CANDIDATE NAME


## CENTRE NUMBER



## MATHEMATICS

0580/42
Paper 4 (Extended)
May/June 2010
2 hours 30 minutes
Candidates answer on the Question Paper.
Additional Materials: Electronic calculator Geometrical instruments
Mathematical tables (optional) Tracing paper (optional)

## READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
DO NOT WRITE IN ANY BARCODES.
Answer all questions.
If working is needed for any question it must be clearly shown below that question.
Electronic calculators should be used.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For $\pi$ use either your calculator value or 3.142 .
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 130 .

1 Alberto and Maria share $\$ 240$ in the ratio $3: 5$.
(a) Show that Alberto receives $\$ 90$ and Maria receives $\$ 150$.

Answer(a)
(b) (i) Alberto invests his $\$ 90$ for 2 years at $r \%$ per year simple interest.

At the end of 2 years the amount of money he has is $\$ 99$.
Calculate the value of $r$.

$$
\text { Answer(b)(i) } r=
$$

(ii) The $\$ 99$ is $60 \%$ of the cost of a holiday.

Calculate the cost of the holiday.

Answer(b)(ii) \$
(c) Maria invests her $\$ 150$ for 2 years at $4 \%$ per year compound interest.

Calculate the exact amount Maria has at the end of 2 years.

## Answer(c) \$

(d) Maria continues to invest her money at $4 \%$ per year compound interest.

After 20 years she has $\$ 328.67$.
(i) Calculate exactly how much more this is than $\$ 150$ invested for 20 years at $4 \%$ per year simple interest.
Answer(d)(i) \$
(ii) Calculate $\$ 328.67$ as a percentage of $\$ 150$.

2 (a) $\mathbf{p}=\binom{3}{2}$ and $\mathbf{q}=\binom{6}{3}$.
(i) Find, as a single column vector, $\mathbf{p}+2 \mathbf{q}$.
Answer(a)(i)
(ii) Calculate the value of $|\mathbf{p}+2 \mathbf{q}|$.
(b)


In the diagram, $C M=M V$ and $O L=2 L V$.
$O$ is the origin. $\overrightarrow{O C}=\mathbf{c}$ and $\overrightarrow{O V}=\mathbf{v}$.
Find, in terms of $\mathbf{c}$ and $\mathbf{v}$, in their simplest forms
(i) $\overrightarrow{C M}$,
Answer(b)(i)
(ii) the position vector of $M$,
Answer(b)(ii)
(iii) $\overrightarrow{M L}$.


The diagram shows a spinner with six numbered sections.
Some of the sections are shaded.
Each time the spinner is spun it stops on one of the six sections.
It is equally likely that it stops on any one of the sections.
(a) The spinner is spun once.

Find the probability that it stops on
(i) a shaded section,
Answer(a)(i)
(ii) a section numbered 1 ,
Answer(a)(ii)
(iii) a shaded section numbered 1 ,
Answer(a)(iii)
(iv) a shaded section or a section numbered 1 .
Answer(a)(iv)
(b) The spinner is now spun twice.

Find the probability that the total of the two numbers is
(i) 20 ,
Answer(b)(i)
(ii) 11 .

> Answer(b)(ii)
(c) (i) The spinner stops on a shaded section.

Find the probability that this section is numbered 2 .
Answer(c)(i)
(ii) The spinner stops on a section numbered 2 .

Find the probability that this section is shaded.

Answer(c)(ii)
(d) The spinner is now spun until it stops on a section numbered 2 .

The probability that this happens on the $n$th spin is $\frac{16}{243}$.
Find the value of $n$.

$$
\operatorname{Answer}(d) n=
$$


(a) On the grid, draw
(i) the translation of triangle $T$ by the vector $\binom{-7}{3}$,
(ii) the rotation of triangle $T$ about $(0,0)$, through $90^{\circ}$ clockwise.
(b) Describe fully the single transformation that maps
(i) triangle $T$ onto triangle $U$,

Answer(b)(i)
(ii) triangle $T$ onto triangle $V$.

