

---

**MATHEMATICS**

**9709/03**

Paper 3

**For examination from 2017**

MARK SCHEME

Maximum Mark: 75

---

**Specimen**

---

This document consists of **13** printed pages and **1** blank page.

**Mark Scheme Notes**

Marks are of the following three types:

- M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B Mark for a correct result or statement independent of method marks.
- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep\*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
  - The symbol  $\surd$  implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.
  - Note: B2 or A2 means that the candidate can earn 2 or 0.  
B2/1/0 means that the candidate can earn anything from 0 to 2.
- The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.
- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
  - For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking  $g$  equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

- AEF Any Equivalent Form (of answer is equally acceptable)
- AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
- BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
- CAO Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
- CWO Correct Working Only – often written by a ‘fortuitous’ answer
- ISW Ignore Subsequent Working
- MR Misread
- PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)
- SOS See Other Solution (the candidate makes a better attempt at the same question)
- SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

### **Penalties**

- MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through  $\checkmark$ ” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
- PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Partial Marks	Guidance
<b>1</b>	<i>EITHER:</i> State or imply non-modular inequality $(2x-5)^2 > (3(2x+1))^2$ , or corresponding quadratic equation, or pair of linear equations $(2x-5) = \pm 3(2x+1)$	1	<b>B1</b>	
	Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations for $x$	1	<b>M1</b>	
	Obtain critical values $-2$ and $\frac{1}{4}$	1	<b>A1</b>	
	State final answer $-2 < x < \frac{1}{4}$	1	<b>A1</b>	
	<i>OR:</i> Obtain critical value $x = -2$ from a graphical method, or by inspection, or by solving a linear equation or inequality	1	<b>(B1)</b>	
	Obtain critical value $x = \frac{1}{4}$ similarly	2	<b>(B2)</b>	
	State final answer $-2 < x < \frac{1}{4}$	1	<b>(B1)</b>	[Do not condone $\leq$ for $<$ ]
		<b>4</b>		

Question	Answer	Marks	Partial Marks	Guidance
2	State or imply $1 + u = u^2$	1	<b>B1</b>	
	Solve for $u$	1	<b>M1</b>	
	Obtain root $\frac{1}{2}(1 + \sqrt{5})$ , or decimal in [1.61, 1.62]	1	<b>A1</b>	
	Use correct method for finding $x$ from a positive root	1	<b>M1</b>	
	Obtain $x = 0.438$ and no other answer	1	<b>A1</b>	
		<b>5</b>		
3	Use $\tan(A \pm B)$ and obtain an equation in $\tan \theta$ and $\tan \phi$	1	<b>M1*</b>	
	Substitute throughout for $\tan \theta$ or for $\tan \phi$	1	<b>DM1</b>	
	Obtain $3 \tan^2 \theta - \tan \theta - 4 = 0$ or $3 \tan^2 \phi - 5 \tan \phi - 2 = 0$ , or 3-term equivalent	1	<b>A1</b>	[Treat answers in radians as a misread. Ignore answers outside the given interval.] [SR: Two correct values of $\theta$ (or $\phi$ ) score A1; then A1 for both correct $\theta, \phi$ pairs.]
	Solve a 3-term quadratic and find an angle	1	<b>M1</b>	
	Obtain answer $\theta = 135^\circ, \phi = 63.4^\circ$	1	<b>A1</b>	
	Obtain answer $\theta = 53.1^\circ, \phi = 161.6^\circ$	1	<b>A1</b>	
		<b>6</b>		

Question	Answer	Marks	Partial Marks	Guidance
<b>4(i)</b>	Evaluate, or consider the sign of, $x^3 - x^2 - 6$ for two integer values of $x$ , or equivalent	1	<b>M1</b>	
	Obtain the pair $x = 2$ and $x = 3$ , with no errors seen	1	<b>A1</b>	
		<b>2</b>		
<b>4(ii)</b>	State a suitable equation, e.g. $x = \sqrt{(x + (6/x))}$	1	<b>B1</b>	
	Rearrange this as $x^3 - x^2 - 6 = 0$ , or work <i>vice versa</i>	1	<b>B1</b>	
		<b>2</b>		
<b>4(iii)</b>	Use the iterative formula correctly at least once	1	<b>M1</b>	
	Obtain final answer 2.219	1	<b>A1</b>	
	Show sufficient iterates to 5 d.p. to justify 2.219 to 3 d.p., or show there is a sign change in the interval (2.2185, 2.2195)	1	<b>A1</b>	
		<b>3</b>		

