 5$	1	B1	OR B2 for $x = 5$ www
			2	

Question	Answer	Marks	Partial Marks	Guidance
6(iii)	$16 - 4(3 - a) = 0$ (applying $b^2 - 4ac = 0$)	1	M1	OR $dy/dx = 2x - 1 \rightarrow 2x - 1 = 3$
	$a = -1$	1	A1	$x = 2$
	$(x - 2)^2 = 0 \rightarrow x = 2$	1	A1	$y = 2^2 - 2 + 3 \rightarrow y = 5$
	$y = 5$	1	A1	$5 = 6 + a \rightarrow a = -1$
			4	
7(i)	$BC^2 = r^2 + r^2 = 2r^2 \rightarrow BC = r\sqrt{2}$	1	B1	AG

Question	Answer	Marks	Partial Marks	Guidance
7(ii)	Area sector $BCFD = \frac{1}{4}\pi(r\sqrt{2})^2$ soi	1	M1	Expect $\frac{1}{2}\pi r^2$
	Area $\triangle BCAD = \frac{1}{2}(2r)r$	1	M1*	Expect r^2 (could be embedded)
	Area segment $CFDA = \frac{1}{2}\pi r^2 - r^2$.oe	1	A1	
	Area semi-circle $CADE = \frac{1}{2}\pi r^2$			
	Shaded area $\frac{1}{2}\pi r^2 - \left(\frac{1}{2}\pi r^2 - r^2\right)$	1	B1	
	or $\pi r^2 - \left(\frac{1}{2}\pi r^2 + \left(\frac{1}{2}\pi r^2 - r^2\right)\right)$	1	DM1	Depends on the area $\triangle BCD$
	$= r^2$	1	A1	
		6		
8(i)	$x^2 - 4x = 12$	1	M1	$4x - x^2 = 12$ scores M1A0
	$x = -2$ or 6	1	A1	
	3 rd term $= (-2)^2 + 12 = 16$ or $6^2 + 12 = 48$	2	A1A1	SC1 for 16, 48 after $x = 2, -6$
		4		

Question	Answer	Marks	Partial Marks	Guidance
8(ii)	$r^2 = \frac{x^2}{4x} \left(= \frac{x}{4} \right)$ soi	1	M1	
	$\frac{4x}{1-\frac{x}{4}} = 8$	1	M1	Accept use of unsimplified $\frac{x^2}{4x}$ or $\frac{4x}{x^2}$ or $\frac{4}{x}$
	$x = \frac{4}{3}$ or $r = \frac{1}{3}$	1	A1	
	3 rd term = $\frac{16}{27}$ (or 0.593)	1	A1	
		4		
	ALTERNATIVE METHOD			
	$\frac{4x}{1-r} = 8 \rightarrow r = 1 - \frac{1}{2}x$ or $\frac{4x}{1-r} = 8 \rightarrow x = 2(1-r)$	1	(M1)	
$x^2 = 4x \left(1 - \frac{1}{2}x \right)$ $r = \frac{2(1-r)}{4}$	1	(M1)		
$x = \frac{4}{3}$ $r = \frac{1}{3}$	1	(A1)		
9(i)	$-(1)(x-3)^2 + 4$	3	B1B1B1	
9(ii)	Smallest (m) is 3	1	B1 [✓]	Accept $m \geq 3$, $m = 3$. Not $x \geq 3$. Ft <i>their b</i>

Question	Answer	Marks	Partial Marks	Guidance
9(iii)	$(x - 3)^2 = 4 - y$	1	M1	Or x/y transposed. Ft <i>their a, b, c</i>
	Correct order of operations	1	M1	
	$f^{-1}(x) = 3 + \sqrt{4 - x}$ cao	1	A1	Accept $y =$ if clear
	Domain is $x \leq 0$	1	B1	
		4		
10(i)	$PM = 2\mathbf{i} - 10\mathbf{k} + \frac{1}{2}(6\mathbf{j} + 8\mathbf{k})$ oe	1	M1	Any valid method
	$PM = 2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$	1	A1	
	$\div \sqrt{4 + 9 + 36}$	1	M1	
	Unit vector = $\frac{1}{7}(2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k})$	1	A1	
		4		

Question	Answer	Marks	Partial Marks	Guidance
10(ii)	AT = 6j + 8k, PT = ai + 6j – 2k soi (or TA and TP)	1	B1	Allow 1 vector reversed at this stage.
	$(\cos ATP) = \frac{(6\mathbf{j} + 8\mathbf{k}) \cdot (\mathbf{ai} + 6\mathbf{j} - 2\mathbf{k})}{\sqrt{36 + 64}\sqrt{a^2 + 36 + 4}}$	1	M1	(AM or MT could be used for AT)
	$= \frac{36 - 16}{\sqrt{36 + 64}\sqrt{a^2 + 36 + 4}}$ $\frac{20}{10\sqrt{a^2 + 40}}$	1	A1 ✓	Ft from their AT and PT
	$\frac{2}{\sqrt{a^2 + 40}} = \frac{2}{7}$ oe and attempt to solve	1	M1	
	a = 3	1	A1	Withheld if only 1 vector reversed
			5	
	ALTERNATIVE METHOD Alt (Cosine Rule) Vectors (AT, PT etc.)	1	(B1)	
	$\cos ATP = \frac{a^2 + 36 + 4 + 36 + 64 - (100 + a^2)}{2\sqrt{(a^2 + 40)}\sqrt{100}}$ then as above	2	(M1A1)	

Question	Answer	Marks	Partial Marks	Guidance
11(i)	$\frac{dy}{dx} = \left[\frac{1}{2}(1+4x)^{-1/2} \right] \times [4]$	2	B1B1	
	At $x = 6$, $\frac{dy}{dx} = \frac{2}{5}$	1	B1	
	Gradient of normal at $P = -\frac{1}{2}$	1	B1 [✓]	OR eqn of norm $y - 5 = \text{their } -\frac{5}{2}(x - 6)$ When $y = 0$, $x = 8$ hence result
	Gradient of $PQ = -\frac{5}{2}$ hence PQ is a normal, or $m_1 m_2 = -1$	1	B1	
		5		

Question	Answer	Marks	Partial Marks	Guidance
11(ii)	Vol for curve = $(\pi) \int (1 + 4x)$ and attempt to integrate y^2	1	M1*	
	= $(\pi)[x + 2x^2]$ ignore '+ c'	1	A1	
	= $(\pi)[6 + 72 - 0]$	1	DM1	Apply limits $0 \rightarrow 6$ (allow reversed if corrected later)
	= $78(\pi)$	1	A1	
	Vol for line = $\frac{1}{3} \times (\pi) \times 5^2 \times 2$	1	M1	OR $(\pi) \left[\frac{\left(-\frac{5}{2}x + 20\right)^3}{3 \times -\frac{5}{2}} \right]_6^8$
	= $\frac{50}{3}(\pi)$	1	A1	
	Total Vol = $78\pi + 50\pi/3 = 94\frac{2}{3}\pi$ (or $284\pi/3$)	1	A1	
		7		

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