Answer(b)(ii)
(c) Find the 2 by 2 matrix which represents the transformation that maps
(i) triangle $T$ onto triangle $U$,

(ii) triangle $T$ onto triangle $V$,

(iii) triangle $V$ onto triangle $T$.



NOT TO SCALE

The diagram shows some straight line distances between Auckland $(A)$, Hamilton $(H)$, Tauranga $(T)$ and Rotorua $(R)$.
$A T=180 \mathrm{~km}, A H=115 \mathrm{~km}$ and $H T=90 \mathrm{~km}$.
(a) Calculate angle $H A T$.

Show that this rounds to $25.0^{\circ}$, correct to 3 significant figures.

Answer(a)
(b) The bearing of $H$ from $A$ is $150^{\circ}$.

Find the bearing of
(i) $T$ from $A$,

> Answer(b)(i)
(ii) $A$ from $T$.
(c) Calculate how far $T$ is east of $A$.
(d) Angle $T H R=30^{\circ}$ and angle $H R T=70^{\circ}$.

Calculate the distance $T R$.

## Answer(d)

km
(e) On a map the distance representing $H T$ is 4.5 cm .

The scale of the map is $1: n$.
Calculate the value of $n$.

6 A spherical ball has a radius of 2.4 cm .
(a) Show that the volume of the ball is $57.9 \mathrm{~cm}^{3}$, correct to 3 significant figures.
[The volume $V$ of a sphere of radius $r$ is $V=\frac{4}{3} \pi r^{3}$.]
Answer(a)
(b)


NOT TO
SCALE

Six spherical balls of radius 2.4 cm fit exactly into a closed box. The box is a cuboid.

Find
(i) the length, width and height of the box,
Answer(b)(i)
$\qquad$ cm, $\qquad$ cm , cm
(ii) the volume of the box,
Answer(b)(ii)

$$
\mathrm{cm}^{3}
$$

(iii) the volume of the box not occupied by the balls,
(iv) the surface area of the box.
(c)


NOT TO SCALE

The six balls can also fit exactly into a closed cylindrical container, as shown in the diagram.
Find
(i) the volume of the cylindrical container,

$$
\text { Answer(c)(i) ................................................. } \mathrm{cm}^{3}
$$

(ii) the volume of the cylindrical container not occupied by the balls,

Answer(c)(ii) $\qquad$ $\mathrm{cm}^{3}$
(iii) the surface area of the cylindrical container.

7200 students were asked how many hours they exercise each week.
The table shows the results.

| Time ( $t$ hours) | $0<t \leqslant 5$ | $5<t \leqslant 10$ | $10<t \leqslant 15$ | $15<t \leqslant 20$ | $20<t \leqslant 25$ | $25<t \leqslant 30$ | $30<t \leqslant 35$ | $35<t \leqslant 40$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of <br> students | 12 | 15 | 23 | 30 | 40 | 35 | 25 | 20 |

(a) Calculate an estimate of the mean.

## Answer(a)

h [4]
(b) Use the information in the table above to complete the cumulative frequency table.

| Time ( $t$ hours $)$ | $t \leqslant 5$ | $t \leqslant 10$ | $t \leqslant 15$ | $t \leqslant 20$ | $t \leqslant 25$ | $t \leqslant 30$ | $t \leqslant 35$ | $t \leqslant 40$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Cumulative frequency | 12 | 27 | 50 | 80 | 120 |  |  | 200 |


(c) On the grid, draw a cumulative frequency diagram to show the information in the table in part (b).
(d) On your cumulative frequency diagram show how to find the lower quartile.
(e) Use your cumulative frequency diagram to find
(i) the median,
Answer(e)(i)
(ii) the inter-quartile range,
Answer(e)(ii)
(iii) the 64th percentile,
Answer(e)(iii)
(iv) the number of students who exercise for more than 17 hours.
Answer(e)(iv)

8 (a) $y$ is 5 less than the square of the sum of $p$ and $q$.
Write down a formula for $y$ in terms of $p$ and $q$.

$$
\begin{equation*}
\text { Answer(a) } y= \tag{2}
\end{equation*}
$$

(b) The cost of a magazine is $\$ x$ and the cost of a newspaper is $\$(x-3)$.