Question	Answer	Marks	Partial Marks	Guidance
<b>5(i)</b>	State or imply that the derivative of $e^{-2x}$ is $-2e^{-2x}$	1	<b>B1</b>	
	Use product or quotient rule	1	<b>M1</b>	
	Obtain correct derivative in any form	1	<b>A1</b>	
	Use Pythagoras	1	<b>M1</b>	
	Justify the given form	1	<b>A1</b>	
			<b>5</b>	
<b>5(ii)</b>	Fully justify the given statement		<b>1 B1</b>	
<b>5(iii)</b>	State answer $x = \frac{1}{4}\pi$		<b>1 B1</b>	
<b>6(i)</b>	Substitute $x = -1$ , equate to zero and simplify at least as far as $-8 + a - b - 1 = 0$	1	<b>B1</b>	
	Substitute $x = -\frac{1}{2}$ and equate the result to 1	1	<b>M1</b>	
	Obtain a correct equation in any form, e.g. $-1 + \frac{1}{4}a - \frac{1}{2}b - 1 = 1$	1	<b>A1</b>	
	Solve for $a$ or for $b$	1	<b>M1</b>	
	Obtain $a = 6$ and $b = -3$	1	<b>A1</b>	
			<b>5</b>	

Question	Answer	Marks	Partial Marks	Guidance
<b>6(ii)</b>	Commence division by $(x + 1)$ reaching a partial quotient $8x^2 + kx$	1	<b>M1</b>	[The M1 is earned if inspection reaches an unknown factor $8x^2 + Bx + C$ and an equation in $B$ and/or $C$ , or an unknown factor $Ax^2 + Bx - 1$ and an equation in $A$ and/or $B$ .] [If linear factors are found by the factor theorem, give B1B1 for $(2x - 1)$ and $(4x + 1)$ , and B1 for the complete factorisation.]
	Obtain quadratic factor $8x^2 - 2x - 1$	1	<b>A1</b>	
	Obtain factorisation $(x + 1)(4x + 1)(2x - 1)$	1	<b>A1</b>	
		<b>3</b>		
<b>7(i)</b>	Use correct method to form a vector equation for $AB$	1	<b>M1</b>	
	Obtain a correct equation, e.g. $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + \lambda(2\mathbf{i} - 2\mathbf{j} + \mathbf{k})$ or $\mathbf{r} = 3\mathbf{i} + \mathbf{k} + \mu(2\mathbf{i} - 2\mathbf{j} + \mathbf{k})$	1	<b>A1</b>	
		<b>2</b>		
<b>7(ii)</b>	Using a direction vector for $AB$ and a relevant point, obtain an equation for $m$ in any form	1	<b>M1</b>	
	Obtain answer $2x - 2y + z = 4$ , or equivalent	1	<b>A1</b>	
		<b>2</b>		



Question	Answer	Marks	Partial Marks	Guidance
7(iii)	Express general point of $AB$ in component form, e.g. $(1 + 2\lambda, 2 - 2\lambda, \lambda)$ or $(3 + 2\mu, -2\mu, 1 + \mu)$	1	<b>B1</b> <sup>✓</sup>	
	Substitute in equation of $m$ and solve for $\lambda$ or for $\mu$	1	<b>M1</b>	
	Obtain final answer $\frac{7}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$ for the position vector of $N$ , from $\lambda = \frac{2}{3}$ or $\mu = -\frac{1}{3}$	1	<b>A1</b>	
	Carry out a correct method for finding $CN$	1	<b>M1</b>	
	Obtain the given answer $\sqrt{13}$	1	<b>A1</b>	[The f.t. is on the direction vector for $AB$ .]
			<b>5</b>	

Question	Answer	Marks	Partial Marks	Guidance
<b>8</b>	Separate variables and integrate one side	1	<b>B1</b>	
	Obtain term $\ln(x + 2)$	1	<b>B1</b>	
	Use $\cos 2A$ formula to express $\sin^2 2\theta$ in the form $a + b \cos 4\theta$	1	<b>M1</b>	
	Obtain correct form $(1 - \cos 4\theta)/2$ , or equivalent	1	<b>A1</b>	
	Integrate and obtain term $\frac{1}{2}\theta - \frac{1}{8}\sin 4\theta$ , or equivalent	1	<b>A1</b> <sup>✓</sup>	
	Evaluate a constant, or use $\theta = 0, x = 0$ as limits in a solution containing terms $c \ln(x + 2), d \sin(4\theta), e\theta$	1	<b>M1</b>	
	Obtain correct solution in any form, e.g. $\ln(x + 2) = \frac{1}{2}\theta - \frac{1}{8}\sin 4\theta + \ln 2$	1	<b>A1</b>	
	Use correct method for solving an equation of the form $\ln(x + 2) = f$	1	<b>M1</b>	
	Obtain answer $x = 0.962$	1	<b>A1</b>	
			<b>9</b>	

Question	Answer	Marks	Partial Marks	Guidance
<b>9(i)</b>	Show $u$ in a relatively correct position	1	<b>B1</b>	
	Show $u^*$ in a relatively correct position	1	<b>B1</b>	
	Show $u^* - u$ in a relatively correct position	1	<b>B1</b>	
	State or imply that $OABC$ is a parallelogram	1	<b>B1</b>	
		<b>4</b>		
<b>9(ii)</b>	<i>EITHER</i> : Substitute for $u$ and multiply numerator and denominator by $3 + i$ , or equivalent	1	<b>M1</b>	
	Simplify the numerator to $8 + 6i$ or the denominator to 10	1	<b>A1</b>	
	Obtain final answer $\frac{4}{5} + \frac{3}{5}i$ , or equivalent	1	<b>A1</b>	
	<i>OR</i> : Substitute for $u$ , obtain two equations in $x$ and $y$ and solve for $x$ or for $y$	1	<b>(M1)</b>	
	Obtain $x = \frac{4}{5}$ or $y = \frac{3}{5}$ , or equivalent	1	<b>(A1)</b>	
	Obtain final answer $\frac{4}{5} + \frac{3}{5}i$ , or equivalent	1	<b>(A1)</b>	
		<b>3</b>		

Question	Answer	Marks	Partial Marks	Guidance
<b>9(iii)</b>	State or imply $\arg(u^*/u) = \tan^{-1}(\frac{3}{4})$	1	<b>B1</b>	
	Substitute exact arguments in $\arg(u^*/u) = \arg u^* - \arg u$	1	<b>M1</b>	
	Fully justify the given statement using exact values	1	<b>A1</b>	
		<b>3</b>		
<b>10(i)</b>	Use the quotient rule	1	<b>M1</b>	
	Obtain correct derivative in any form	1	<b>A1</b>	
	Equate derivative to zero and solve for $x$	1	<b>M1</b>	
	Obtain answer $x = \sqrt[3]{2}$ , or exact equivalent	1	<b>A1</b>	
		<b>4</b>		

Question	Answer	Marks	Partial Marks	Guidance
<b>10(ii)</b>	State or imply indefinite integral is of the form $k \ln(1 + x^3)$		<b>M1</b>	
	State indefinite integral $\frac{1}{3} \ln(1 + x^3)$		<b>A1</b>	
	Substitute limits correctly in an integral of the form $k \ln(1 + x^3)$		<b>M1</b>	
	State or imply that the area of $R$ is equal to $\frac{1}{3} \ln(1 + p^3) - \frac{1}{3} \ln 2$ , or equivalent		<b>A1</b>	
	Use a correct method for finding $p$ from an equation of the form $\ln(1 + p^3) = a$ or $\ln((1 + p^3)/2) = b$		<b>M1</b>	
	Obtain answer $p = 3.40$		<b>A1</b>	
		<b>6</b>		

**BLANK PAGE**