The total cost of 6 magazines and 9 newspapers is $\$ 51$.
Write down and solve an equation in $x$ to find the cost of a magazine.
(c) Bus tickets cost $\$ 3$ for an adult and $\$ 2$ for a child.

There are $a$ adults and $c$ children on a bus.
The total number of people on the bus is 52 .
The total cost of the 52 tickets is $\$ 139$.
Find the number of adults and the number of children on the bus.
$9 \quad$ (a)


The lines $A B$ and $C D E$ are parallel.
$A D$ and $C B$ intersect at $X$.
$A B=9 \mathrm{~cm}, C D=6 \mathrm{~cm}$ and $D X=3 \mathrm{~cm}$.
(i) Complete the following statement.

Triangle $A B X$ is $\qquad$ to triangle $D C X$.
(ii) Calculate the length of $A X$.

$$
\text { Answer(a)(ii) } A X=
$$

(iii) The area of triangle $D C X$ is $6 \mathrm{~cm}^{2}$.

Calculate the area of triangle $A B X$.

Answer(a)(iii)
$\mathrm{cm}^{2}$
(iv) Angle $B A X=x^{\circ}$ and angle $A B X=y^{\circ}$.

Find angle $A X B$ and angle $X D E$ in terms of $x$ and/or $y$.

$$
\begin{equation*}
\text { Answer(a)(iv) Angle } A X B= \tag{2}
\end{equation*}
$$

$\qquad$
Angle $X D E=$
(b)

$P, Q, R$ and $S$ lie on a circle, centre $O$.
Angle $O P S=42^{\circ}$ and angle $P R Q=35^{\circ}$.
Calculate
(i) angle $P O S$,

$$
\begin{equation*}
\text { Answer(b)(i) Angle } P O S= \tag{1}
\end{equation*}
$$

(ii) angle $P R S$,

$$
\text { Answer(b)(ii) Angle } P R S=
$$

(iii) angle $S P Q$,

$$
\text { Answer(b)(iii) Angle } S P Q=
$$

$\qquad$
(iv) angle $P S Q$.
Answer(b)(iv) Angle PSQ =
(c) The interior angle of a regular polygon is 8 times as large as the exterior angle.

Calculate the number of sides of the polygon.


The diagrams show some polygons and their diagonals.
(a) Complete the table.

| Number of sides | Name of polygon | Total number of diagonals |
| :---: | :---: | :---: |
| 3 | triangle | 0 |
| 4 | quadrilateral | 2 |
| 5 | hexagon | 5 |
| 6 | heptagon | 9 |
| 7 |  | 14 |
| 8 |  |  |

(b) Write down the total number of diagonals in
(i) a decagon (a 10-sided polygon),
Answer(b)(i)
(ii) a 12-sided polygon.
Answer(b)(ii)
(c) A polygon with $n$ sides has a total of $\frac{1}{p} n(n-q)$ diagonals, where $p$ and $q$ are integers.
(i) Find the values of $p$ and $q$.

$$
\begin{aligned}
\operatorname{Answer}(c)(\mathrm{i}) p & =. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . ~ \\
q & = \\
& . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . ~
\end{aligned}
$$

(ii) Find the total number of diagonals in a polygon with 100 sides.

Answer(c)(ii)
[1]
(iii) Find the number of sides of a polygon which has a total of 170 diagonals.

## Answer(c)(iii)

(d) A polygon with $n+1$ sides has 30 more diagonals than a polygon with $n$ sides. Find $n$.

Answer (d) $n=$

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